



Model-Based Verification and Testing

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MURI Review Meeting

Frameworks and Tools for High-Confidence Design of Adaptive, Distributed Embedded Control Systems

Berkeley, CA

September 6, 2007

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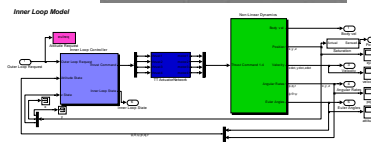
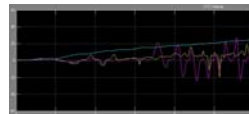
Model-Based Verification

Timing in Networked Control Systems

(Bhave, Krogh)

- impact on performance and stability of feedback loops
- analytical and simulation tools

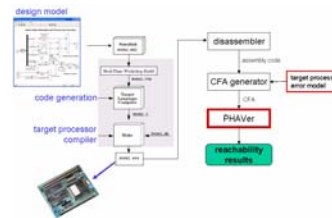
$$\left| \frac{P_{alias}(\omega)C(e^{j\omega})}{1 + P_{ZOH}(e^{j\omega})C(e^{j\omega})} \right| < \frac{1}{N|e^{j\omega} - 1|}$$



Verification of Numerical Code

(Maka, Freshe, Krogh)

- verification for target environment
- reachability with polyhedral domains
- widening for iterative computations



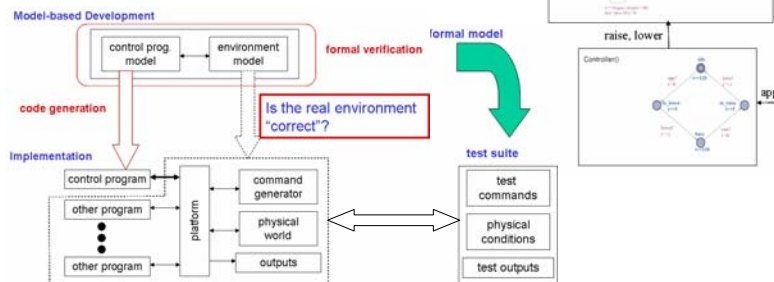
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Model-Based Testing Generation

Testing Environmental Assumptions

(Ivol, Kaynar, Krogh)

- new application of conformance testing
- initial implementation with timed i/o automata



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Timing Analysis in Networked Control Systems

- *Timing Variations in Feedback Control Loops*
- *Analytical Tools*
- *Co-Simulation Tool*
- *Future Work*

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Timing Variations in Feedback Control Loops

- **Sources**
 - single processor: interrupt routines, task scheduling, hardware (e.g., ADC)
 - multi-processor: buses, shared memory and DMA
 - physical interface: networked I/O*, sensors, actuators
- **Types**
 - jitter (sampling and input-output)*
 - skipped/lost packets (noise and data rate limits)
 - race conditions
- **Impact**
 - performance degradation*
 - loss of stability*
 - incorrect decisions

* *Focus of first year work*

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Analytical Tools

- **Time delay robustness is a large and extensively studied area with numerous theoretical results [Gu, K., Niculescu S.]**
- **Results categorized into**
 - delay dependent or delay independent stability criteria
 - frequency domain or time domain approaches
- **Majority of time domain results based on extensions of Lyapunov methods to infinite-dimensional systems**
 - Razumikhin method allows for bounded but arbitrarily time varying delays
 - Krasovskii method allows for delays bounded in both length and time variation
- **Most results are sufficient conditions**
 - may be excessively conservative or complex for practical application

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Frequency Domain Stability Analysis

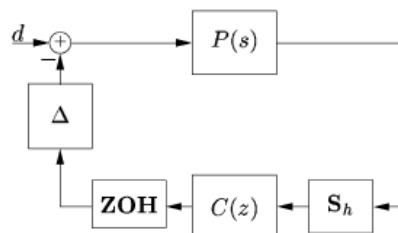
- Criteria for sampled-data, LTI control systems [Kao, Lincoln]
- Uses the Small Gain Theorem with delay modeled as uncertainty in the loop
- Current analysis done on SISO, stable, strictly proper continuous plant and discrete controller
- Extendable to MIMO via Integral Quadratic Constraint (IQC) approach with LMI [Kao, Rantzer]
- Delay is bounded with known worst-case bound but otherwise arbitrarily varying
- Stability is easily checked using closed-loop Bode plot

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Frequency Domain Stability Analysis

Formula for Stability:

$$\left| \frac{P_{\text{alias}}(\omega)C(e^{j\omega})}{1 + P_{\text{ZOH}}(e^{j\omega})C(e^{j\omega})} \right| < \frac{1}{N|e^{j\omega} - 1|}, \quad \forall \omega \in [0, \infty]$$



$0 \leq \text{Delay } \Delta \leq Nh$ where h is the sampling period

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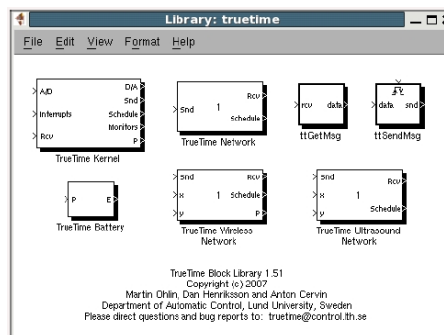
Co-Simulation: TrueTime Simulink Blockset

- **Co-simulation of**
 - Control task execution
 - Network communication
 - Plant dynamics
- **Investigate timing behavior of control loops implemented on digital computers**
- **System is subject to delays, jitter and lost samples**
- **Created at the Dept. of Automatic Control, Lund University**

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TrueTime – cont'd.

- **A Kernel block, 3 Network blocks, 1 Battery block**
 - Simulink S-functions written in C++
 - Event-based execution using zero-crossing functions
 - Portable to other simulation environments



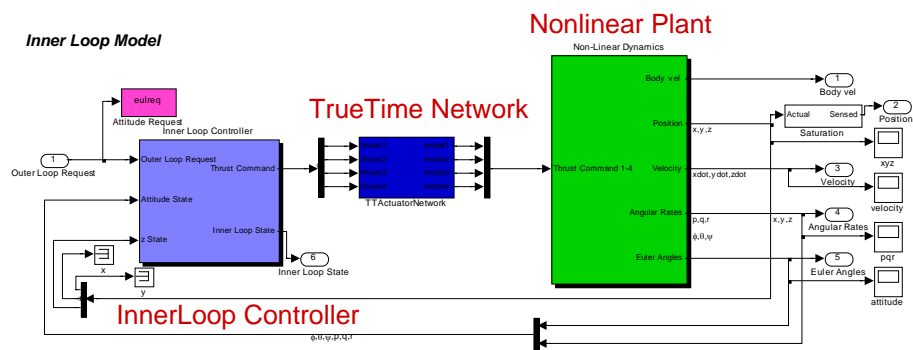
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STARMAC Example Scenario

- Attitude/Altitude controller connected to nonlinear plant through 10 Mbit Ethernet
- Motor thrust commands transmitted with jitter because of the random backoff network algorithm
- Pre/post processing of packets explicitly simulated with random delays at each node
- Each motor receives control signal at differing intervals which leads to deterioration in tracking
- Extreme jitter (order of plant dynamics) can cause instability of system

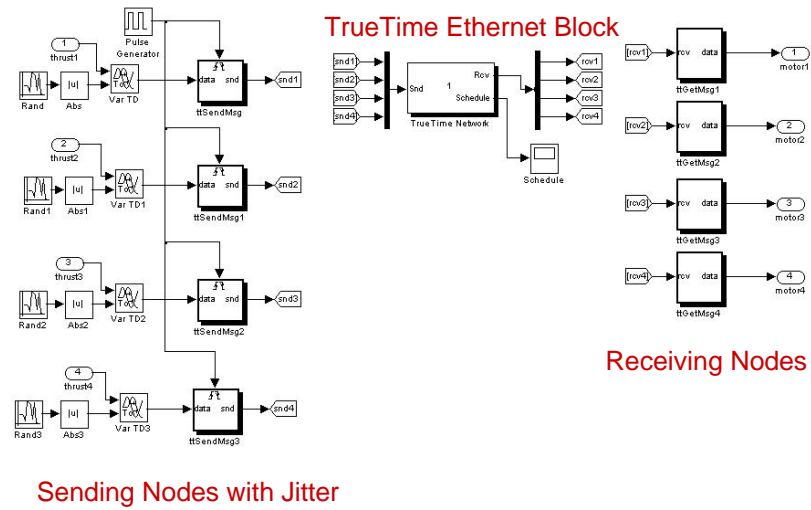
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STARMAC Vehicle Model



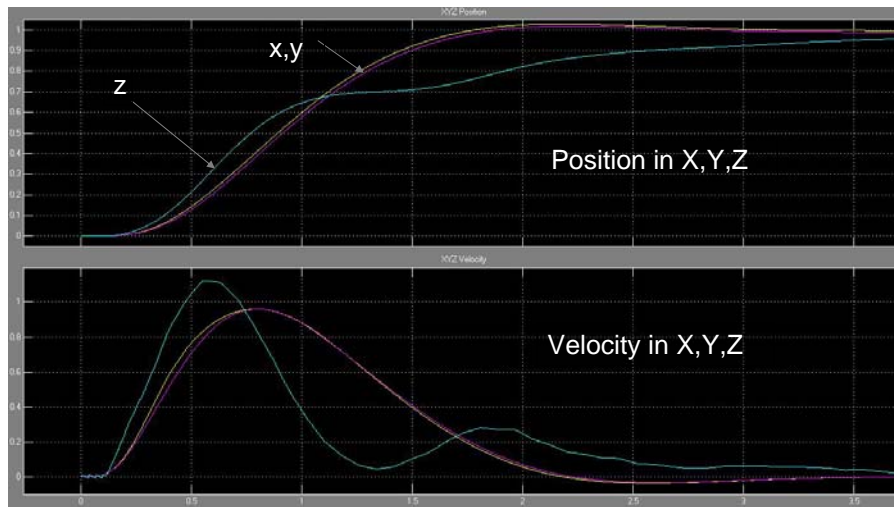
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Actuator Network



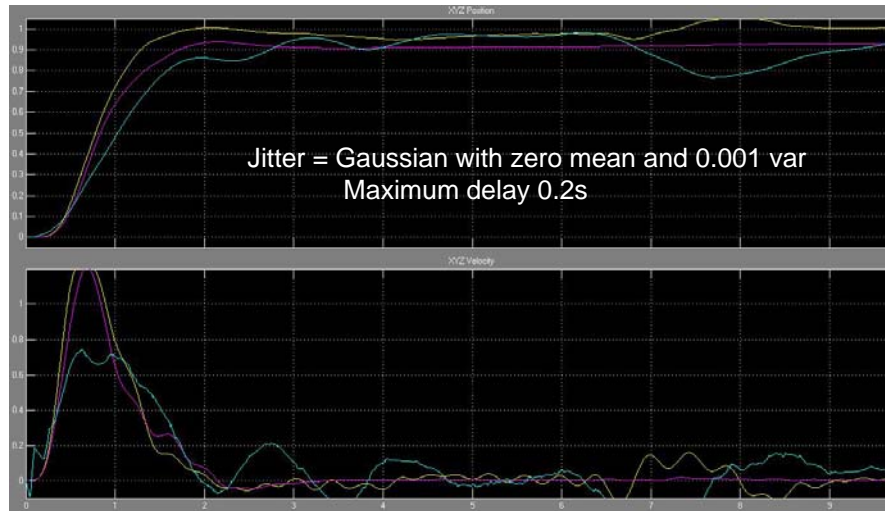
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STARMAC Step Response



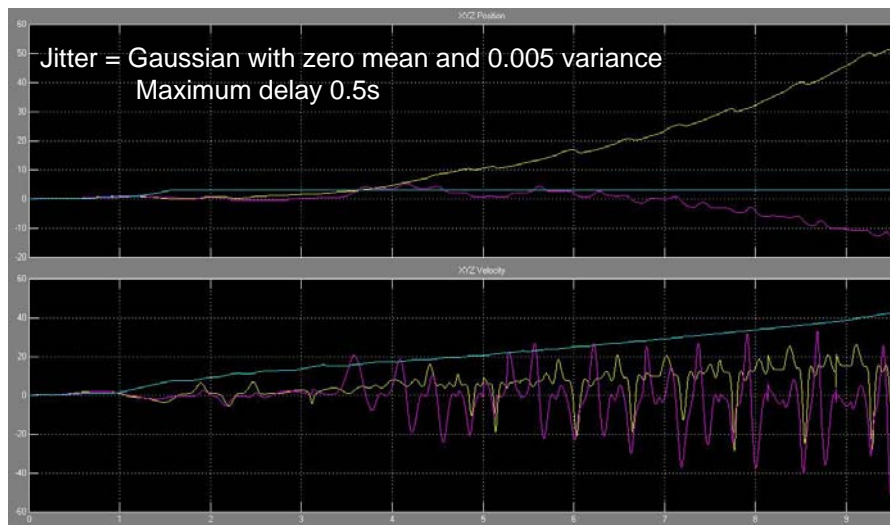
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STARMAC Response with Jitter



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STARMAC Response with Instability



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References – Timing Analysis & Simulation

- Lincoln, B. (2002), "A simple stability criterion for control systems with varying delays", In Proceedings of the 15th IFAC World Congress, Barcelona.
- Kao, C.-Y., & Lincoln, B. (2004), "Simple stability criteria for systems with time-varying delays", In Automatica 40 (2004), 1429-1434.
- Kao, C.-Y., & Rantzer, A. (2003). Stability criteria for systems with bounded uncertain time-varying delays. In Proceedings of the 2003 European Control Conference. Cambridge, UK.
- Gu, K., Niculescu S.I., "Survey on recent results in the stability and control of time-delay systems", Journal of Dynamic Systems, Measurement, and Control, vol. 125, pp. 158- 165, June 2003.
- Cervin, A., "How does control timing affect performance", IEEE Contr. Syst Mag., vol. 2, no. 2, pp. 16–30, June 2003.
- K. Gu, V. L. Kharitonov, and J. Chen, "Stability of Time-Delay Systems", 1st ed. Boston, MA: Birkhauser, 2003

Verification of Numerical Code

- **Numerical code verification**
- **Verification for target processors using polyhedra**
- **Widening for iterative computations**
- **Future work**

Issues in Numerical Code

- **numerical representations**
 - integer, fixed point, floating point
- **round-off error**
- **error accumulation**
- **divide-by-zero, overflow**
- **different results for different environments**
 - compiler (optimizations)
 - OS (exception handling)
 - processor (word length, instruction set)

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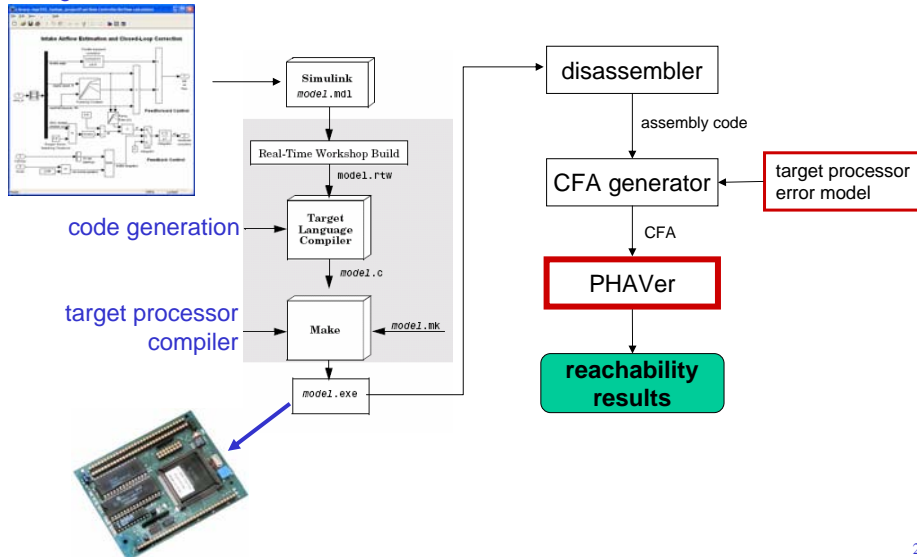
Verifying Numerical Code

- **error models**
 - interval arithmetic
 - ellipsoids
 - affine arithmetic
 - octagonal abstract domain (Cousot et al.)
- **control-flow automaton (CFA)**
 - states: precede program instructions (control locations)
 - edges: program guards (conditions)/actions (operations)
 - error model introduced in the actions
- **verification (static analysis)**
 - given initial ranges of input variables
 - propagate sets of variable values at each control location
 - check safety conditions

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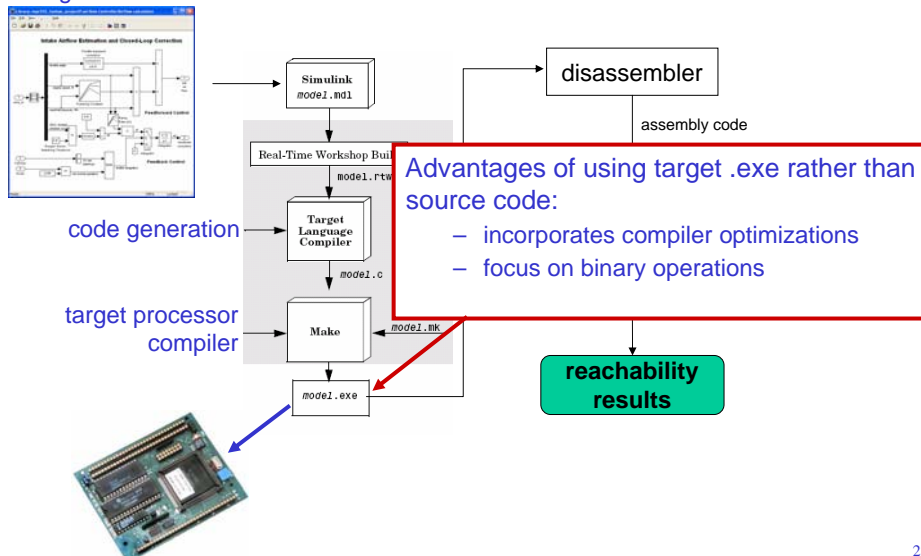
Verification for Target Processors

design model



Verification for Target Processors

design model



Verification for Target Processors

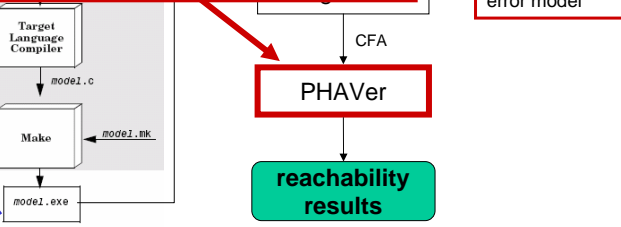
design model

Polyhedral reachability computations*

- full variable-space representation
- infinite precision implementation (PPL)
- currently for linear operations

code generation

target processor compiler



* G. Frehse, PHAVer: Algorithmic Verification of Hybrid Systems past HyTech, *Software Tools for Technology Transfer*, to appear

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Widening for Iterative Computations

For a lattice \mathbb{L} with preorder relation \sqsubseteq and join \sqcup ,
 $\nabla : \mathbb{L} \times \mathbb{L} \rightarrow \mathbb{L}$ is a **widening operator** if:

- For any $P, Q \in \mathbb{L}$, $P \sqcup Q \sqsubseteq P \nabla Q$.
 (overapproximation)
- For any increasing sequence $Q_1 \sqsubseteq Q_2 \sqsubseteq Q_3 \dots$ the increasing sequence defined by $P_0 = Q_0$ and $P_{i+1} = P_i \nabla Q_{i+1}$ is not strictly increasing. (termination)

Accelerates convergence to a fixed point.

P. Cousot and R. Cousot. Abstract interpretation: A unified lattice model for static analysis of programs by construction or approximation of fixpoints. In *Proceedings of the Fourth Annual ACM Symposium on Principles of Programming Languages*, pp. 238-252, New York, 1977. ACM Press.

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A New Widening Operator

For polyhedra P, Q represented as linear inequalities with integer coefficients

$$P \nabla_{CL} Q = \text{coflimit}(P \sqcup Q, k),$$

where for a polyhedron R , $\text{coflimit}(R, k)$ is a polyhedron that contains R with all coefficients with less than k bits.

- performs better than standard widening when reachable sets are contracting
- this is often the case in iterative numerical computations

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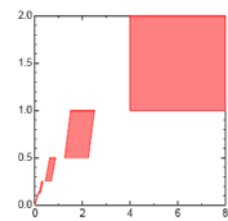
Example

Program 1

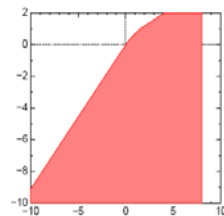
```

1:  $x \in [4, 8]$ 
2:  $y \in [1, 2]$ 
3: while true do
4:    $x = 0.25x + 0.25y$ 
5:    $y = 0.5y$ 
6: end while

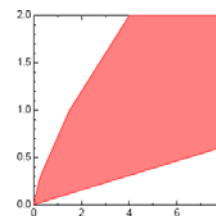
```



Reachable Region of Prog.1



Standard widening.



CL widening.

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Rate-Limiter Application*

Program 4

```

1: Y ← 0
2: while true do
3:   X ← [-128, 128]
4:   D ← [0, 16]
5:   S ← Y
6:   R ← X - S
7:   Y ← X
8:   if R ≤ -D then
9:     Y ← S - D
10:  else if D ≤ R then
11:    Y ← S + D
12:  end if
13: end while

```

- Y tries to follow the value of X with the step size of D.
- X and D change on each iteration
- Verify bounds on Y

Results:

- Using standard widening (Miné & Cousot) : $|Y| < 144$
- Using CL widening (Maka & Frehse) : $|Y| < 128.046$

* Antoine Miné. Relational abstract domains for the detection of floating-point run-time errors. In David A. Schmidt, editor, ESOP, volume 2986 of Lecture Notes in Computer Science, pages 3–17. Springer, 2004.

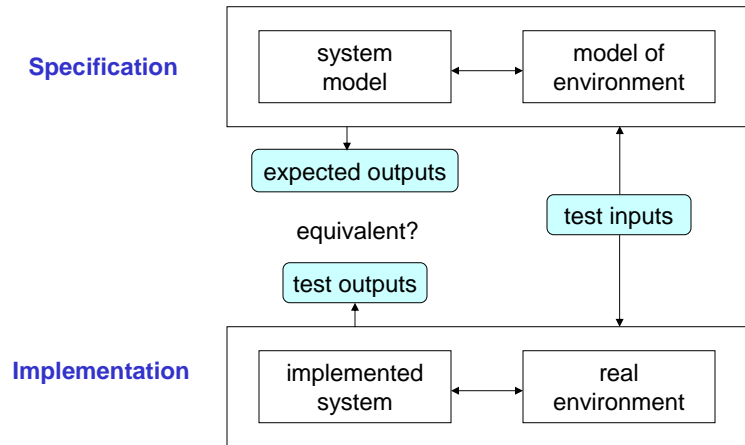
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Model-Based Test Generation

- Standard conformance testing (SCT)
- Testing environmental assumptions
- Mapping SCT to our problem
- Test generation for timed-automata
- Current implementation
- Next steps

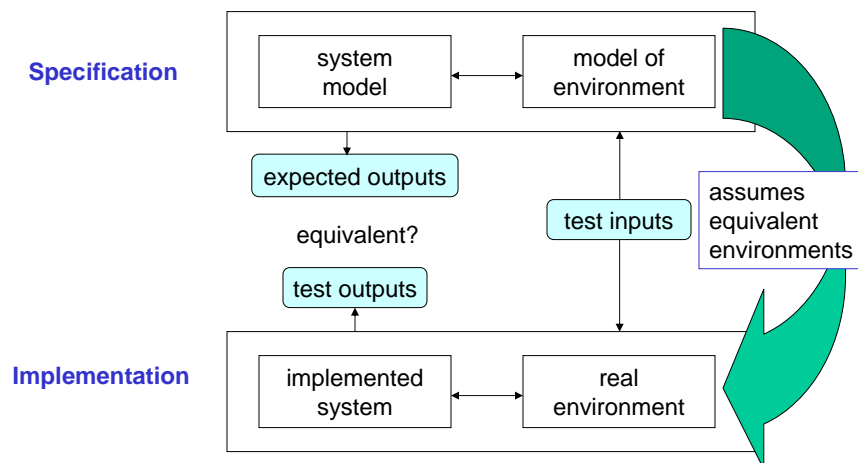
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Standard Conformance Testing



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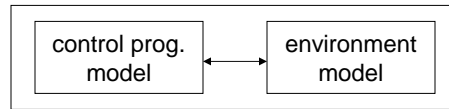
Conformance Testing



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Our Scenario

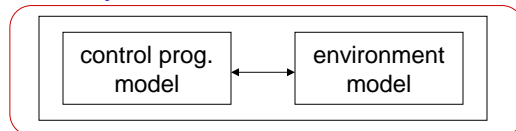
Model-based Development



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Our Scenario

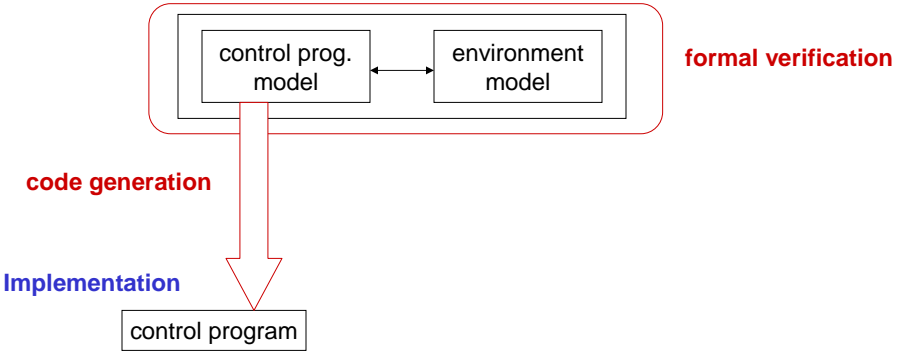
Model-based Development

**formal verification**

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Our Scenario

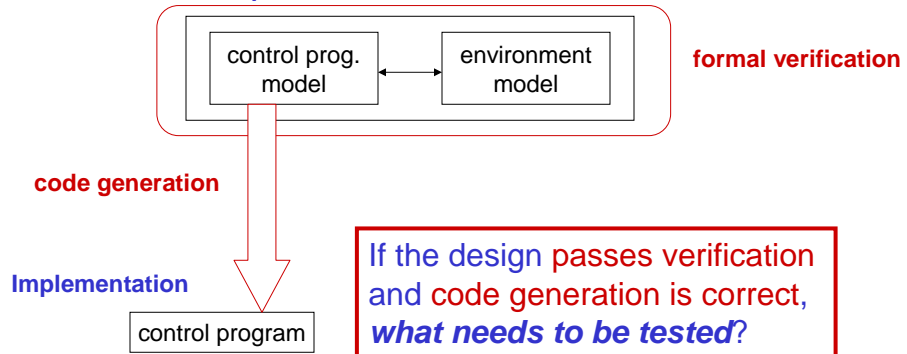
Model-based Development



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Our Scenario

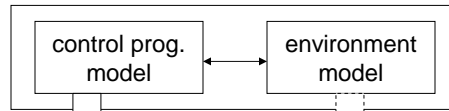
Model-based Development



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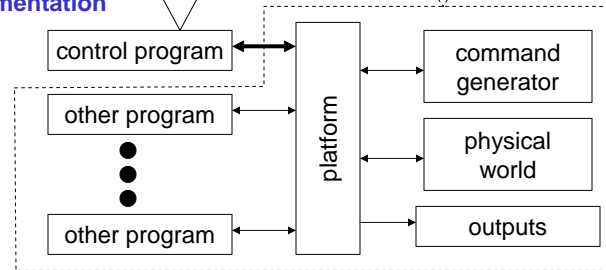
Our Scenario

Model-based Development



Is the real environment
"correct"?

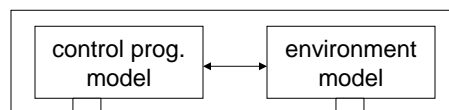
Implementation



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Testing Environmental Assumptions

Model-based Development

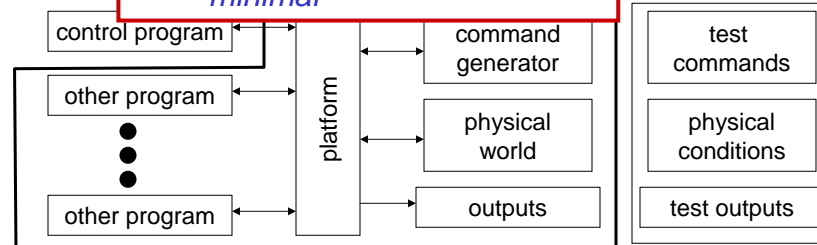


formal model

Goal: Generate a test suite that is

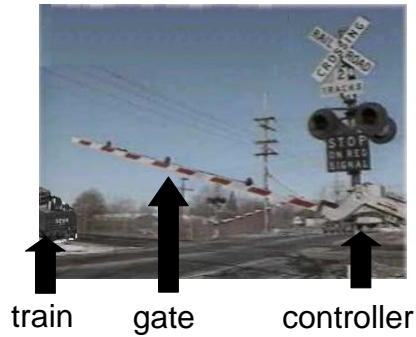
- complete
- sound
- *minimal*

Implementation



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Train Gate Example



Train-Gate Specification

- train sends *approach* signal
- controller sends *lower* command to gate
- train sends *exit* signal
- controller sends *raise* command

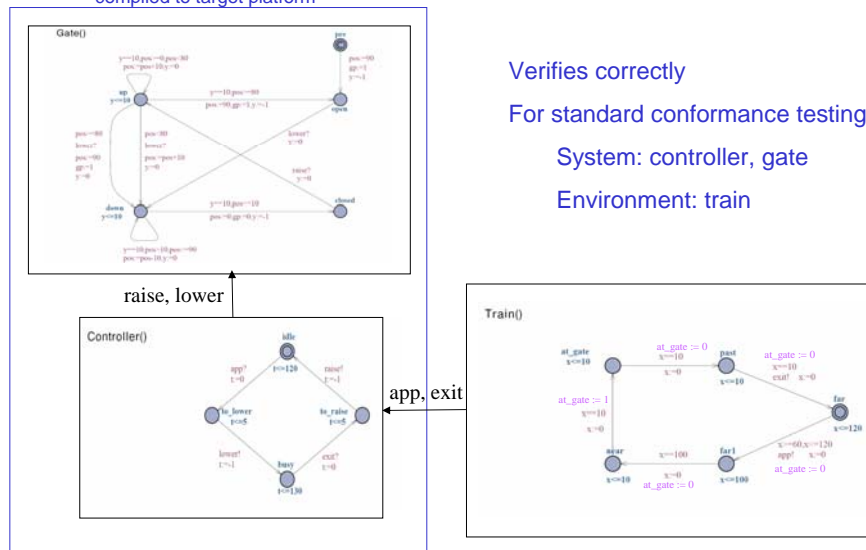
Safety Requirement

Gate must be down when a train is passing.

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Train Gate Timed I/O Automata Model

compiled to target platform



Verifies correctly

For standard conformance testing:

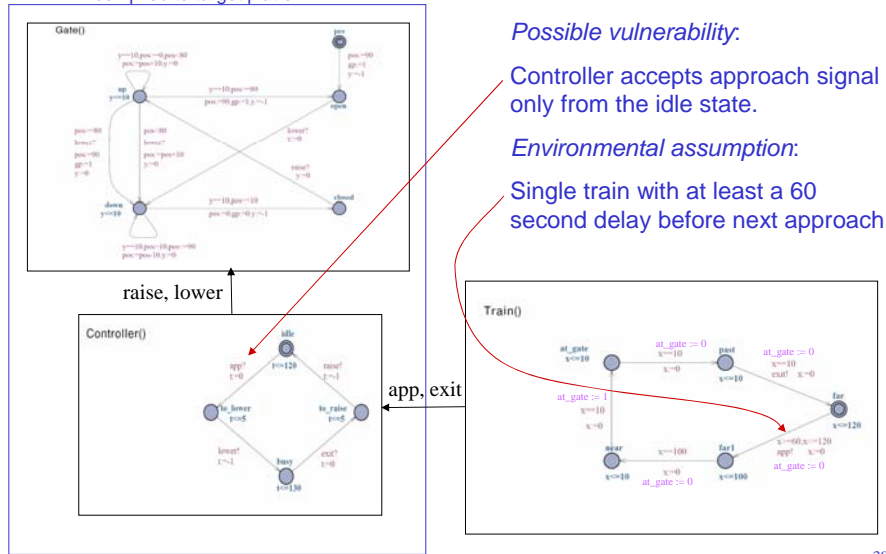
System: controller, gate

Environment: train

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Train Gate Timed I/O Automata Model

compiled to target platform



Possible vulnerability:

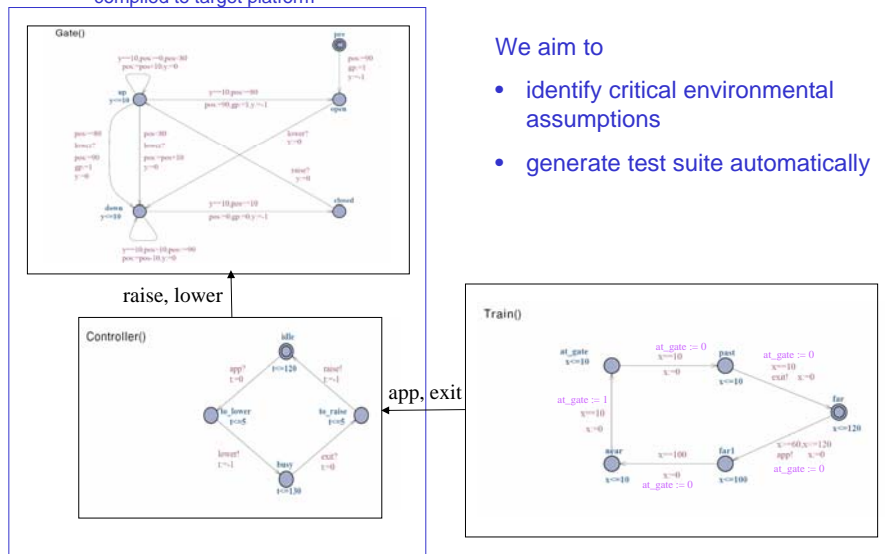
Controller accepts approach signal only from the idle state.

Environmental assumption:

Single train with at least a 60 second delay before next approach

Train Gate Timed I/O Automata Model

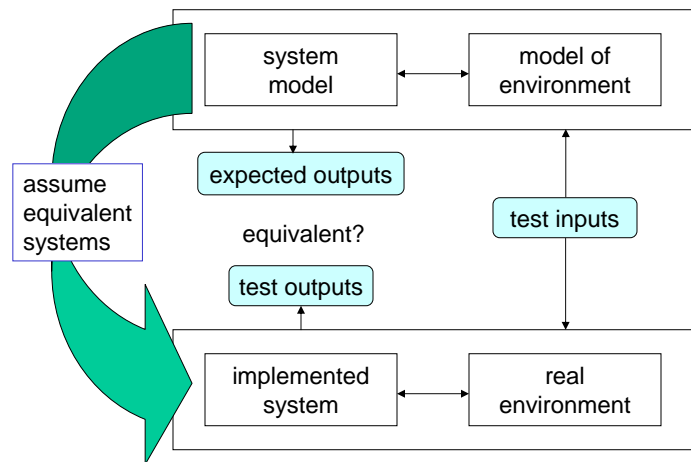
compiled to target platform



We aim to

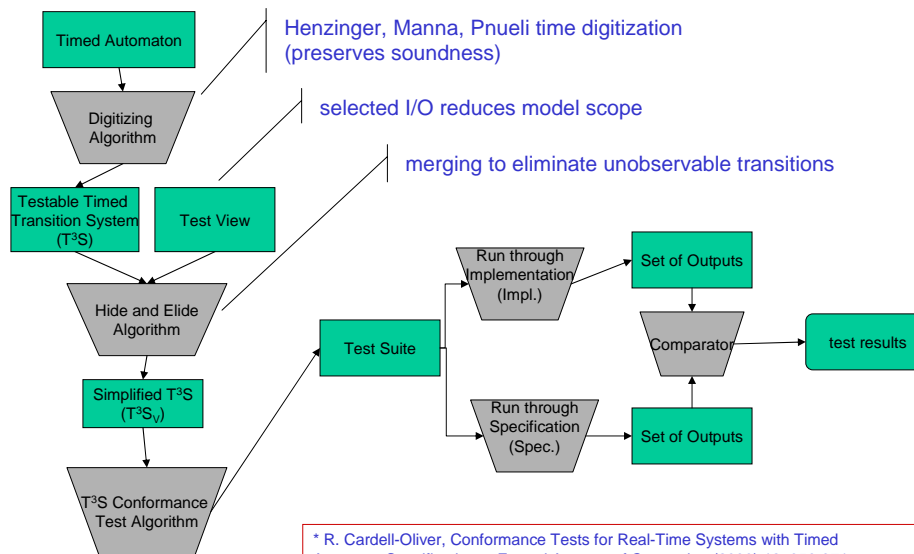
- identify critical environmental assumptions
- generate test suite automatically

Approach: Apply conformance testing to environmental assumptions



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Conformance Testing for TIOA*



* R. Cardell-Oliver, Conformance Tests for Real-Time Systems with Timed Automata Specifications, Formal Aspects of Computing (2000) 12: 350-371

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Tool Development

- **Specifications - UPPAAL TIOA**
- **XML interface to test generation program (C)**
- **Completed: digitization, test view hide/elide**
- **Currently debugging test suite generation**

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Model-Based Verification and Testing - Next Steps -

Timing in Networked Control Systems

- extensions to distributed systems
- develop analytical/simulation toolbox

Verification of Numerical Code

- extensions to nonlinear computations
- verification of Simulink/Stateflow designs

Testing Environmental Assumptions

- test suite reduction
- extensions to richer dynamics

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