EECS 20N: Structure and Interpretation of Signals and Systems QUIZ 4 Sol.
Department of Electrical Engineering and Computer Sciences 5 December 2005
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name Expert FIRST Name Frequency

Lab Time 365/24/7

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take you up to 20 minutes to complete. You will be given at least 20 minutes—up to the end of today's lecture hour—to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- **The quiz printout consists of pages numbered 1 through 5. The last page is left blank; you may use it only for scratch work.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the five numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2	20	20
Total	**45**	45

Q4.1 (20 Points) Consider a discrete-time system

$$F : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}].$$

Let x denote an arbitrary input signal and y the corresponding output signal. The output y is characterized by:

$$\forall x \in [\mathbb{Z} \to \mathbb{R}] \text{ and } \forall n \in \mathbb{Z}, \quad y(n) = \begin{cases} 0 & \text{if } n \text{ is an odd integer} \\ 1 & \text{if } n \text{ is an even integer.} \end{cases}$$

For each part below, *explain your reasoning succinctly, but clearly and convincingly.*

(a) **Memorylessness:** Select the strongest assertion from the choices below.

- (I) The system must be memoryless.
- (II) The system could be memoryless, but does not have to be.
- **[(III)]** The system cannot be memoryless.

Consider an input signal x that has at least two equal adjacent sample values, e.g., $x(n_0) = x(n_0 + 1)$ for some $n_0 \in \mathbb{Z}$. Clearly, the corresponding adjacent values of the output signal y are not equal; if n_0 is an even integer, then $y(n_0) = 1$ and $y(n_0 + 1) = 0$, and if n_0 is an odd integer, then $y(n_0) = 0$ and $y(n_0+1) = 1$. This violates memorylessness.

(b) **Causality:** Select the strongest assertion from the choices below.

- **[(I)]** The system must be causal.
- (II) The system could be causal, but does not have to be.
- (III) The system cannot be causal.

Any two signals x_1 and x_2 that are equal up to, and including, an arbitrary sample $n_0 \in \mathbb{Z}$ (i.e., $x_1(n) = x_2(n), \forall n \leq n_0$) produce corresponding output signals y_1 and y_2 which are identical up to, and including, n_0. In fact, any two arbitrary input signals produce identical output signals for *all* samples $n \in \mathbb{Z}$. Therefore, the system must be causal.

(c) **Time Invariance:** Select the strongest assertion from the choices below.

(I) The system must be time invariant.

(II) The system could be time invariant, but does not have to be.

[(III)] The system cannot be time invariant.

Shifting the input signal x by one sample does not produce a corresponding shift in the output signal y. Therefore, the system cannot be time invariant.

(d) **Linearity:** Select the strongest assertion from the choices below.

(I) The system must be linear.

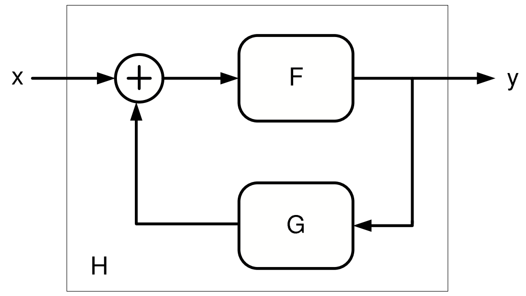
(II) The system could be linear, but does not have to be.

[(III)] The system cannot be linear.

Scaling the input signal x by a constant does not correspondingly scale the output signal y. Also note that the output signal y is non-zero, even if the input signal x is zero, i.e., $x(n) = 0, \forall n \in \mathbb{Z}$. Therefore, the system cannot be linear.

You may use the blank space below (on this page) for scratch work. Nothing written below this line will be considered in grading.

Q4.2 (20 points) System H, shown below, is a feedback composition of real, discrete-time LTI systems F and G:



The impulse response $f : \mathbb{Z} \to \mathbb{R}$ of system F is given by:

$$\forall n \in \mathbb{Z}, \quad f(n) = \begin{cases} \frac{1}{4} & \text{if } n = -1 \\ \frac{1}{2} & \text{if } n = 0 \\ \frac{1}{4} & \text{if } n = +1 \\ 0 & \text{elsewhere.} \end{cases}$$

The frequency response $G : \mathbb{R} \to \mathbb{C}$ of system G is given by:

$$\forall \omega \in \mathbb{R}, \quad G(\omega) = \frac{1 - e^{i\omega}}{\frac{1}{4}e^{i\omega} + \frac{1}{2} + \frac{1}{4}e^{-i\omega}} .$$

For each part of this problem, *explain your reasoning succinctly, but clearly and convincingly.*

(a) Is the system F causal?

No. The system cannot be causal. A causal LTI system must have an impulse response that is zero for negative samples; the impulse response f of the LTI system F has the nonzero value $f(-1) = 1/4$.

(b) Determine an expression for the frequency response H of system H. Express—in a very simple form—the impulse response h of system H in terms of the impulse response f of system F?

The frequency response of system F is:

$$\forall \omega \in \mathbb{R}, \quad F(\omega) = \frac{1}{4}e^{i\omega} + \frac{1}{2} + \frac{1}{4}e^{-i\omega}.$$

The loop gain FG is:

$$\forall \omega \in \mathbb{R}, \quad F(\omega)\, G(\omega) = 1 - e^{i\omega} .$$

The frequency response of the feedback system H is given by:

$$H(\omega) = \frac{F(\omega)}{1 - F(\omega)\, G(\omega)} = \frac{F(\omega)}{1 - (1 - e^{i\omega})} = e^{-i\omega}\, F(\omega).$$

Therefore, the impulse response of the composite feedback system H is given by:

$$\forall n \in \mathbb{Z}, \quad h(n) = f(n-1).$$

(c) Is the composite feedback system H causal?

Yes. The composite system H is causal, because $h(n) = 0, \forall n \in \mathbb{Z}$. In particular,

$$\forall n \in \mathbb{Z}, \quad h(n) = \frac{1}{4}\,\delta(n) + \frac{1}{2}\,\delta(n-1) + \frac{1}{4}\,\delta(n-2).$$

You may use the blank space below (on, and on the back of, this page) for scratch work. Nothing written beyond this line will be considered in grading.