Discrete-Event Systems:
Generalizing Metric Spaces and Fixed-Point Semantics

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Discrete-Event (DE) Systems

- **Traditional Examples**
  - VHDL
  - OPNET Modeler
  - NS-2

- **Distributed systems**
  - TeaTime protocol in Croquet

(two players vs. the computer)
Introduction to DE Systems

- In DE systems, concurrent objects (processes) interact via signals
What is the semantics of DE?

- Simultaneous events may occur in a model
  - VHDL Delta Time

- Simultaneity absent in traditional formalisms
  - Yates
  - Chandy/Misra
  - Zeigler
Time in Software

- Traditional programming language semantics lack time
- When a physical system interacts with software, how should we model time?
- One possibility is to assume some computations take zero time, e.g.
  - Synchronous language semantics
  - GIOTTO logical execution time
Simultaneity in Hardware

• Simultaneity is common in synchronous circuits

• Example:

    | Time |
    |------|
    | 0    |
    | 1    |
    | 0    |
    | 0    |

This value changes instantly
Simultaneity in Physical Systems

[Biswas]
Our Contributions

• We generalize DE semantics to handle simultaneous events

• We generalize metric space concepts to handle our model of time

• We give uniqueness conditions and conditions for avoidance of Zeno behavior
Models of Time

- **Time (real time)**

- **Superdense time [Maler, Manna, Pnueli]**

Superdense time increases first in this direction (sequence time) then in this direction (real time)
Zeno Signals

- Definition: Zeno Signal
  infinite events in finite real time

Chattering Zeno [Ames]

Genuinely Zeno [Ames]
Source of Zeno Signals

- Feedback can cause Zeno
Genuinely Zeno

- A source of genuinely Zeno signals
Simple Processes

• Definition: *Simple Process*

  Non-Zeno Signal → Process → Non-Zeno Signal

• *Merge* is simple, but it has Zeno feedback solutions

• When are compositions of simple processes simple?
Cantor Metric for Signals

“Distance” between two signals

\[ d(s_1, s_2) = \frac{1}{2^t} \]

First time at which the two signals differ.
Tetrics: Extending Metric Spaces

- Cantor metric doesn’t capture simultaneity
- We can capture simultaneity with a tetric
- Tetrics are generalized metrics
- We generalized metric spaces with “tetric spaces”
- Our tetric allows us to deal with simultaneity
Our Tetric for signals

"Distance" between two signals:

\[ d(s_1, s_2) = \left( \frac{1}{2^t}, \frac{1}{2^n} \right) \]

First time at which the two signals differ:

\[ d_1(s_1, s_2) = \frac{1}{2^t} \]

First sequence number at which the two signals differ:

\[ d_2(s_1, s_2) = \frac{1}{2^n} \]
Delta Causal

Definition: *Delta Causal*
Input signals agree up to time $t$ implies output signals agree up to time $t + \Delta$
What Delta Causal Means

• Signals which delay their response to input events by delta will have non-Zeno fixed points
Extending Delta Causal

• The system should be allowed to chatter

• As long as time eventually advances by delta
Definition: *Tetric Delta Causal*

1) Input signals agree up to time $(t,n)$ implies output signals agree up to time $(t, n + 1)$

2) If $n$ is large enough, this also implies output signals agree up to time $(t + \Delta, 0)$
**Definition: Causal**

If input signals agree up to supertime \((t, n)\) then the output signals agree up to supertime \((t, n)\).
Result 1

- Every tetric delta causal process has a unique feedback solution
Result 2

- Every network of simple, causal processes is a simple causal process, provided in each cycle there is a delta causal process.
- Example
Conclusions

- We broadened DE semantics to handle superdense time
- We invented tetric spaces to measure the distance between DE signals
- We gave conditions under which systems will have unique fixed-point solutions
- We provided sufficient conditions under which this solution is non-Zeno
- http://ptolemy.eecs.berkeley.edu/papers/05/DE_Systems
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