EECS 20N: Structure and Interpretation of Signals and Systems — QUIZ 2
Department of Electrical Engineering and Computer Sciences — 13 March 2007
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name De Cay FIRST Name Expo
Lab Time Mon 12AM-3AM

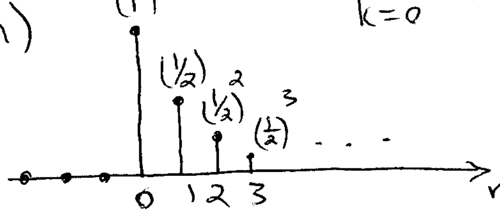
- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 30 minutes to complete. You will be given at least 30 minutes, up to a maximum of 40 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2	20	20
Total	**45**	45

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

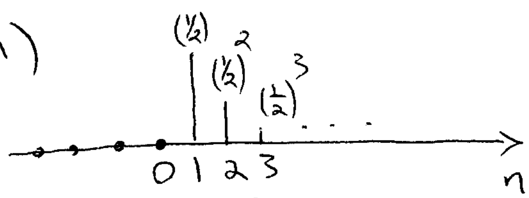
Q2.1 (b): Graphical method of determining the step response: $s(n) = \sum_{k=0}^{\infty} h_k(n)$

$h_0(n)$



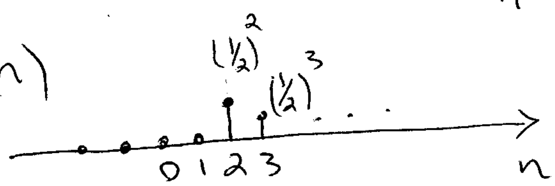
+

$h_1(n)$



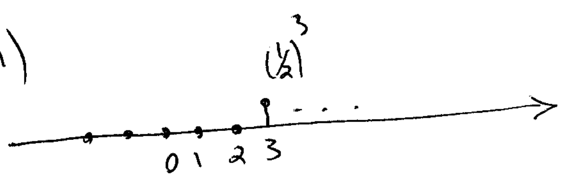
+

$h_2(n)$



+

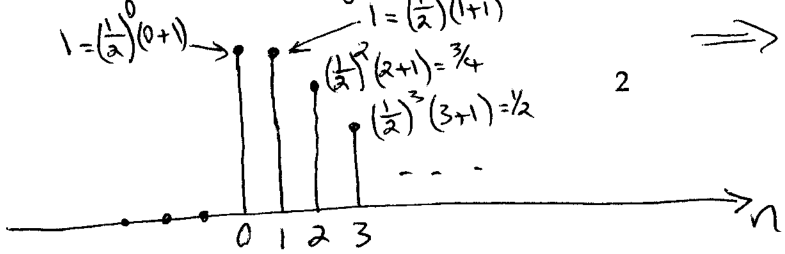
$h_3(n)$



+

⋮

Pointwise addition yields



$\Longrightarrow$ By inspection:

$$s(n) = \left(\frac{1}{2}\right)^n (n+1)$$

Q2.1 (20 Points) Consider a *linear*, discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ shown below:

$$\delta_k \longrightarrow \boxed{\;\mathsf{F}\;} \longrightarrow h_k$$

If the input signal is the shifted impulse δ_k,

$$\forall n \in \mathbb{Z}, \quad \delta_k(n) = \delta(n-k)\,,$$

then the output signal is the one-sided exponential h_k,

$$\forall n \in \mathbb{Z}, \quad h_k(n) = \left(\frac{1}{2}\right)^n u(n-k),$$

for every $k \in \mathbb{Z}$.

(a) (8 Points) Select the strongest true assertion from the list below.

(i) The system must be time invariant.

(ii) The system could be time invariant, but does not have to be.

$\boxed{\text{(iii) The system cannot be time invariant.}}$

Explain your reasoning succinctly, but clearly and convincingly.

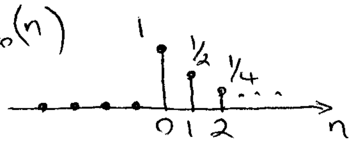
$$\delta(n) \longrightarrow h_0(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$$\delta(n-1) \longrightarrow h_1(n) = \left(\tfrac{1}{2}\right)^n u(n-1) \neq \left(\tfrac{1}{2}\right)^{n-1} u(n-1) = h_0(n-1)$$

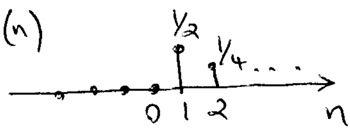
More generally, for this problem,

$$h_k(n) \neq h_0(n-k)$$

$h_0(n)$



$h_1(n)$



Note that $h_1(n)$ is not a shifted version of $h_0(n)$. Rather, it is a truncated version of $h_0(n)$.

$\boxed{\text{F is a linear, time-varying system}}$

(b) (12 Points) Determine the *step response* of the system F.

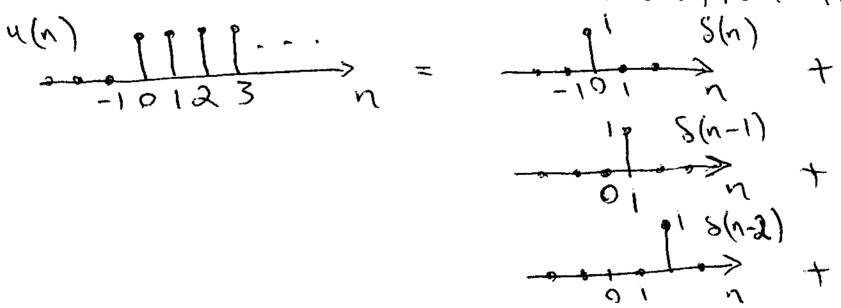
$$u \longrightarrow \boxed{\;F\;} \longrightarrow s$$

In particular, determine a simple, closed form expression for the output s of the system in response to the unit-step signal u. Recall that

$$u(n) = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$

$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

The unit-step function is a sum of shifted impulses



We're told F is a linear system, so the step response can be written as the sum of responses to the individual, shifted impulses $\delta(n-k)$

$$\delta_k \longrightarrow h_k$$

$$s(n) = \sum_{k=0}^{\infty} h_k(n) = \sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^n u(n-k) = \left(\tfrac{1}{2}\right)^n \sum_{k=0}^{\infty} u(n-k)$$

$$= \left(\tfrac{1}{2}\right)^n \underbrace{\sum_{k=0}^{n} 1}_{\text{"n+1" Terms}} = \left(\tfrac{1}{2}\right)^n (n+1) \implies \underline{s(n) = \left(\tfrac{1}{2}\right)^n (n+1)}$$

For a graphical method of determining $s(n) = \sum_{k=0}^{\infty} h_k(n)$,

see p. 2.

Q2.2 (20 Points) Some traders on Wall Street treat stock price data as discrete-time signals. They make buy-sell-hold decisions based on certain statistics that they compute from the data.

One indicator of stock price movement, which traders commonly glean from the price data, is the *exponentially-weighted moving average (EWMA)*.

The following equation depicts a model of the relationship between the input and output of the EWMA filter:

$$y(n) = C \sum_{l=0}^{+\infty} \alpha^l x(n-l). \tag{1}$$

The EWMA can be thought of as the output of a DT-LTI system; the input signal x and the output signal y denote the stock price and the EWMA values, respectively.

The sample $n \in \mathbb{Z}$ denotes the instance of interest; it can represent a day, an hour, a minute, or whatever time scale the trader wishes to use for trading.

Note that stock price values corresponding to "older" samples in time enjoy diminishing importance in the computation of the current value of y; that is, the farther in the past the stock price value at a particular time instance is, the less weight it carries in computing $y(n)$.

The real constant C is a normalization factor; if it is chosen to be $C = 1 - \alpha$, then

$$C \sum_{l=0}^{+\infty} \alpha^l = 1.$$

That is, the sum of all the weights would equal 1.

NOTE: Although the answers to some of the following parts may be useful in solving others, it is possible to tackle each part independently.

(a) (5 Points) Explain why Equation 1 can NOT represent a practical implementation of the EWMA filter.

Equation 1 can not represent a practical implementation because it requires infinite memory (i.e., storage of input signal values from n to $-\infty$).

(b) (5 Points) Determine a simple expression for $h(n)$, the impulse response values of the EWMA filter.

Method 1: We have $y(n) = \sum_{l=0}^{\infty} C\alpha^{l} x(n-l)$, which can be written as:

$$y(n) = \sum_{l=-\infty}^{\infty} \underbrace{C\alpha^{l} u(l)}_{h(l)} x(n-l)$$, which is in the form of a convolution sum.

Note the lowerlimit $\longrightarrow$

The impulse response is $h(n) = C\alpha^{n} u(n)$

Method 2: Let $x(n) = \delta(n) \Rightarrow y(n) = h(n) = C\sum_{l=0}^{\infty} \alpha^{l}\delta(n-l) = C\alpha^{n}$ $n \geq 0$, $h(n) = 0$, $n < 0$.

(c) (5 Points) Determine a simple expression for the frequency response $H(\omega)$, $\forall \omega \in [-\pi, \pi]$, and provide a well-labeled sketch for its magnitude $|H(\omega)|$.

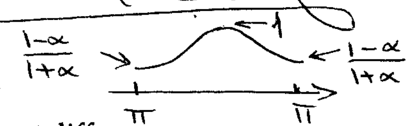
Method 1: Let $x(n) = e^{i\omega n} \Rightarrow x(n-l) = e^{+i\omega n} e^{-i\omega l}$, $y(n) = H(\omega) e^{i\omega n}$

Plug into Eqn. (1): $y(n) = H(\omega)\cancel{e^{i\omega n}} = C\left(\sum_{l=0}^{\infty} \alpha^{l} e^{-i\omega l}\right)\cancel{e^{i\omega n}} \Rightarrow$

$H(\omega) = C\sum_{l=0}^{\infty} (\alpha e^{-i\omega})^{l} = \dfrac{C}{1-\alpha e^{-i\omega}}$ since $|\alpha| < 1$. (see the normalization equation).

$H(\omega) = \dfrac{C}{1-\alpha e^{-i\omega}} = \dfrac{1-\alpha}{1-\alpha e^{-i\omega}} = \dfrac{e^{i\omega}(1-\alpha)}{e^{i\omega} - \alpha}$

This is the prototypical DT low-pass Filter



Method 2: $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n} = C\sum_{n=0}^{\infty} \alpha^{n} e^{-i\omega n} = \dfrac{C}{1-\alpha e^{-i\omega}}$

(d) (5 Points) Determine a finite-order, linear, constant-coefficient difference equation that is consistent with the input-output relationship between x and y.

Method 1: $H(\omega) = \dfrac{C}{1-\alpha e^{-i\omega}} \Rightarrow (1-\alpha e^{-i\omega}) H(\omega) = C \Rightarrow$

By inspection (and matching the coefficients)

$$y(n) - \alpha y(n-1) = C x(n)$$

$$y(n) = \alpha y(n-1) + C x(n)$$

Better Method

Ad Hoc Method

Method 2: $y(n) = C\sum_{l=0}^{\infty} \alpha^{l} x(n-l) \Rightarrow \left\{ y(n-1) = C\sum_{l=0}^{\infty} \alpha^{l} x(n-1-l) \right\} \Rightarrow$

let $m = l+1 \Rightarrow l = m-1$

$y(n-1) = C\sum_{m=1}^{\infty} \alpha^{m-1} x(n-m) = \dfrac{1}{\alpha}\underbrace{\sum_{m=0}^{\infty} C\alpha^{m} x(n-m)}_{y(n)} - \dfrac{C}{\alpha} x(n) \Rightarrow$

$y(n-1) = \dfrac{1}{\alpha} y(n) - \dfrac{C}{\alpha} x(n) \Longrightarrow y(n) = \alpha y(n-1) + C x(n)$