Multidimensional Dataflow

Ptolemy ILP Conference

March 14, 1997

Praveen Murthy,
Dept. of EECS,
UC Berkeley

Multidimensional Dataflow Extension

Balance equations:

\[ r_{A,1} O_{A,1} = r_{B,1} I_{B,1} \]
\[ r_{A,2} O_{A,2} = r_{B,2} I_{B,2} \]

Solve for the smallest integers \( r_{X,i} \), which then give the number of repetitions of actor \( X \) in dimension \( i \).

Higher dimensionality follows similarly.
**Generalization to Arbitrary Lattices**

- MDSDF handles only rectangularly sampled signals.
- GMDSDF handles signals on arbitrary lattices, *without sacrificing compile-time schedulability.*

**Non-rectangular Sampling**

Definition: The set of all sample points given by \( \hat{i} = V \hat{n} \), \( \hat{n} \in \mathbb{R} \) is called the *lattice* generated by \( V \). It is denoted \( LAT(V) \).
The Fundamental Parallelepiped

The fundamental parallelepiped, denoted by $FPD(V)$, is the set of points given by $Vx$ where $x = [x_1, x_2]^T$ with $0 \leq x_1, x_2 < 1$.

**Definition:** The set of integer points in $FPD(V)$ is denoted as $N(V)$.

**Lemma:** $J(V) = |N(V)| = |\text{det}(V)|$ for an integer matrix $V$.

$L = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$.

Multidimensional Decimators

M-D decimation is given by the relationship:

$y(\hat{n}) = x(\hat{n}), \hat{n} \in LAT(V_I M)$

where $x$ is defined on the points $V_I k$, $V_I$ being the sampling matrix.

$M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$

$V_I = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

Decimation ratio: $|\text{det}(M)|$
**Multidimensional Expanders**

M-D expander:

\[
y(n) = \begin{cases} 
  x(n) & n \in \text{LAT}(V_I) \\
  0 & \text{otherwise} 
\end{cases} \quad \forall n \in \text{LAT}(V_I L^{-1})
\]

where \( x \) is defined at the points \( V_I k \), \( V_I \) being the sampling matrix.

![Rectangular expansion vs Non-rectangular expansion]

- **Rectangular expansion**
- **Non-rectangular expansion**

\[
L = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}
\]

Renumbered samples from the expander output

\[
L = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}
\]

Expansion ratio:

\[ |\det(L)| \]

**Genarlized MDSDF (GMDSDF): Sources**

*Definition:* The containability condition: let \( X \) be a set of integer points in \( \mathbb{R}^m \). We say that \( X \) satisfies the containability condition if there exists an \( m \times m \) matrix \( W \) such that \( N(W) = X \).

*Definition:* We will assume that any source actor in the system produces data in the following manner. A source \( S \) will produce a set of samples \( \zeta \) on each firing such that each sample in \( \zeta \) will lie on the lattice \( \text{LAT}(V_S) \). We assume that the renumbered set \( \tilde{\zeta} \) satisfies the containability condition.

\[
V_S = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}
\]

\[
Q = \begin{bmatrix} 3 & 1.5 \\ 3 & 1.5 \end{bmatrix}
\]

\[
\tilde{\zeta} = \{ V_S^{-1} x : x \in \zeta \}, \tilde{\zeta} = N(Q)
\]
**Concise Problem Statement**

**MDSDF**
- Rectangular lattice
- Regions of data produced = rectangular arrays
- Rectangular arrays specified concisely by tuples of produced/consumed.
- Coordinate axes for dataflow along arcs orthogonal to each other (x and y axes).

**GMDSDF**
- Arbitrary lattice
- Regions of data produced = parallelograms
- Parallelograms specified concisely as the set of integer points inside a support matrix.
- Coordinate axes for dataflow along arcs not necessarily orthogonal.

**Support Matrices**

Want to describe regions where the data is contained.
- In MDSDF, these are ordinary arrays
- In the extension, these are *support matrices*.

![Diagram](image)

**Theorem:**

For the decimator,

\[ V_f = V_e M \]  and  \[ W_f = M^{-1} W_e. \]

For the expander,

\[ V_f = V_e L^{-1} \]  and  \[ W_f = LW_e. \]
Semantics of GMDSDF

$V_{SA} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$L = \begin{bmatrix} 2 & -2 \\ 3 & -2 \end{bmatrix}$, $|L| = 5 \times 2$

$M = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$, $|M| = 2 \times 2$

$W_{SA} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$A$ consumes (1,1) and produces (5,2).

$B$ consumes (2,2) and produces (1,1) on average.

$T$ consumes (1,1)

GMDSDF — Balance Equations

• We don’t know yet exactly how many samples on each firing the decimator will produce.

• Idea: Assume that it produces (1,1) and solve balance equations:

  $3r_{S,1} = 1r_{A,1}$  $5r_{A,1} = 2r_{B,1}$  $r_{B,1} = r_{T,1}$
  $3r_{S,2} = 1r_{A,2}$  $2r_{A,2} = 2r_{B,2}$  $r_{B,2} = r_{T,2}$

• Solution:

  $r_{S,1} = 2$, $r_{S,2} = 1$
  $r_{A,1} = 6$, $r_{A,2} = 3$
  $r_{B,1} = 15$, $r_{B,2} = 3$
  $r_{T,1} = 15$, $r_{T,2} = 3$
Balance equations cont’d

Question: Have we really “balanced”?

No: by counting the number of samples that have been kept in the previous slide.

More systematically:

\[ W_{SA} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} r_{S,1} & 0 \\ 0 & r_{S,2} \end{bmatrix} = \begin{bmatrix} 3r_{S,1} & 0 \\ 0 & 3r_{S,2} \end{bmatrix} \]

\[ W_{AB} = LW_{SA} = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3r_{S,1} & 0 \\ 0 & 3r_{S,2} \end{bmatrix} = \begin{bmatrix} 6r_{S,1} & -6r_{S,2} \\ 9r_{S,1} & 6r_{S,2} \end{bmatrix} \]

\[ W_{BT} = M^{-1}W_{AB} = \frac{1}{4} \begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 6r_{S,1} & -6r_{S,2} \\ 9r_{S,1} & 6r_{S,2} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 21r_{S,1} & -6r_{S,2} \\ 3r_{S,1} & -18r_{S,2} \end{bmatrix} \]
Balance equations cont’d

Want to know if

\[ |N(W_{BT})| = \frac{|N(W_{AB})|}{|M|} \]

We have

\[ |N(W_{AB})| = |det(W_{AB})| = 90r_{S,1}r_{S,2} \]

The right hand side becomes

\[ \frac{90r_{S,1}r_{S,2}}{4} = \frac{45r_{S,1}r_{S,2}}{2} \]

Therefore, we need

\[ r_{S,1}r_{S,2} = 2k \quad k = 0, 1, 2, … \]

The balance equations gave us \( r_{S,1} = 2, r_{S,2} = 1 \).

With these values, we get

\[ W_{BT} = \begin{bmatrix} 21/2 & -3/2 \\ 3/2 & -9/2 \end{bmatrix} \]

This matrix has 47 points inside its FPD (determined by drawing it out).

\[ \Rightarrow \text{Balance equation solution is not quite right.} \]

Augmented Balance Equations

To get the correct balance, take into account the constraint given by

\[ |N(W_{BT})| = \frac{|N(W_{AB})|}{|M|} \]

**Sufficiency**: force \( W_{BT} \) to be an integer matrix.

\[ \Rightarrow r_{S,1} = 4k, k = 1, 2, … \]

\[ \Rightarrow r_{S,2} = 2k, k = 1, 2, … \]

Therefore,

\[ r_{S,1} = 4, r_{S,2} = 2. \]

- So decimator produces (1,1) on average but has cyclostatic behavior.

Production sequence: 2,1,1,2,1,0,1,1,0,1,2,1,1,2,1,....

**Theorem**: Always possible to solve these **augmented** balance equations.
**Effect of Different Factorizations**

Suppose we let $|\text{det}(M)| = 1 \times 4$ instead. Balance equations give:

\[
\begin{align*}
    r_{S,1} &= 1, r_{S,2} = 2 \\
    r_{A,1} &= 3, r_{A,2} = 6 \\
    r_{B,1} &= 15, r_{B,2} = 3 \\
    r_{T,1} &= 15, r_{T,2} = 3
\end{align*}
\]

Also,

\[
W_{BT} = \begin{bmatrix} \frac{21}{4} & -3 \\ \frac{3}{4} & -9 \end{bmatrix}
\]

It turns out that

\[
|N(W_{BT})| = 45
\]

as required.

=> Lower number of overall repetitions with this factoring choice.

---

**Dataspace on Arc AB**

[Diagram showing a 1x4 rectangle consumed by a decimator, with original samples produced by source, samples retained by decimator, samples added by expander, and discarded by decimator highlighted.]
**Summary of Extended Model**

- Each arc has associated with it a lattice-generating matrix, and a support matrix.
- The source actor for an arc establishes the ordering of the data on that arc.
- Expander: consumes $(1,1)$ and produces $FPD(L)$, ordered as an $(L_1, L_2)$ rectangle where $L_1 L_2 = |\text{det}(L)|$.
- Decimator: consumes an $(M_1, M_2)$ rectangle, where $M_1 M_2 = |\text{det}(M)|$ and produces $(1,1)$ on average.
- Write down balance equations.
- Additional equations for support matrices on decimator outputs.
- The above two sets are simultaneously solved to determine the smallest non-zero number of times each node is to be invoked in a periodic schedule.
- Actors are then scheduled as in SDF or MDSDF.

**Aspect Ratio Conversion**

Format conversion of 2:1 interlaced video from 4/3 aspect ratio to 16/9 aspect ratio.

Format conversion of 2:1 interlaced video from 4/3 aspect ratio to 16/9 aspect ratio.

Format conversion of 2:1 interlaced video from 4/3 aspect ratio to 16/9 aspect ratio.
**Concrete Data Structures**

- “Cells” can have specific “Values”
- Enabling relationship says when a cell can be filled.
- “Cell” dependency partial order can be arbitrary
- Formalizes most forms of “real-world” data structures: lists, trees, arrays etc.
- Kahn-Plotkin sequential functions on CDS provide an elegant model of computation with many formal properties, like full abstraction.
- CDS approach has been mostly semantic; need to sort out operational issues (like scheduling).

**Array-OL**

- Array-oriented language developed at Thomson
- Graphical syntax for specifying “array access patterns”
  - In many multidimensional programs, manipulating data aligned in various dimensions is a challenge. For example: Transpose.
  - Patterns specified by “paving” and “tiling” relationships.
- Combine with MDSDF...