9.1 Exercises 6 and 7 in Chapter 8 of Lee & Varaiya.

9.2 Two complex numbers \( z_1 \) and \( z_2 \) are described below:

\[
z_1 = 1 + i\sqrt{3} \quad \quad z_2 = \exp \left( i \frac{2\pi}{3} \right).
\]

(a) Identify each of the following complex numbers as points (or vectors) on the complex plane, using a well-labeled sketch: \( z_1 \), \( z_2 \), \( z_1^* \), \( z_2^* \), \( 1/z_1 \), \( 1/z_2 \), \( 1/z_1^* \), and \( 1/z_2^* \).

(b) Determine each of the sums \( z_1 + 2z_2 \), \( z_1^2 + z_2 \), and \( \frac{1}{2}z_1 + z_2^* \).

(c) Determine each of the magnitudes \(|z_1 z_2|\), \(|z_1 z_2^*|\), \(|1/z_1|\), and \(|1/z_2|\).

(d) Determine each of the following powers of \( z_1 \) and \( z_2 \):

(i) \( z_1^2 \)  
(ii) \( z_1^3 \)  
(iii) \( z_1^6 \)  
(iv) \( z_2^4 \).

(e) Determine \( z_2^{1/4} \). Be mindful of how many fourth roots \( z_2 \) has and identify each of them graphically on a well-labeled sketch of the complex plane.

Express each of your answers in Cartesian form \((a + ib)\), in polar form \((re^{i\theta})\), where \( r > 0 \), as a real number, as an imaginary number, or graphically in a well-labeled complex-plane diagram, whichever form is less cluttered and more appropriate.

9.3 With little algebraic manipulation, determine each of the following sums:

(i) \[ \sum_{n=0}^{N} \cos(n\theta) \]

(ii) \[ \sum_{n=1}^{N} \sin(n\theta) \]

Hint: You may find geometric series useful.

9.4 Consider the following sixth-order equation:

\[
z^6 - 2\sqrt{3}z^4 + 4z^2 = 0.
\]
Determine the six solutions (roots) of the equation, and express each root in both a simple rectangular and a simple polar form. Explain your work succinctly, but clearly and convincingly. Also, plot these solutions on a single, well-labeled diagram of the complex plane.

9.5 For each set defined below, provide a well-labeled diagram identifying all the points on the complex plane that belong to it. \( \mathbb{C} \) refers to the set of complex numbers, \( \mathbb{R} \) refers to the set of real numbers, and \( \mathbb{Z} \) refers to the set of integers.

(a) \( \{ z \in \mathbb{C} \mid |z - i| = |z + i| \} \)
(b) \( \{ z \in \mathbb{C} \mid \text{Im}(z) > \text{Re}(z) \} \)
(c) \( \{ z \in \mathbb{C} \mid 0 < \angle z < \pi/4 \} \)
(d) \( \{ z \in \mathbb{C} \mid 1 < |z - 2i| < 3 \} \)
(e) \( \{ z \in \mathbb{C} \mid z + z^* = 0 \} \)
(f) \( \{ z \in \mathbb{C} \mid z = e^{i(2\pi/3)t}, t \in \mathbb{R} \} \)
(g) \( \{ z \in \mathbb{C} \mid z = e^{i(2\pi/3)n}, n \in \mathbb{Z} \} \)
(h) \( \{ z \in \mathbb{C} \mid \text{Re}(z) > \text{Re}(i') \} \)