Problem 9.18

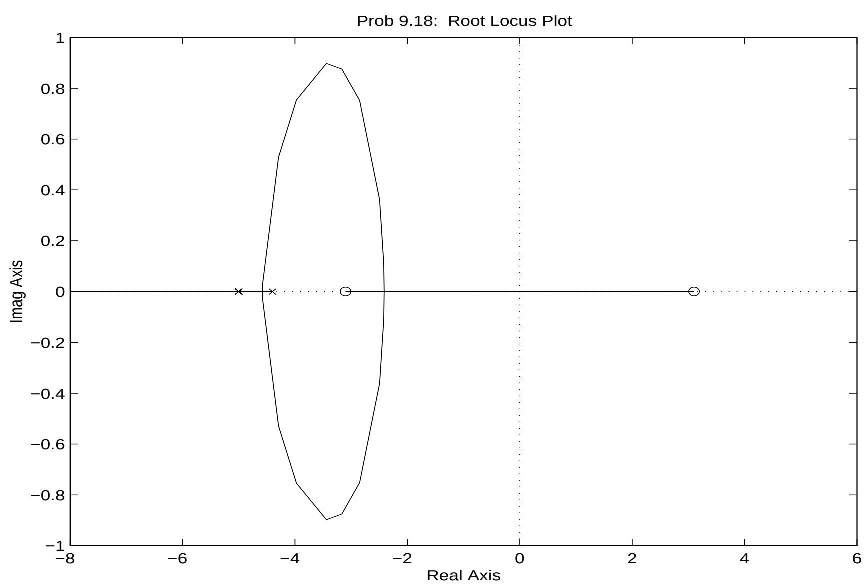
When we are using the proportional controller, we have $L(s) = \frac{K_p(s+3.1)(s-3.1)}{s^2(s+4.4)(s-4.4)}$.

The characteristic equation is $s^4 + (K_p - 4.4^2)s^2 - K_p 3.1^2 = 0$.

$$\begin{array}{lll} 1 & K_p - 4.4^2 & -K_p 3.1^2 \\ 0 & 0 & \\ 0 & 0 & \\ 0 & & \end{array}$$

Using the Routh test, we see that the system is not stable for any K_p.

If we let $H(s) = \frac{Ks^2(s-4.4)}{(s+5)^2}$, $L(s) = \frac{K(s+3.1)(s-3.1)}{(s+4.4)(s+5)^2}$. Below is the root locus plot, it is obviously stable for some K.



Problem 9.20

Let $F(s) = 1 + KG(s)H(s)$ and multiply $\frac{1}{K}$ on both side of the equation, we get $R(s) = \frac{F(s)}{K} = \frac{1}{K} + G(s)H(s)$.

Now as we traverse the contour C clockwise, instead of investigating how many time $KG(s)H(s)$ encircles the -1, we want to see how many times $G(s)H(s)$ encircles $-\frac{1}{K}$.

Thus, (# of times $G(s)H(s)$ encircles $-\frac{1}{K}$ clockwise)
= (# of times $KG(s)H(s)$ encircles -1 clockwise)
= (# of times $\frac{1}{K} + G(s)H(s)$ encircles 0 clockwise)
= (# of zeros of $\frac{1}{K} + G(s)H(s)$ inside C) - (# of poles of $\frac{1}{K} + G(s)H(s)$ inside C)
= (# of zeros of $\frac{1}{K} + G(s)H(s)$ inside C) - (# of poles of $G(s)H(s)$ inside C)

The above equation is derived from the fact that poles of $\frac{1}{K} + G(s)H(s)$ are poles of $G(s)H(s)$.

Since we take the contour C to be the entire right half of s-plane except for poles at the $j\omega$ axis,
(# of zeros of $\frac{1}{K} + G(s)H(s)$ inside C)
= (# of zeros of $\frac{1}{K} + G(s)H(s)$ in the right half of s-plane)
= (# of poles of the closed loop transfer function in the right half of the s-plane)

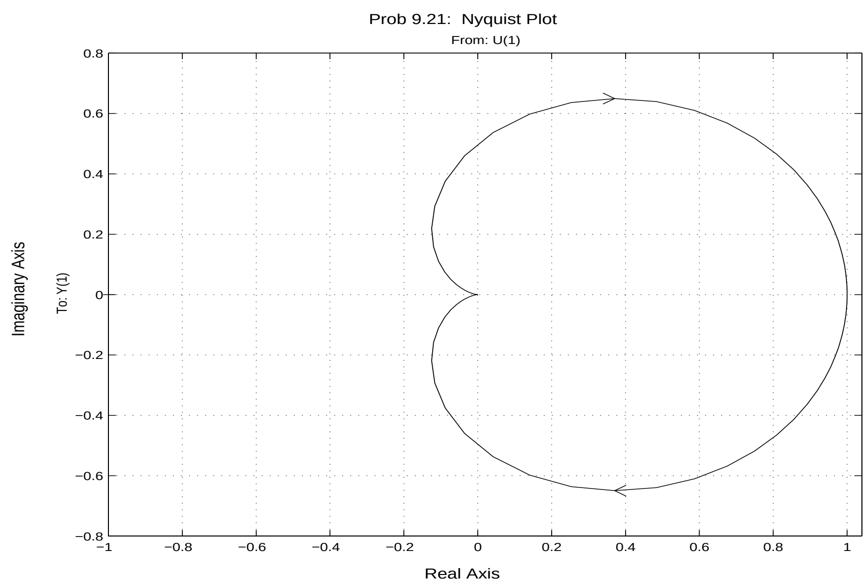
and, (# of poles of $G(s)H(s)$ inside C)
= (# of poles of $G(s)H(s)$ inside the right half of the s-plane)
= (# of poles of $\frac{1}{K} + G(s)H(s)$ inside the right half of the s-plane)

Thus, (# of times $G(s)H(s)$ encircles $-\frac{1}{K}$ clockwise)
= (# of poles of the closed loop transfer function in the right half of the s-plane) - (# of poles of $G(s)H(s)$ inside the right half of the s-plane)

If we want a system to be stable, we always want (# of poles of the closed loop transfer function in the right half of the s-plane) $= 0$.

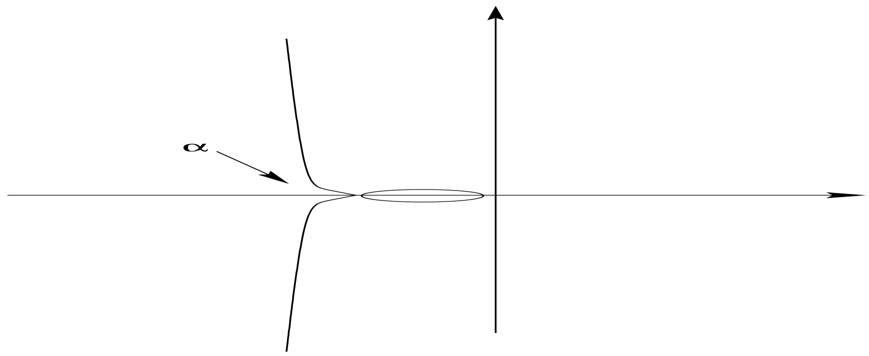
In this problem, we have $L(s) = \frac{K}{(s+1)^2} \Rightarrow G(s)H(s) = \frac{1}{(s+1)^2}$. Obviously, we see that $G(s)H(s)$ has two poles at $s = -1$. Thus, (# of poles of $G(s)H(s)$ inside the right half of the s-plane) $= 0$. From above equation, we get (# of times $G(s)H(s)$ encircles $-\frac{1}{K}$ clockwise) should be zero.

From the figure below, we can see that for $K > 0$, $-\frac{1}{K}$ will never be inside the loop. Therefore, the system is stable for all $K > 0$.



Problem 9.25

(a) From the problem, $L(s) = \frac{K}{s(s+1)(s+2)}$ and $G(s)H(s) = \frac{1}{s(s+1)(s+2)}$. We first plot the nyquist diagram (in matlab, you cannot see the loop unless you zoom in.)



We also know that the (# of poles of $G(s)H(s)$ inside the right half of the s-plane) = 0. So we want (# of times $G(s)H(s)$ encircles $-\frac{1}{K}$ clockwise) = 0 also. This implies that $-\frac{1}{K}$ should not be in the loop. We need to calculate the exact value of point α in the figure. To do that, we see α is a real value, i.e. $Im\{\alpha\} = 0$.

$G(j\omega)H(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)} = \frac{j\omega(-\omega^2-3j\omega+2)}{-\omega^2(\omega^2+1)(\omega^2+4)}$

Then, $Im\{G(j\omega)H(j\omega)\} = \frac{2\omega-\omega^3}{-\omega^2(\omega^2+1)(\omega^2+4)} = 0 \Rightarrow 2\omega = \omega^3 \Rightarrow \omega = \sqrt{2}, -\sqrt{2}$.

$\alpha = Re\{G(j\omega)H(j\omega)\}|_{\omega=\sqrt{2}} = -\frac{6}{36} = -\frac{1}{6}$.

Obviously, for $0 < K < 6$, $-\frac{1}{K}$ will not be in the loop; thus, the system is stable.

Using Routh Criteria, the closed loop characteristic equation is $s^3 + 3s^2 + 2s + K = 0$.

1	2
3	K
$\frac{6-K}{3}$	0
K	0

For stability, we need $\frac{6-K}{3} > 0$ and $K > 0$. That is $0 < K < 6$.

(b) For $K = 2$,$L(s) = \frac{2}{s(s+1)(s+2)}$. To find the gain margin, we need to find K such that the following equation holds true.

$\frac{2K}{j\omega_1(j\omega_1+1)(j\omega_1+2)} = -1$ where ω_1 satisfies the equation $-[\frac{\pi}{2} + tan^{-1}(\omega_1) + tan^{-1}(\frac{\omega_1}{2})] = -\pi$. We get that $\omega_1 = 1.414 \Rightarrow$ gain margin $= \frac{1}{|L(j\omega_1)|} = 3 = 9.5dB$.

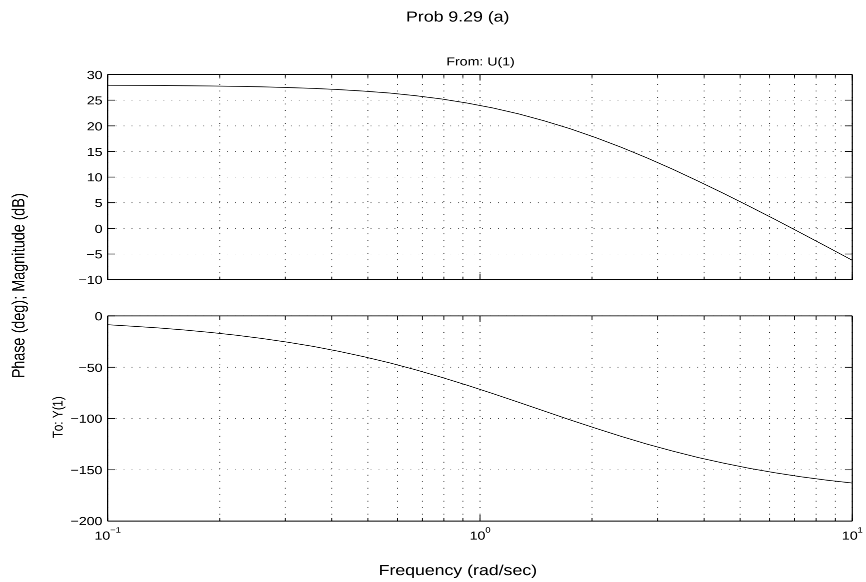
To find the phase margin, we want $e^{-j\phi}L(j\omega_2) = -1$ where ω_2 satisfies the equation $|L(j\omega_2)| = 1$. We get $\omega_2 = 0.749$. So $tan^{-1}(0.749) + tan^{-1}(\frac{0.749}{2}) + \frac{\pi}{2} = 147.36°$. phase margin $= 180 - 147.36 = 32.6°$.

(c) Here the phase margin $= 20° = 0.349$. That is $-0.349 + angle\{L(j\omega)\} = -\pi \Rightarrow angle\{L(j\omega)\} = -2.792rad \Rightarrow \frac{\pi}{2} + tan^{-1}(\omega) + tan^{-1}(\frac{\omega}{2}) = 2.792 \Rightarrow \omega = 1$. Since we want $\frac{K}{|j\omega(j\omega+1)(j\omega+2)|} = 1$, we get $K = |j\omega(j\omega+1)(j\omega+2)| = 3$ in which $\omega = 1$.

Problem 9.29

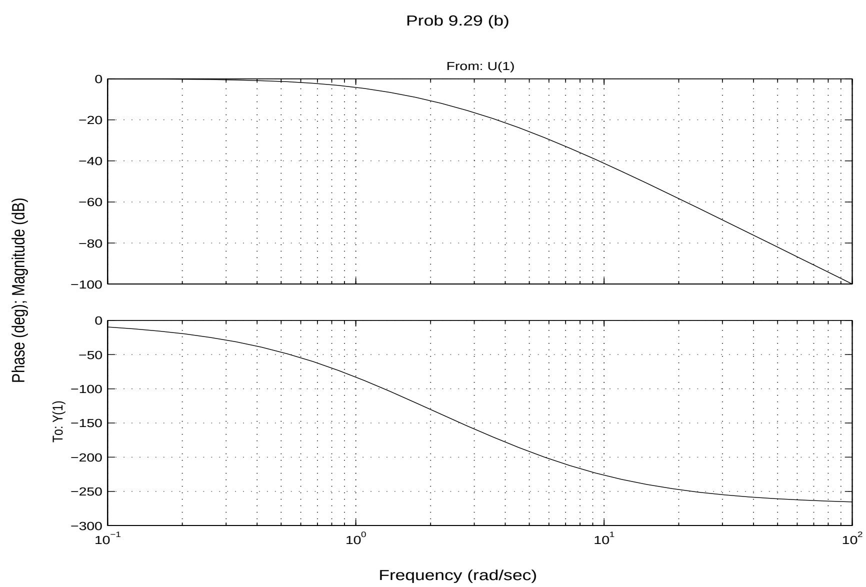
```
% Problem 9.29(a)

sys = tf([50],[1 3 2]);
bode(sys);
title('Prob 9.29 (a)');
```



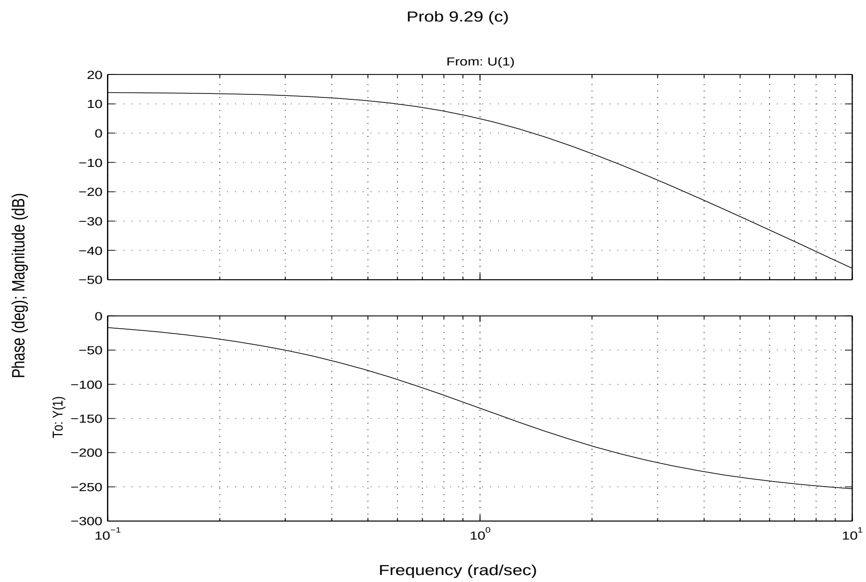

```
% Problem 9.29(b)

sys = tf([10],[1 8 17 10]);
bode(sys);
title('Prob 9.29 (b)');
```




```
% Problem 9.29(c)

sys = tf([5],[1 3 3 1]);
bode(sys);
title('Prob 9.29 (c)');
```




```
% Problem 9.29(d)

sys = tf([10 5],[1 8 17 10]);
bode(sys);
title('Prob 9.29 (d)');
```