Controller Coefficient Truncation Using Lyapunov Performance Certificates

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Specifier/implementer interface

- specifier (control/system designer) and implementer (hardware/ software/network implementer) interact through an interface
- specifier provides *reference design* and *performance certificate*
- implementer uses any method, provided implementation passes certification.
- system designer warrants that over all system will work, if implemented controller passes certification
- this work: implementing controller with fixed-point coefficients, with Lyapunov/ LMI certificate

State feedback controller with LQR cost specification

- plant: $x(t+1) = Ax(t) + Bu(t)$; controller: $u(t) = Kx(t)$
- reference controller $K^{\rm nom}$ is LQR optimal
- goal: minimize $\Phi(K)$, total number of bits in K, while guaranteeing LQR cost is ϵ-suboptimal
- controller K passes certification if

$$(1+\epsilon)(A+BK)^T P^{\rm nom}(A+BK) - (1+\epsilon)P^{\rm nom} + Q + K^T RK \succeq 0$$

where $P^{\rm nom}$ is solution of DARE

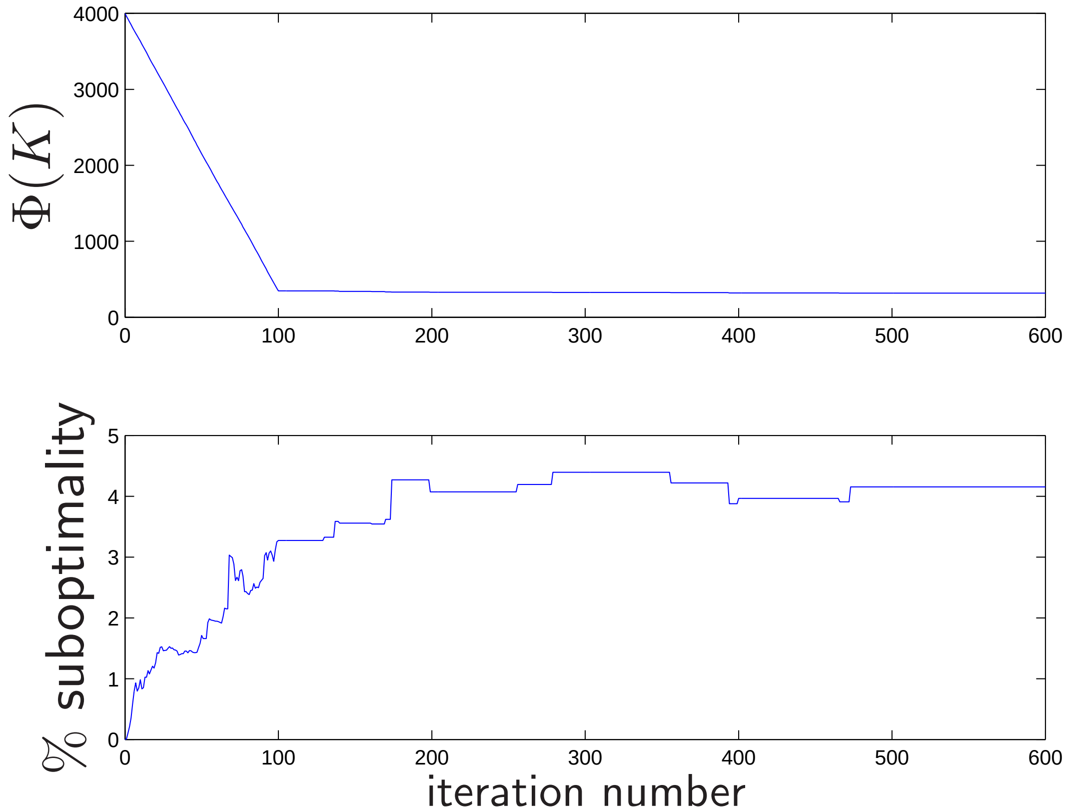
$$(A+BK^{\rm nom})^T P(A+BK^{\rm nom}) - P + Q + K^{\rm nom}TRK^{\rm nom} = 0$$

Algorithm

- initialize with nominal design (truncated to 40-bit coefficients)
- at each step choose an index pair (i, j) at random; fix all other entries in K
- use Lyapunov certificate to find an interval $[l, u]$ of admissible values for K_{ij} (by solving convex optimization problem)
- truncate K_{ij} to value in interval with fewest bits
- stop when design does not change over one pass over all coefficients
- run algorithm several times; take best design found as final choice

Example run

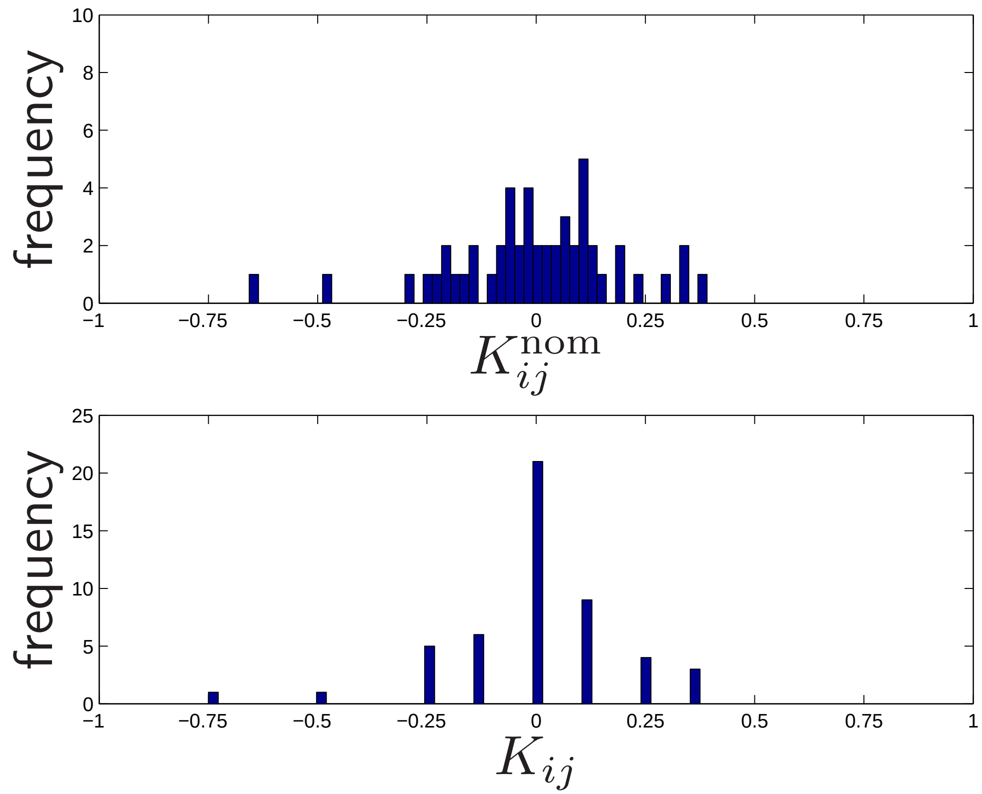
10 states, 5 inputs; certificate requires 15% suboptimality



final controller is 4.1%-suboptimal, with 6.3 bits/coeff.

Final design

best design in 100 runs is 14.9%-suboptimal, has only 1.5 bits/coeff.



Conclusions & observations

the method described

- extends to many other problems
  - dynamic controllers
  - nonlinear effects (saturation, underflow)
  - filters
- often achieves extremely aggressive coefficient truncation, with little decrease in performance (when implementation is over parametrized)

next: handling timing errors (jitter, late/lost packets, . . .)