Problem session, week 13

9.4 (Revised) **T** This exercise discusses **amplitude modulation** or **AM**. AM is a technique that is used to convert low frequency signals into high frequency signals for transmission over a radio channel. Conversion of the high frequency signal back to a low frequency signal is called **demodulation**. The system structure is depicted in figure 1. The transmission medium (air, for radio signals) is approximated here as a medium that passes the signal y unaltered. Suppose your AM radio station is allowed to transmit signals at a carrier frequency of 740 kHz (this is the frequency of KCBS in San Francisco). Suppose you want to send the audio signal x : *Reals* $\rightarrow$ *Reals*. The AM signal that you would transmit is given by, for all $t \in$ *Reals*,

$$y(t) = x(t)\cos(\omega_c t),$$

where $\omega_c = 2\pi \times 740,000$ is the **carrier frequency** (in radians per second). Suppose $X(\omega)$ is the Fourier transform of an audio signal with magnitude as shown in figure 2.

(a) What is the Fourier transform Y of y in terms of X?

(b) Carefully sketch $|Y(\omega)|$ and note the important magnitudes and frequencies on your sketch.

Note that if $X(\omega) = 0$ for $|\omega| > 2\pi \times 10,000$, then $Y(\omega) = 0$ for $||\omega| - |\omega_c|| > 2\pi \times 10,000$. In words, if the signal x being modulated is bandlimited to less than 10 kHz, then the modulated signal is bandlimited to frequencies that are withing 10 kHz of the carrier frequency. Thus, an AM radio station only needs to occupy 20 kHz of the radio spectrum in order to transmit audio signals up to 10 kHz.

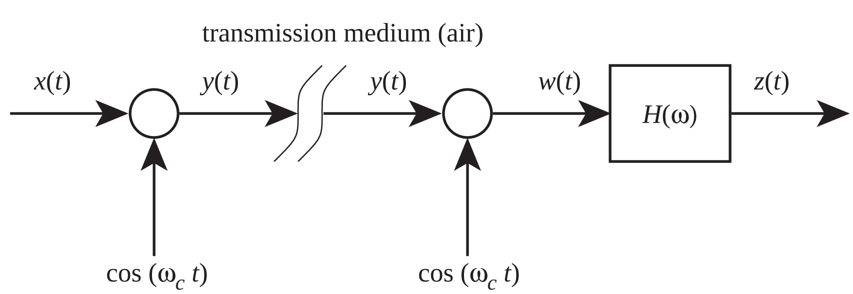


Figure 1: AM transmission. In the figure, attenuation and noise in the transmission medium are neglected, so the received signal is the same as the transmitted signal, y.

(c) At the receiver, the problem is to recover the audio signal x from y. One way is to demodulate by multiplying y by a sinewave at the carrier frequency to obtain the signal w, where

$$w(t) = y(t)\cos(\omega_c t).$$

What is the Fourier transform W of w in terms of X? Sketch $|W(\omega)|$ and note the important magnitudes and frequencies.

(d) After performing the demodulation of part (c), an AM receiver will filter the received signal through a low-pass filter with frequency response $H(\omega)$ such that $H(\omega) = 1$ for $|\omega| \leq 2\pi \times 10,000$ and $|H(\omega)| = 0$ for $|\omega| > 2\pi \times 20,000$. Let z be the filtered signal, as shown in figure 1. What is the Fourier transform Z of z? What is the relationship between z and x?

Solution

(a) Write

$$\begin{aligned} y(t) &= x(t)cos(\omega_c t) \\ &= (x(t)/2)e^{i\omega_c t} + (x(t)/2)e^{-i\omega_c t}. \end{aligned}$$

Thus,

$$Y(\omega) = X(\omega - \omega_c)/2 + X(\omega + \omega_c)/2.$$

(b) The original Fourier transform is shifted up and down by ω_c and scaled by half, as shown below:

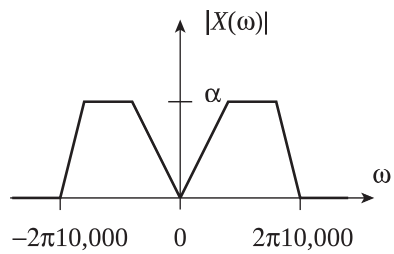
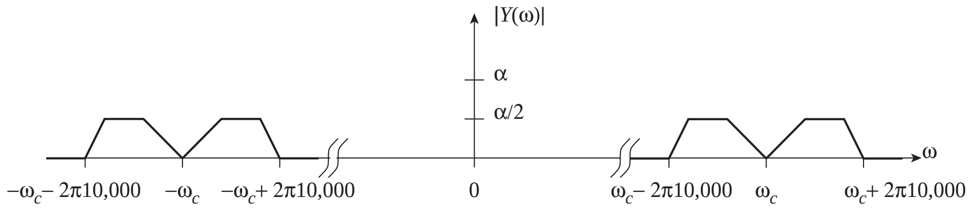


Figure 2: Magnitude of the Fourier transform of an example audio signal.



(c) Note that

$$\begin{aligned} w(t) &= y(t)cos(\omega_c t) \\ &= (y(t)/2)e^{i\omega_c t} + (y(t)/2)e^{-i\omega_c t}. \end{aligned}$$

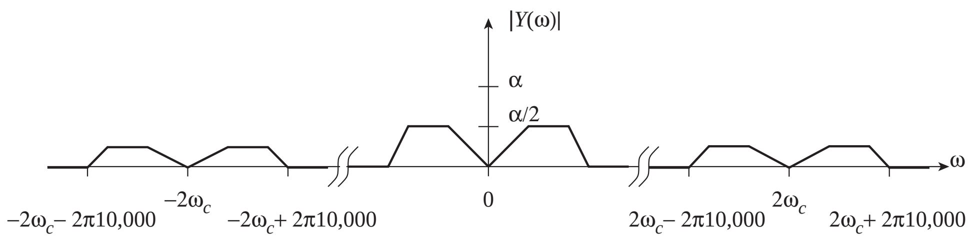
Thus,

$$W(\omega) = Y(\omega - \omega_c)/2 + Y(\omega + \omega_c)/2.$$

Using Y from part (a),

$$W(\omega) = (X(\omega - 2\omega_c)/2 + X(\omega) + X(\omega + 2\omega_c)/2)/2.$$

This is sketched below:



(d) The lowpass filter will eliminate all but the center portion of the figure above, so

$$Z(\omega) = X(\omega)/2$$

or

$$z(t) = x(t)/2.$$

The original audio signal is recovered, albeit attenuated by a factor of 2.