SOLUTIONS

EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 3
Department of Electrical Engineering and Computer Sciences 9 November 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take you up to 15 minutes to complete. You will be given at least 15 minutes—up to a maximum of 20 minutes—to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

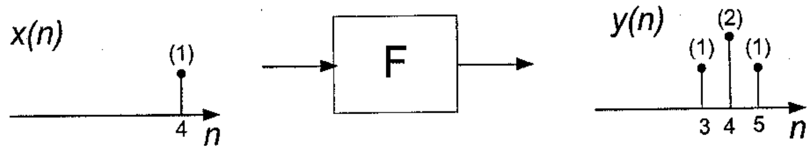
Problem	Points	Your Score
Name	5	
1	10	
2	20	
3	10	
Total	**45**	

Q3.1 (10 Points) Consider a discrete-time system

$$F : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}].$$

It is known that F is linear. It is *not* known whether it is time invariant.

The following input-output signal pair (x, y) is a behavior of the system. The signals x and y are zero outside the regions shown.



[1] correct answer

[4] (a) Could the system F be causal? Explain your reasoning succinctly, but clearly and convincingly.

F is linear $\Rightarrow$ zero input produces zero output. If $z(n) = 0 \;\; \forall n$ is the zero input, then $y_z(n) = 0 \;\; \forall n$ is the output.

[3] correct explanation

$z(n) = x(n) \;\; \forall n \leq 3$, but $y_z(n) \neq y(n)$ @ $n = 3$. Therefore, F cannot be causal.

[1] — [3] (b) Could the system be memoryless? Explain your reasoning succinctly, but clearly and convincingly.

correct answer

We know that all memoryless systems are causal, i.e.,

Memoryless $\Rightarrow$ Causal.

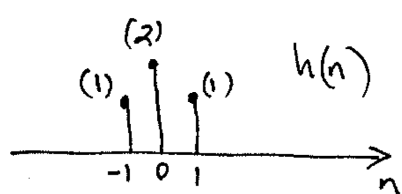
[2] correct explanation

Hence, Not Causal $\Rightarrow$ Not Memoryless. F is not memoryless.

[3] (c) For this part only, suppose F is *also* time invariant. Determine the impulse response f of F by providing a well-labeled plot of the impulse response sample values $f(n)$, or explain why it is not possible to determine the impulse response based on what is known about the system.

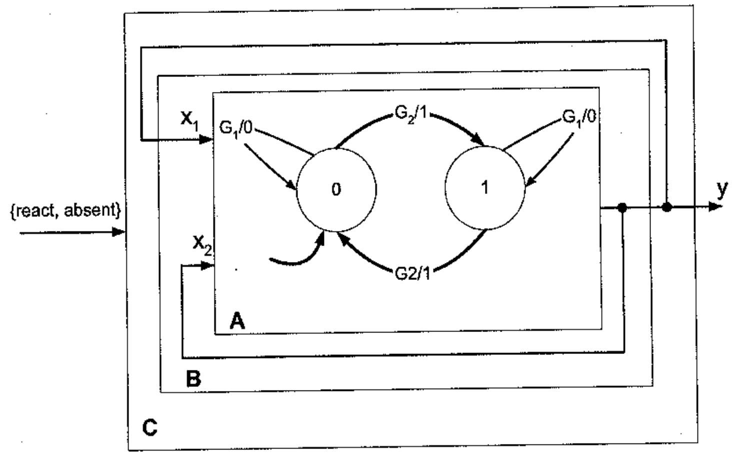
[-1] no plot but correct

If F is time invariant, then $\delta(n) = x(n+4)$ means that $h(n) = y(n+4)$, i.e.,



Q3.2 Version 1

Q3.2 (20 points) Consider the finite-state machine composition shown below:



Let the set D $= \{0, 1, \text{absent}\}$ denote an alphabet. For every pair $(x_1(n), x_2(n)) \in$ D^2, $x_1(n)$ and $x_2(n)$ denote the top and bottom input symbols in the figure, respectively. The n^{th} output symbol $y(n) \in$ D.

For *each* of the following guard sets G_1, G_2, and for each of the machines B and C, determine whether the machine is well-formed (WF) or not well-formed (NWF) by circling one choice (WF or NWF) in each entry of the table below? No explanation will be considered. No partial credit will be given.

$$(I)\begin{cases} G_1 = \{(1,0)\} \\ G_2 = \{(1,1)\} \end{cases} \qquad (II)\begin{cases} G_1 = \{(0,0),(1,0)\} \\ G_2 = \{\} \end{cases}$$

$$(III)\begin{cases} G_1 = \{(1,0)\} \\ G_2 = \{(0,1)\} \end{cases} \qquad (IV)\begin{cases} G_1 = \{(1,1),(0,1)\} \\ G_2 = \{(0,0),(1,0)\} \end{cases}$$

Guard Set	Machine B	Machine C
(I)	WF NWF	WF NWF
(II)	WF NWF	WF NWF
(III)	WF NWF	WF NWF
(IV)	WF NWF	WF NWF

More than one nonstuttering fixed point.

Unique nonstuttering fixed pt for B & C.

Unique non-stuttering fxd pt for each x_1 ⇒ B is WF.

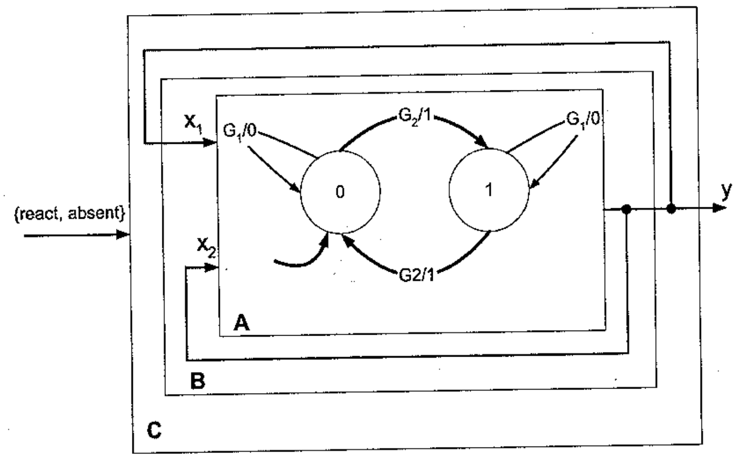
No nonstuttering fxd pt for B or C.

No nonstuttering fxd pt for "react" input to C ⇒ C is NWF

NOTE: The answer to (I) shows that B not well-formed DOES NOT IMPLY C is not well-formed.

Q3.2 Version 2

Q3.2 (20 points) Consider the finite-state machine composition shown below:



Let the set D $= \{0, 1, \text{absent}\}$ denote an alphabet. For every pair $(x_1(n), x_2(n)) \in$ D^2, $x_1(n)$ and $x_2(n)$ denote the top and bottom input symbols in the figure, respectively. The n^{th} output symbol $y(n) \in$ D.

For *each* of the following guard sets G_1, G_2, and for each of the machines B and C, determine whether the machine is well-formed (WF) or not well-formed (NWF) by circling one choice (WF or NWF) in each entry of the table below? No explanation will be considered. No partial credit will be given.

$$(I)\begin{cases} G_1 = \{(1,0)\} \\ G_2 = \{(0,1)\} \end{cases} \qquad (II)\begin{cases} G_1 = \{(0,0),(1,0)\} \\ G_2 = \{(0,1),(1,1)\} \end{cases}$$

$$(III)\begin{cases} G_1 = \{(1,1)\} \\ G_2 = \{(0,0)\} \end{cases} \qquad (IV)\begin{cases} G_1 = \{(0,0),(1,0)\} \\ G_2 = \{\} \end{cases}$$

No nonstuttering fxd pt for "react" input to C ⇒ C is NWF.

Guard Set	Machine B	Machine C
(I)	WF NWF	WF NWF
(II)	WF NWF	WF NWF
(III)	WF NWF	WF NWF
(IV)	WF NWF	WF NWF

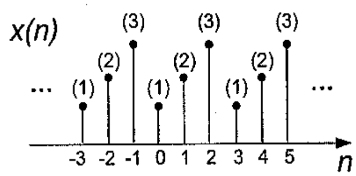
Unique nonstuttering fxd pt for each x_1 ⇒ B is WF

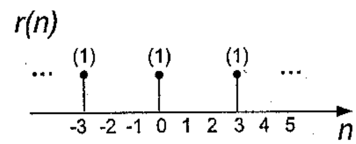
>1 nonstuttering fxd pt C & B ⇒ Both NWF.

No nonstuttering fxd pt

Unique nonstuttering fxd pt for B & C ⇒ Each is WF.

Q3.3 (10 points) Consider a real, discrete-time periodic signal x, which is modulated by a real, periodic impulse-train signal r. The signals x and r are shown in the figure below.





Let q denote the resulting signal. The sample values $q(n)$ of the signal q are given by:

$$q(n) = r(n)x(n).$$

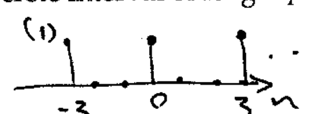
Determine the period p, the fundamental frequency ω_0, and the Fourier series coefficients Q_k, $k = 0, 1, \ldots, p-1$, of the signal q.

The following complex exponential Fourier series expressions for a periodic discrete-time signal having period p may be of potential use to you:

$$q(n) = \sum_{k=\langle p\rangle} Q_k \, e^{ik\omega_0 n} \qquad\qquad Q_k = \frac{1}{p} \sum_{n=\langle p\rangle} q(n)\, e^{-ik\omega_0 n}\,,$$

where $\omega_0 = \dfrac{2\pi}{p}$ and $\langle p\rangle$ denotes a suitable contiguous discrete interval of length p.

Note that $q(n) = r(n) = \sum_{\ell=-\infty}^{\infty} \delta(n-3\ell)$



$p = 3$, $\omega_0 = \frac{2\pi}{3}$

$Q_k = \frac{1}{p} \sum_{n=0}^{2} q(n) e^{-ik\omega_0 n}$

$= \frac{1}{3}\left(e^{0} + 0\cdot e^{i\omega_0 k} + 0\cdot e^{-i2\omega_0 k}\right) = \frac{1}{3} \Rightarrow Q_k = \frac{1}{3}$ $k = 0, 1, 2$