# Expressions 

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## 1 Introduction

In Ptolemy II, models specify computations by composing actors. Many computations, however, are awkward to specify this way. A common situation is where we wish to evaluate a simple algebraic expression, such as " $\sin (2 \pi(x-1))$." It is possible to express this computation by composing actors in a block diagram, but it is far more convenient to give it textually. actions in state machines, and for the calculation performed by the Expression actor. In fact, the expression language is part of the generic infrastructure in Ptolemy II, and it can be used by programmers extending the Ptolemy II system. In this chapter, we describe how to use expressions from the perspective of a user rather than a programmer.

### 1.1 Expression Evaluator

Vergil provides an interactive expression evaluator, which is accessed through the File:New menu. This operates like an interactive command shell, and is shown in figure 1. It supports a command history. To access the previously entered expression, type the up arrow or Control-P. To go back, type the down arrow or Control-N. The expression evaluator is useful for experimenting with expressions.

## 2 Simple Arithmetic Expressions

### 2.1 Constants and Literals

The simplest expression is a constant, which can be given either by the symbolic name of the constant, or by a literal. By default, the symbolic names of constants supported are PI, pi, E, e,


Figure 1: The Expression Evaluator
true, false, i, j, NaN, Infinity, PositiveInfinity, NegativeInfinity, MaxUnsignedByte, MinUnsignedByte, MaxShort, MinShort, MaxInt, MinInt, MaxLong, MinLong, MaxFloat, MinFloat, MaxDouble, MinDouble. For example,

PI/2.0
is a valid expression that refers to the symbolic name "PI" and the literal " 2.0 ." The constants $i$ and $j$ are the imaginary number with value equal to the square root of -1 . The constant NaN is "not a number," which for example is the result of dividing 0.0/0.0. The constant Infinity is the result of dividing 1.0/0.0. The constants that start with "Max" and "Min" are the maximum and minimum values for their corresponding types.

Numerical values without decimal points, such as " 10 " or " -3 " are integers (type int). Numerical values with decimal points, such as " 10.0 " or " 3.14159 " are of type double. Numerical values followed by " f " or " F " are of type float. Numerical values without decimal points followed by the character "l" (el) or "L" are of type long. long. Numerical values without decimal points followed by the character " $s$ " or " S " are of type short. Unsigned integers followed by "ub" or "UB" are of type , unsignedByte, as in "5ub". An unsignedByte has a value between 0 and 255; note that it not quite the same as the Java byte, which has a value between -128 and 127. Numbers of type int, long, short or unsignedByte can be specified in decimal, octal, or hexadecimal. Numbers beginning with a leading " 0 " are octal numbers. Numbers beginning with a leading " 0 x " are hexadecimal numbers. For example, "012" and "0xA" are both equal to the integer 10.

A complex is defined by appending an " i " or a " j " to a double for the imaginary part. This gives a purely imaginary complex number which can then leverage the polymorphic operations in the Token classes to create a general complex number. Thus $2+3 i$ will result in the expected complex number. You can optionally write this $2+3 * i$.

Literal string constants are also supported. Anything between double quotes, "...", is interpreted as a string constant. The following built-in string-valued constants are defined:

Table 1: String-valued constants defined in the expression language

| Variable <br> name | Meaning | Property name | Example under Windows |
| :--- | :--- | :--- | :--- |
| PTII | The directory in <br> which Ptolemy II <br> is installed | ptolemy.ptII.dir | c: $\backslash$ tmp |
| HOME | The user home di- <br> rectory | user.home | c: $\backslash$ Documents and Settings $\backslash \backslash$ you |
| CWD | The current work- <br> ing directory | user.dir | c: $\backslash$ ptII |
| TMPDIR | The temporary di- <br> rectory | java.io.tmpdir | c:\Documents and Settings $\backslash \backslash$ you $\backslash$ Local Settings $\backslash$ Temp $\backslash$ |

The value of these variables is the value of the Java virtual machine property, such as user.home. The properties user. dir and user. home are standard in Java. Their values are platform dependent; see the documentation for the java.lang.System.getProperties() method for details. Note that user.dir and user.home are usually not readable in unsigned applets, in which case, attempts to use these variables in an expression will result in an exception. Vergil will display all the Java properties if you invoke JVM Properties in the View menu of a Graph Editor.

The ptolemy.ptII.dir property is set automatically when Vergil or any other Ptolemy II executable is started up. You can also set it when you start a Ptolemy II process using the java command by a syntax like the following:

```
java -Dptolemy.ptII.dir=${PTII} classname
```

where classname is the full class name of a Java application. The constants() utility function returns a record with all the globally defined constants. If you open the expression evaluator and invoke this function, you will see that its value is something like:

```
CLASSPATH = "xxxxxxCLASSPATHxxxxxx", CWD = "/Users/Ptolemy/ptII",
E = 2.718281828459, HOME = "/Users/cxh", Infinity = Infinity,
MaxDouble = 1.7976931348623E308, MaxFloat = 3.4028234663853E38,
MaxInt = 2147483647, MaxLong = 9223372036854775807L,
MaxShort = 32767s, MaxUnsignedByte = 255ub, MinDouble = 4.9E-324,
MinFloat = 1.4012984643248E-45, MinInt = -2147483648,
MinLong = -9223372036854775808L, MinShort = -32768s,
MinUnsignedByte = Oub, NaN = NaN, NegativeInfinity = -Infinity,
PI = 3.1415926535898, PTII = "/Users/Ptolemy/ptII",
```

```
PositiveInfinity = Infinity,
TMPDIR = "/var/folders/7f/7f-o2nyjFgewH67h0keKu++++TI/-Tmp-/",
boolean = false, complex = 0.0 + 0.0i, double = 0.0,
e = 2.718281828459, false = false, fixedpoint = fix(0,2,2),
float = 0.0f, general = present, i = 0.0 + 1.0i, int = 0,
j = 0.0 + 1.0i, long = OL, matrix = [], nil = nil,
null = object(null), object = object(null), pi = 3.1415926535898,
scalar = present, short = 0s, string = "", true = true,
unknown = present, unsignedByte = Oub, xmltoken = null
```


### 2.2 Variables

Expressions can contain identifiers that are references to variables within the scope of the expression. For example,

PI*x/2.0
is valid if " $x$ " a variable in scope. In the expression evaluator, the variables that are in scope include the built-in constants plus any assignments that have been previously made. For example,

```
>> x = pi/2
1.5707963267949
>> sin(x)
1.0
```

In the context of Ptolemy II models, the variables in scope include all parameters defined at the same level of the hierarchy or higher. So for example, if an actor has a parameter named "x" with value 1.0 , then another parameter of the same actor can have an expression with value " $\mathrm{PI}{ }^{*} \mathrm{x} / 2.0$ ", which will evaluate to $\pi / 2$.

Consider a parameter P in actor X which is in turn contained by composite actor Y . The scope of an expression for P includes all the parameters contained by X and Y , plus those of the container of Y , its container, etc. That is, the scope includes any parameters defined above in the hierarchy.

You can add parameters to actors (composite or not) by right clicking on the actor, selecting "Configure" and then clicking on "Add", or by dragging in a parameter from the utilities library. Thus, you can add variables to any scope, a capability that serves the same role as the "let" construct in many functional programming languages.

Occasionally, it is desirable to access parameters that are not in scope. The expression language supports a limited syntax that permits access to certain variables out of scope. In particular, if in place of a variable name x in an expression you write $\mathrm{A}:: \mathrm{x}$, then instead of looking for x in scope, the interpreter looks for a container named $A$ in the scope and a parameter named $x$ in $A$. This allows reaching down one level in the hierarchy from either the current container or any of its containers.

### 2.3 Operators

The arithmetic operators are $+,-, *, /,{ }_{\wedge}$ and $\%$. Most of these operators operate on most data types, including arrays, records, and matrices. The ôperator computes "to the power of" or exponentiation where the exponent can only be an int, short, or an unsignedByte.

The unsignedByte, short, int and long types can only represent integer numbers. Operations on these types are integer operations, which can sometimes lead to unexpected results. For instance, $1 / 2$ yields 0 if 1 and 2 are integers, whereas $1.0 / 2.0$ yields 0.5 . The exponentiation operator " $\stackrel{\wedge}{ }$ " when used with negative exponents can similarly yield unexpected results. For example, 2^-1 is 0 because the result is computed as $1 /(2 \hat{1})$.

The \% operation is a modulo or remainder operation. The result is the remainder after division. The sign of the result is the same as that of the dividend (the left argument). For example,

```
>> 3.0 % 2.0
1.0
>> -3.0 % 2.0
-1.0
>> -3.0 % -2.0
-1.0
>> 3.0 % -2.0
1.0
```

The magnitude of the result is always less than the magnitude of the divisor (the right argument). Note that when this operator is used on doubles, the result is not the same as that produced by the remainder() function (see Table 10). For instance,

```
>> remainder(-3.0, 2.0)
1.0
```

The remainder() function calculates the IEEE 754 standard remainder operation. It uses a rounding division rather than a truncating division, and hence the sign can be positive or negative, depending on complicated rules (see 8.0.4). For example, counter intuitively,

```
>> remainder(3.0, 2.0)
-1.0
```

When an operator involves two distinct types, the expression language has to make a decision about which type to use to implement the operation. If one of the two types can be converted without loss into the other, then it will be. For instance, int can be converted losslessly to double, so $1.0 / 2$ will result in 2 being first converted to 2.0 , so the result will be 0.5 . Among the scalar types, unsignedByte can be converted to anything else, short can be converted to int, int can be converted
to double, float can be converted to double and double can be converted to complex. Note that long cannot be converted to double without loss, nor vice versa, so an expression like 2.0 / 2 L yields the following error message:

```
Error evaluating expression "2.0/2L"
in .Expression.evaluator
Because:
divide method not supported between ptolemy.data.DoubleToken '2.0'
and ptolemy.data.LongToken '2L' because the types are incomparable.
```

Just as long cannot be cast to double, int cannot be cast to float and vice versa.
All scalar types have limited precision and magnitude. As a result of this, arithmetic operations are subject to underflow and overflow.

- For double numbers, overflow results in the corresponding positive or negative infinity. Underflow (i.e. the precision does not suffice to represent the result) will yield zero.
- For integer types and fixedpoint, overflow results in wraparound. For instance, while the value of MaxInt is 2147483647, the expression MaxInt + 1 yields -2147483648 . Similarly, while MaxUnsignedByte has value 255 ub , MaxUnsignedByte +1 ub has value 0ub. Note, however, that MaxUnsignedByte +1 yields 256, which is an int, not an unsignedByte. This is because MaxUnsignedByte can be losslessly converted to an int, so the addition is int addition, not unsignedByte addition.

The bitwise operators are \&, ।, \# and ~. They operate on boolean, unsignedByte, short, int and long (but not fixedpoint, float, double orcomplex). The operator $\&$ is bitwise AND, $\sim$ is bitwise NOT, and । is bitwise OR, and \# is bitwise XXOR ( exclusive or, after MATLAB).

The relational operators are $<,<=,>,>=,==$ and $!=$. They return type boolean. Note that these relational operators check the values when possible, irrespective of type. So, for example,

```
1 == 1.0
```

returns true. If you wish to check for equality of both type and value, use the equals() method, as in

```
>> 1.equals(1.0)
false
```

Boolean-valued expressions can be used to give conditional values. The syntax for this is

```
boolean ? value1 : value2
```

If the boolean is true, the value of the expression is value 1 ; otherwise, it is value2. The logical boolean operators are $\& \&,| |,!, \&$ and $\mid$. They operate on type boolean and return type boolean. The difference between logical $\& \&$ and and logical \& is that \& evaluates all the operands regardless of whether their value is now irrelevant. Similarly for logical \|। and \|. This approach is borrowed from Java. Thus, for example, the expression false $\& \& \times$ will evaluate to false irrespective of whether $x$ is defined. On the other hand, false \& $x$ will throw an exception.
The << and >> operators performs arithmetic left and right shifts respectively. The >>> operator performs a logical right shift, which does not preserve the sign. They operate on unsignedByte, short, int, and long.

### 2.4 Comments

In expressions, anything inside / * . . * / is ignored, so you can insert comments.

## 3 Uses of Expressions

### 3.1 Parameters

The values of most parameters of actors can be given as expressions. The variables in the expression refer to other parameters that are in scope, which are those contained by the same container or some container above in the hierarchy. They can also reference variables in a scope-extending attribute, which includes variables defining units, as explained below in section 12. Adding parameters to actors is straightforward, as explained in the previous chapter.

### 3.2 Port Parameters

It is possible to define a parameter that is also a port. Such a PortParameter provides a default value, which is specified like the value of any other parameter. When the corresponding port receives data, however, the default value is overridden with the value provided at the port. Thus, this object functions like a parameter and a port. The current value of the PortParameter is accessed like that of any other parameter. Its current value will be either the default or the value most recently received on the port.

A PortParameter might be contained by an atomic actor or a composite actor. To put one in a composite actor, drag it into a model from the utilities library, as shown in figure 2. The resulting icon is actually a combination of two icons, one representing the port, and the other representing the

[^0]

Figure 2: A portParameter is both a port and a parameter. To use it in a composite actor, drag it into the actor, change its name to something meaningful and set its default value.
parameter. These can be moved separately, but doing so might create confusion, so we recommend selecting both by clicking and dragging over the pair and moving both together.

To be useful, a PortParameter has to be given a name (the default name, "portParameter," is not very compelling). To change the name, right click on the icon and select "Customize Name," as shown in figure 2. In the figure, the name is set to "noiseLevel." Then set the default value by either double clicking or selecting "Configure." In the figure, the default value is set to 10.0.

An example of a library actor that uses a PortParameter is the Sinewave actor, which is found in the sources library in Vergil. It is shown in figure 3. If you double click on this actor, you can set the default values for frequency and phase. But both of these values can also be set by the corresponding ports, which are shown with grey fill.

### 3.3 String Parameters

Some parameters have values that are always strings of characters. Such parameters support a simple string substitution mechanism where the value of the string can reference other parameters in scope by name using the syntax \$name, where name is the name of the parameter in scope. For example, the StringCompare actor in figure 4 has as the value of firstString"The answer is $\$ \mathrm{PI}$ ". This references the built-in constant PI. The value of secondString is "The answer is 3.1415926535898 ". As shown in the figure, these two strings are deemed to be equal because \$PI is replaced with the value of PI.


Figure 3: Sinewave actor, showing its port parameters, and their use at the lower level of hierarchy


Figure 4: String parameters are indicated in the parameter editor boxes by a light blue background. A string parameter can include references to variables in scope with \$name, where name is the name of the variable. In this example, the built-in constant $\$$ PI is referenced by name in the first parameter.

### 3.4 Expression Actor

Expression actor is a particularly useful actor found in the math library. By default, it has one output and no inputs, as shown in Figure 53.5(a). The first step in using it is to add ports, as shown in (b) and (c), resulting in a new icon as shown in (d). Note: In (c) when you click on Add, you will be prompted for a Name (pick one) and a Class. Leave the Class entry blank and click OK. You then specify an expression using the port names, as shown in (e), resulting in the icon shown in (f).

### 3.5 State Machines

Expressions give the guards for state transitions, as well as the values used in actions that produce outputs and actions that set values of parameters in the refinements of destination states. This mechanism was explained in the previous chapter.

## 4 Composite Data Types

### 4.1 Arrays

Arrays are specified with curly brackets, e.g., " $\{1,2,3\}$ " is an array of int, while "\{"x", "y", "z"\}" is an array of string. The types are denoted "\{int\}" and "\{string\}" respectively. An array is an ordered list of tokens of any type, with the only constraint being that


Figure 5: Illustration of the Expression actor.
the elements all have the same type. If an array is given with mixed types, the expression evaluator will attempt to losslessly convert the elements to a common type. Thus, for example,
$\{1,2.3\}$
has value

```
{1.0, 2.3}
```

Its type is $\{$ double $\}$. The common type might be scalar, which is a union type (a type that can contain multiple distinct types). For example,

```
{1, 2.3, true}
```

has value

```
{1, 2.3, true}
```

The value is unchanged, although the type of the array is now scalar.
The elements of the array can be given by expressions, as in the example " $\{2 \star \mathrm{pi}, 3 \star \mathrm{pi}\}$." Arrays can be nested; for example, " $\{\{1,2\},\{3,4,5\}\} "$ is an array of arrays of integers. The elements of an array can be accessed as follows:

```
>> {1.0, 2.3}(1)
2.3
```

which yields 2.3. Note that indexing begins at 0 . Of course, if name is the name of a variable in scope whose value is an array, then its elements may be accessed similarly, as shown in this example:

```
>> x = {1.0, 2. 3}
{1.0, 2.3}
>> x(0)
1.0
```

Arithmetic operations on arrays are carried out element-by-element, as shown by the following examples:

```
>> {1, 2}*{2, 2}
{2, 4}
```

```
>> {1, 2}+{2, 2}
{3, 4}
>> {1, 2}-{2, 2}
{-1, 0}
>> {1, 2}\^2
{1, 4}
>> {1, 2}\%{2, 2}
{1, 0}
```

Addition, subtraction, multiplication, division, and modulo of arrays by scalars is also supported, as in the following examples:

```
>> {1.0, 2.0} / 2.0
{0.5, 1.0}
>> 1.0 / {2.0, 4.0}
{0.5, 0.25}
>> 3*{2, 3}
{6, 9}
>> 12 / {3, 4}
{4, 3}
```

Arrays of length 1 are equivalent to scalars, as illustrated below:

```
>> {1.0, 2.0} / {2.0}
{0.5,1.0}
>> {1.0} / {2.0, 4.0}
{0.5, 0.25}
>> {3} * {2, 3}
{6, 9}
>> {12} / {3, 4}
{4, 3}
```

A significant subtlety arises when using nested arrays. Note the following example

```
>>{{1.0, 2.0},{3.0, 1.0}} / {0.5, 2.0}
{{2.0, 4.0}, {1.5,0.5}}
```

In this example, the left argument of the divide is an array with two elements, and the right argument is also an array with two elements. The divide is thus elementwise. However, each division is the division of an array by a scalar. An array can be checked for equality with another array as follows:
$\gg\{1,2\}==\{2,2\}$

```
false
>> {1, 2}!={2, 2}
true
```

For other comparisons of arrays, use the compare() function (see Table 10). As with scalars, testing for equality using the $==$ or $!=$ operators tests the values, independent of type. For example,

```
>> {1, 2}=={1.0, 2.0}
true
```

You can extract a subarray by invoking the subarray() method as follows:

```
>> {1, 2, 3, 4}.subarray(2, 2)
{3, 4}
```

The first argument is the starting index of the subarray, and the second argument is the length.
You can also extract non-contiguous elements from an array using the extract() method. This method has two forms. The first form takes a boolean array of the same length as the original array which indicates which elements to extract, as in the following example:

```
>> {'`red'','`green'', '`blue''}.extract({true,false,true})
{'`red'', '`blue''}
```

The second form takes an array of integers giving the indices to extract, as in the following example:

```
>> {'`red'', ''green'', '`blue''}.extract({2,0,1,1})
{``blue'', '`red'', '`green'', '`green''}
```

You can create an empty array with a specific element type using the emptyArray() function. For example, to create an empty array of integers, use:

```
>> emptyArray(int)
{}
```

You can combine arrays into a single array using the concatenate() function. For example,

```
>> concatenate({1, 2}, {3})
{1, 2, 3}
```


### 4.2 Matrices

In Ptolemy II, arrays are ordered sets of tokens. Ptolemy II also supports matrices, which are more specialized than arrays. They contain only certain primitive types, currently boolean, complex, double, fixedpoint, int, and long. Currently float, short and unsignedByte matrices are not supported. Matrices cannot contain arbitrary tokens, so they cannot, for example, contain matrices. They are intended for data intensive computations. Matrices are specified with square brackets, using commas to separate row elements and semicolons to separate rows. E.g., " $[1,2,3 ; 4,5,5+1]$ " gives a two by three integer matrix ( 2 rows and 3 columns). Note that an array or matrix element can be given by an expression. A row vector can be given as " $[1,2,3]$ " and a column vector as " $[1 ; 2$; $3]$ ". Some MATLAB-style array constructors are supported. For example, "[1:2:9]" gives an array of odd numbers from 1 to 9 , and is equivalent to " $[1,3,5,7,9]$." Similarly, " $[1: 2: 9 ; 2: 2: 10]$ " is equivalent to " $[1,3,5,7,9 ; 2,4,6,8,10]$." In the syntax " $[\mathrm{p}: \mathrm{q}: \mathrm{r}]$ ", $p$ is the first element, $q$ is the step between elements, and $r$ is an upper bound on the last element. That is, the matrix will not contain an element larger than $r$. If a matrix with mixed types is specified, then the elements will be converted to a common type, if possible. Thus, for example, " $[1.0,1]$ " is equivalent to " $[1.0,1.0]$," but " $[1.0,1 \mathrm{~L}]$ " is illegal (because there is no common type to which both elements can be converted losslessly).

Reference to elements of matrices have the form "matrix( $n, m$ )" or "name $(n, m)$ " where name is the name of a matrix variable in scope, $n$ is the row index, and $m$ is the column index. Index numbers start with zero, as in Java, not 1 , as in MATLAB. For example,

```
>> [1, 2; 3, 4] (0,0)
1
>> a = [1, 2; 3, 4]
[1, 2; 3, 4]
>> a(1,1)
4
```

Matrix multiplication works as expected. For example, as seen in the expression evaluator (see figure 1),

```
>> [1, 2; 3, 4]*[2, 2; 2, 2]
[6, 6; 14, 14]
```

Of course, if the dimensions of the matrix don't match, then you will get an error message. To do element wise multiplication, use the multipyElements() function (see Table 10). Matrix addition and subtraction are element wise, as expected, but the division operator is not supported. Element wise division can be accomplished with the divideElements() function, and multiplication by a matrix inverse can be accomplished using the inverse() function (see Table 10). A matrix can be raised to an int, short or unsignedByte power, which is equivalent to multiplying it by itself some number of times. For instance,

```
> [3, 0; 0, 3]\^3
[27, 0; 0, 27]
```

A matrix can also be multiplied or divided by a scalar, as follows:

```
>> [3, 0; 0, 3]*3
[9, 0; 0, 9]
```

A matrix can be added to a scalar. It can also be subtracted from a scalar, or have a scalar subtracted from it. For instance,

```
>> 1-[3, 0; 0, 3]
[-2, 1; 1, -2]
```

A matrix can be checked for equality with another matrix as follows:

```
>> [3, 0; 0, 3]!=[3, 0; 0, 6]
true
>> [3, 0; 0, 3]==[3, 0; 0, 3]
true
```

For other comparisons of matrices, use the compare() function (see Table 10). As with scalars, testing for equality using the $==$ or $!=$ operators tests the values, independent of type. For example,

```
>> [1, 2]==[1.0, 2.0]
true
```

To get type-specific equality tests, use the equals() method, as in the following examples:

```
>> [1, 2].equals([1.0, 2.0])
false
>> [1.0, 2.0].equals([1.0, 2.0])
true
```


### 4.3 Records

A record token is a composite type containing named fields, where each field has a value. The value of each field can have a distinct type. Records are delimited by curly braces, with each field given a name. For example, " $\left\{a=1, b=^{\prime}\right.$ foo' ' \}" is a record with two fields, named "a" and "b", with values 1 (an integer) and "foo" (a string), respectively. The value of a field can be an arbitrary expression, and records can be nested (a field of a record token may be a record token).

Ordered records behave similarly to normal records except that they preserve the original ordering of the labels rather than alphabetizing them. Ordered records are delimited using square brackets rather than curly braces. For example, $[b=" f \circ \circ ", a=1]$ is an ordered record token in which 'b' will remain the first label.

Fields may be accessed using the period operator. For example,

$$
\{a=1, b=2\} \cdot a
$$

yields 1 . You can optionally write this as if it were a method call:

$$
\{a=1, b=2\} \cdot a()
$$

The arithmetic operators $+,-*^{*}, /$, and $\%$ can be applied to records. If the records do not have identical fields, then the operator is applied only to the fields that match, and the result contains only the fields that match. Thus, for example,

```
{foodCost=40, hotelCost=100} + {foodCost=20, taxiCost=20}
```

yields the result

```
{foodCost=60}
```

You can think of an operation as a set intersection, where the operation specifies how to merge the values of the intersecting fields. You can also form an intersection without applying an operation. In this case, using the intersect() function, you form a record that has only the common fields of two specified records, with the values taken from the first record. For example,

```
>> intersect({a=1, c=2}, {a=3, b=4})
{a=1}
```

Records can be joined (think of a set union) without any operation being applied by using the merge() function. This function takes two arguments, both of which are record tokens. If the two record tokens have common fields, then the field value from the first record is used. For example,

```
merge({a=1, b=2}, {a=3, c=3})
```

yields the result $\{a=1, b=2, c=3\}$.
Records can be compared, as in the following examples:

```
>> {a=1,b=2}!={a=1,b=2}
false
>> {a=1, b=2}!={a=1, c=2}
true
```

Note that two records are equal only if they have the same field labels and the values match. As with scalars, the values match irrespective of type. For example:

```
>> {a=1,b=2}=={a=1.0, b=2.0+0.0i}
true
```

The order of the fields is irrelevant for normal (unordered) records. Hence

```
>> {a=1, b=2}=={b=2, a=1}
true
```

Moreover, normal record fields are reported in alphabetical order, irrespective of the order in which they are defined. For example,

```
>> {b=2,a=1}
{a=1, b=2 }
```

Equality comparisons for ordered records respect the original order of the fields. For example,

```
>> [a=1, b=2]==[b=2, a=1]
false
```

Additionally, ordered record fields are always reported in the order in which they are defined. For example,

```
>> [b=2, a=1]
[b=2, a=1]
```

To get type-specific equality tests, use the equals() method, as in the following examples:

```
>> {a=1, b=2}.equals({a=1.0, b=2.0+0.0i})
false
>> {a=1, b=2}.equals({b=2, a=1})
true
```

Finally, You can create an empty record using the emptyRecord() function:

```
>> emptyRecord()
{}
```


## 5 Invoking Methods

Every element and subexpression in an expression represents an instance of the Token class in Ptolemy II (or more likely, a class derived from Token). The expression language supports invocation of any method of a given token, as long as the arguments of the method are of type Token and the return type is Token (or a class derived from Token, or something that the expression parser can easily convert to a token, such as a string, double, int, etc.). The syntax for this is (token ).methodName (args), where methodName is the name of the method and args is a comma-separated set of arguments. Each argument can itself be an expression. Note that the parentheses around the token are not required, but might be useful for clarity. As an example, the ArrayToken and RecordToken classes have a length() method, illustrated by the following examples:

```
{1, 2, 3}.\index{length()}
{a=1, b=2, c=3}.length()
```

each of which returns the integer 3 .
The MatrixToken classes have three particularly useful methods, illustrated in the following examples:

```
[1, 2; 3, 4; 5, 6].\index{getRowCount()}
```

which returns 3, and

```
[1, 2; 3, 4; 5, 6].getColumnCount()
```

which returns 2 , and

```
[1, 2; 3, 4; 5, 6].toArray()
```

which returns $1,2,3,4,5,6$. The latter function can be particularly useful for creating arrays using MATLAB-style syntax. For example, to obtain an array with the integers from 1 to 100 , you can enter:

```
[1:1:100].toArray()
```


## 6 Casting

The cast function can be used to explicitly cast a value into a type.

When the cast function is invoked with cast (type, value), where type is the target type and value is the value to be cast, a new value is returned (if a predefined casting is applicable) that is in the specified type. For example, cast (long, 1) yields 1L, which is equal to 1 but is in the long data type, and cast (string, 1) yields " 1 ", which is in the string data type.

### 6.1 Object Types

An object token encapsulates a Java object. Methods defined in the Java class of that object can be invoked in an expression. For example, in a model that contains an actor named C, C in an expression may refer to that actor in an object token.

An object token has a type, which is an object type that is specific for the class of the encapsulated Java object or any class that is a superclass of that class. For example, with C being a Const actor, expression C is of an object type that is specific to the Java class ptolemy.actor.lib.Const.

An object type specific to Java class A can be specified with object ("A"), and its value is null. Comparison between object tokens is by reference with the Java objects that they encapsulate. Therefore, object ("A") == object ("B") is always true, because the values in both tokens are null.

### 6.2 Relationship between Object Types

An object type A is more specific than object type B if the Java class represented by A is a subclass of that represented by B.

For example, object ("ptolemy.actor.TypedIOPort") is more specific than object ("ptolemy.actor. and object("ptolemy.kernel.Port").

The most general object type is object (without any argument). Conceptually it encapsulates a null class (which, of course, does not exist in Java). The most specific object type is ob ject ("ptolemy . data.type (which is not very useful in practice). The family of object types forms a lattice.

### 6.3 Object Tokens

Object tokens are tokens of object types. A predefined object token is null, which has null as the value and the null class as the Java class. Its type is ob ject. It is special in that it can be cast into any object type. For example, you can cast it into a port with cast (object ("ptolemy.kernel.Port"), null). If you enter this in the expression evaluator, you shall see the stringification of the token as "object(null: ptolemy.kernel.Port)".
(Notice that the string here is not a valid Ptolemy expression. In fact, most object tokens do not have string representations that are valid expressions, and therefore, they cannot be stored permanently
in a Ptolemy model.)
Except for null, for a Ptolemy expression that evaluates to an object token, the Java class represented by that token's type is always the most specific class. For example, if C is a Const actor, then $C$ in an expression refers to an object token that has actor $C$ as its value, and ob ject ("ptolemy. actor. lib. Const as its type. You can cast this type into a more general actor type by doing cast (object ("ptolemy. actor. Actor C).

### 6.4 Casting between Object Types

An object token can be cast into a different object type, as long as the target object type represents a Java class that the encapsulated object is in. That class need not always be a superclass of the class that the object type represents.

For example, again let C be a Const actor. As discussed, the following expression casts it into a more general actor type:

```
cast(object("ptolemy.actor.Actor"), C)
```

The result of the cast is another object token with C as its value and object ("ptolemy.actor.Actor") as its type. That token can be casted back into one of a more specific object type:

```
cast(object("ptolemy.actor.TypedAtomicActor"),
cast(object("ptolemy.actor.Actor"), C))
```

This is valid because the value C is in any of the mentioned classes.
As mentioned, null is a special object token that can be cast into any object type. Any object token can also be cast into the most general object type, which is ob ject. The only object that can be cast into the most specific object type, object ("ptolemy.data.type. ObjectType\$BottomClass"), is null.

### 6.5 Method Invocation

Native Java methods may be invoked on the objects encapsulated in object tokens.
For example, if C is a Const actor, C.portList () returns a list of its ports. The returned list itself is a Java object in the class java.util.List, so it is encapsulated in an object token. You may further invoke C.portList().isEmpty () to test whether the list is empty. In that case, the isEmpty method is invoked on the returned list. The isEmpty method returns a Java boolean value, and the boolean type corresponds to the Ptolemy boolean data type, so the value is converted into the latter type.

## 7 Defining Functions

The expression language supports definition of functions. The syntax is:

```
function(arg1:Type, arg2:Type...)
    function body
```

where function' is the keyword for defining a function. The type of an argument can be left unspecified, in which case the expression language will attempt to infer it. The function body gives an expression that defines the return value of the function. The return type is always inferred based on the argument type and the expression. For example:

```
function(x:double) x*5.0
```

defines a function that takes a double argument, multiplies it by 5.0, and returns a double. The return value of the above expression is the function itself. Thus, for example, the expression evaluator yields:

```
>> function(x:double) x*5.0
(function(x:double) (x*5.0))
```

To apply the function to an argument, simply do

```
>> (function(x:double) x*5.0) (10.0)
50.0
```

Alternatively, in the expression evaluator, you can assign the function to a variable, and then use the variable name to apply the function. For example,

```
>> f = function(x:double) x*5.0
(function(x:double) (x*5.0))
>> f(10)
50.0
```

Functions can be passed as arguments to certain higher-order functions" that have been defined (see Table 19) . For example, the iterate() function takes three arguments, a function, an integer, and an initial value to which to apply the function. It applies the function first to the initial value, then to the result of the application, then to that result, collecting the results into an array whose length is given by the second argument. For example, to get an array whose values are multiples of 3, try

```
>> iterate(function(x:int) x+3, 5, 0)
{0, 3, 6, 9, 12}
```



Figure 6: Example of a function being passed from one actor to another

The function given as an argument simply adds three to its argument. The result is the specified initial value ( 0 ) followed by the result of applying the function once to that initial value, then twice, then three times, etc.

Another useful higher-order function is the map() function. This one takes a function and an array as arguments, and simply applies the function to each element of the array to construct a result array. For example,

```
>> map(function(x:int) x+3, {0, 2, 3})
{3, 5, 6}
```

A typical use of functions in a Ptolemy II model is to define a parameter in a model whose value is a function. Suppose that the parameter named $f$ has value function ( $x:$ double) $x * 5.0$. Then within the scope of that parameter, the expression $f(10.0)$ will yield result 50.0 .

Functions can also be passed along connections in a Ptolemy II model. Consider the model shown in figure 6. In that example, the Const actor defines a function that simply squares the argument. Its output, therefore, is a token with type function. That token is fed to the " f " input of the Expression actor. The expression uses this function by applying it to the token provided on the " $y$ " input. That token, in turn, is supplied by the Ramp actor, so the result is the curve shown in the plot on the right.

A more elaborate use is shown in figure 7 In that example, the Const actor produces a function, which is then used by the Expression actor to create new function, which is then used by Expression2 to perform a calculation. The calculation performed here adds the output of the Ramp to the square of the output of the Ramp.
Functions can be recursive, as illustrated by the following (rather arcane) example:

```
>> fact = function(x:int,f:(function(x,f) int)) (x<1?1:x*f(x-1,f))
(function(x:int, f:function(a0:general, a1:general) int) (x<1)?1:(x*f((x-1), f)))
>> factorial = function(x:int) fact(x,fact)
(function(x:int) (function(x:int, f:function(a0:general, a1:general) int)
    (x<1) ?1:(x*f((x-1), f))) (x, (function(x:int,
```



Figure 7: More elaborate example with functions passed between actors

```
f:function(a0:general, a1:general) int) (x<1)?1:(x*f((x-1), f)))))
>> map(factorial, [1:1:5].toArray())
{1, 2, 6, 24, 120}
```

The first expression defines a function named "fact" that takes a function as an argument, and if the argument is greater than or equal to 1 , uses that function recursively. The second expression defines a new function "factorial" using "fact." The final command applies the factorial function to an array to compute factorials.

## 8 Built-In Functions

The expression language includes a set of functions, such as $\sin (), \cos ()$, etc. The functions that are built in include all static methods of the classes shown in Table 2, which together provide a rich set . The functions currently available are shown in the tables in the appendix, which also show the argument types and return types.

In most cases, a function that operates on scalar arguments can also operate on arrays and matrices. Thus, for example, you can fill a row vector with a sine wave using an expression like
$\sin ([0.0: P I / 100: 1.0])$

Or you can construct an array as follows,

[^1]```
\(\sin (\{0.0,0.1,0.2,0.3\})\)
```

Functions that operate on type double will also generally operate on int, short, or unsignedByte, because these can be losslessly converted to double, but not generally on long or complex. Tables of available functions are shown in the appendix. For example, Table 4 shows trigonometric functions. Note that these operate on double or complex, and hence on int, short and unsignedByte, which can be losslessly converted to double. The result will always be double. For example,

```
>> cos(0)
1.0
```

These functions will also operate on matrices and arrays, in addition to the scalar types shown in the table, as illustrated above. The result will be a matrix or array of the same size as the argument, but always containing elements of type double

Table 10 shows other arithmetic functions beyond the trigonometric functions. As with the trigonometric functions, those that indicate that they operate on double will also work on int, short and unsignedByte, and unless they indicate otherwise, they will return whatever they return when the argument is double. Those functions in the table that take scalar arguments will also operate on matrices and arrays. For example, since the table indicates that the max() function can take int, int as arguments, then by implication, it can also take int, int. For example,

```
>> max({1, 2}, {2, 1})
{2, 2}
```

Notice that the table also indicates that max() can take int as an argument. E.g.

```
>> max({1, 2, 3})
3
```

Table 2: The classes whose static methods are available as functions in the expression langauge

| java.lang.Math | ptolemy.math.IntegerMatrixMath |
| :--- | :--- |
| java.lang.Double | ptolemy.math.DoubleMatrixMath |
| java.lang.Integer | ptolemy.math.ComplexMatrixMath |
| java.lang.Long | ptolemy.math.LongMatrixMath |
| java.lang.String | ptolemy.math.IntegerArrayMath |
| ptolemy.data.MatrixToken. | ptolemy.math.DoubleArrayStat |
| ptolemy.data.RecordToken. | ptolemy.math.ComplexArrayMath |
| ptolemy.data.expr.UtilityFunctions | ptolemy.math.LongArrayMath |
| ptolemy.data.expr.FixPointFunctions | ptolemy.math.SignalProcessing |
| ptolemy.math.Complex | ptolemy.math.FixPoint |
| ptolemy.math.ExtendedMath | ptolemy.data.ObjectToken |

In the former case, the function is applied pointwise to the two arguments. In the latter case, the returned value is the maximum over all the contents of the single argument.

Table 10 shows functions that only work with matrices, arrays, or records (that is, there is no corresponding scalar operation). Recall that most functions that operate on scalars will also operate on arrays and matrices. Table 11 shows utility functions for evaluating expressions given as strings or representing numbers as strings. Of these, the eval() function is the most flexible.

A few of the functions have sufficiently subtle properties that they require further explanation. That explanation is here.

### 8.0.1 eval() and traceEvaluation()

The built-in function eval() will evaluate a string as an expression in the expression language. For example,
eval("[1.0, 2.0; 3.0, 4.0]")
will return a matrix of doubles. The following combination can be used to read parameters from a file:

```
eval(readFile("filename"))
```

where the filename can be relative to the current working directory (where Ptolemy II was started, as reported by the property user.dir), the user's home directory (as reported by the property user.home), or the classpath, which includes the directory tree in which Ptolemy II is installed. Note that if eval() is used in an Expression actor, then it will be impossible for the type system to infer any more specific output type than general. If you need the output type to be more specific, then you will need to cast the result of eval(). For example, to force it to type double:

```
>> cast(double, eval("pi/2"))
1.5707963267949
```

The traceEvaluation() function evaluates an expression given as a string, much like eval(), but instead of reporting the result, reports exactly how the expression was evaluated. This can be used to debug expressions, particularly when the expression language is extended by users.

### 8.0.2 random(), gaussian()

random() and gaussian() shown in Table 10 return one or more random numbers. With the minimum number of arguments (zero or two, respectively), they return a single number. With one additional
argument, they return an array of the specified length. With a second additional argument, they return a matrix with the specified number of rows and columns.

There is a key subtlety when using these functions in Ptolemy II. In particular, they are evaluated only when the expression within which they appear is evaluated. The result of the expression may be used repeatedly without re-evaluating the expression. Thus, for example, if the value parameter of the Const actor is set to random (), then its output will be a random constant, i.e., it will not change on each firing. The output will change, however, on successive runs of the model. In contrast, if this is used in an Expression actor, then each firing triggers an evaluation of the expression, and consequently will result in a new random number.

### 8.0.3 property()

The property() function accesses system properties by name. Some possibly useful system properties are:

- ptolemy.ptII.dir: The directory in which Ptolemy II is installed.
- ptolemy.ptII.dirAsURL: The directory in which Ptolemy II is installed, but represented as a URL.
- user.dir: The current working directory, which is usually the directory in which the current executable was started.


### 8.0.4 remainder()

This function computes the remainder operation on two arguments as prescribed by the IEEE 754 standard, which is not the same as the modulo operation computed by the $\%$ operator. The result of remainder $(\mathrm{x}, \mathrm{y})$ is $x-y n$, where $n$ is the integer closest to the exact value of $x / y$. If two integers are equally close, then $n$ is the integer that is even. This yields results that may be surprising, as indicated by the following examples:

```
>> remainder(1,2)
1.0
>> remainder (3,2)
-1.0
```

Compare this to

```
>> 3%2
1
```

which is different in two ways. The result numerically different and is of type int, whereasremainder () always yields a result of type double. The remainder () function is implemented by the java.lang.Math class, which calls it IEEEremainder(). The documentation for that class gives the following special cases:

- If either argument is NaN , or the first argument is infinite, or the second argument is positive zero or negative zero, then the result is NaN .
- If the first argument is finite and the second argument is infinite, then the result is the same as the first argument.


### 8.0.5 DCT() and IDCT()

The DCT function can take one, two, or three arguments. In all three cases, the first argument is an array of length $N>0$ and the DCT returns an

$$
\begin{equation*}
X_{k}=s_{k} \sum_{n=0}^{N-1} x_{n} \cos \left((2 n+1) k \frac{\pi}{2 D}\right) \tag{1}
\end{equation*}
$$

for $k$ from 0 to $D-1$, where $N$ is the size of the specified array and $D$ is the size of the DCT. If only one argument is given, then $D$ is set to equal the next power of two larger than $N$. If a second argument is given, then its value is the order of the DCT, and the size of the DCT is $2^{\text {order }}$. If a third argument is given, then it specifies the scaling factors $s_{k}$ according to the following table:

Table 3: Normalization options for the DCT function

| Name | Third argument | Normalization |
| :--- | :--- | :--- |
| Normalized | 0 | $s_{k}= \begin{cases}\frac{1}{\sqrt{2}} ; & k=0 \\ 1, & \text { otherwise }\end{cases}$ |
| Unnormalized | 1 | $s_{k}=1$ |
| Orthonormal | 2 | $s_{k}= \begin{cases}\frac{1}{7} \sqrt{D} ; & k=0 \\ \sqrt{\frac{2}{D}} ; & \text { otherwise }\end{cases}$ |

The default, if a third argument is not given, is "Normalized." The IDCT function is similar, and can also take one, two, or three arguments. The formula in this case is

$$
\begin{equation*}
x_{n}=\sum_{k=0}^{N-1} s_{k} X_{k} \cos \left((2 n+1) k \frac{\pi}{2 D}\right) \tag{2}
\end{equation*}
$$

## 9 Folding

Ptolemy II supports a fold function, which can be used to program a loop in an expression.
The fold function is invoked in the form of $f o l d(f, x, l)$, where $f$ is a function itself, $x$ is the starting value, and $l$ is a collection of values.

- f must take two arguments. x must be valid as the first argument, and any element in collection 1 must be valid as the second argument. Moreover, the return value of $f$ must also be valid as the first argument.
- x is the starting value used to invoke f the first time by the fold function. It can be in any type supported by Ptolemy II.
- I must be either an array in the form of " $\{1,2,3, \ldots\}$," or a Java collection obtained from a Java method call such as "C.output.connectedPortList()" (assuming C is a Const actor here).

The pseudocode for computing the result of the fold function $f o l d(f, x, l)$ is as follows:

1. let $\mathrm{y}=\mathrm{x}$
2. for each element e in 1
3. let $y=f(y, e)$
4. return y

## Examples:

```
- fold(
    function(x : int, e : int) x + 1,
    0, {1, 2, 3}
    )
```

This computes the length of array $\{1,2,3\}$. The result is 3 , which is equal to $\{1,2$, 3\}.length(). Function $f$ here is defined with anonymous function function ( $x$ : int, $e$ : int) $x+1$. Given $x$ and arbitrary element $e$, it returns $x+1$. It is invoked the number of times equal to the number of elements in array $\{1,2,3\}$. Therefore, $x$ is increased 3 times from the starting value 0 .

- fold (
function(x : int, $e$ : int) $x+e$, $0,\{1,2,3\}$
)
This computes the sum of all elements in array $\{1,2,3\}$.
- fold(
function(x : arrayType(int), e : int) e \% $2=0$ ? $x$ : x.append $(\{e\})$,
$\},\{1,2,3,4,5\}$
)
This computes a subarray of array $\{1,2,3,4,5\}$ that contains only odd numbers. The result is $\{1,3,5\}$.
- Let C be an actor.

```
fold(
    function(list : arrayType(string),
        port : object("ptolemy.kernel.Port"))
        port.connectedPortList().isEmpty() ?
            list.append({port}) : list,
    {}, C.portList()
)
```

This returns a list of C's ports that are not connected to any other port (with connectedPortList () being empty). Each port in the returned list is encapsulated in an ObjectToken.

## 10 Nil Tokens

Null or missing tokens are common in analytical systems like R and SAS where they are used to handle sparsely populated data sources. In database parlance, missing tokens are sometimes called null tokens. Since null is a Java keyword, we use the term "nil". Nil tokens are useful for analyzing real world data such as temperature where the value was not measured during every interval. In principle, an as yet unimplemented function such as average() method could properly handle nil tokens - when the average() method sees a nil token, it should be ignored. Note that this can lead to uncertainty. For example, if average() is expecting 30 values and 29 of them are nil, then the average will not be very accurate.

If an operation such as add(), divide(), modulo(), multiply(), one(), subtract(), zero() or their corresponding "reverse" operations includes a nil token, then the output is nil. If one of the arguments for isCloseTo() or isEqualTo() is nil, then the method returns false. Methods that return a nil token return a nil token with a specific type so that type safety is preserved. The following tokens have NIL values defined: ArrayToken, BooleanToken, ComplexToken, DoubleToken, FloatToken IntToken, LongToken, ShortToken, StringToken, Token, UnsignedByteToken. There is no nil token for the various matrix tokens because the underlying matrices are java native type matrices that do not support nil.

The expression language defines a constant named nil that refers to the Token.NIL field. The cast() expression language function can be used to generate references to the NIL fields of the other
classes. For example, "cast(int, nil)" will return a reference to the IntToken.NIL field.

## 11 Fixed Point Numbers

Ptolemy II includes a preliminary fixed point data type. We represent a fixed point value in the expression language using the following format:

```
fix(value, totalBits, integerBits)
```

Thus, a fixed point value of 5.375 that uses 8 bit precision of which 4 bits are used to represent the (signed) integer part can be represented as:

```
fix(5.375, 8, 4)
```

The value can also be a matrix of doubles. The values are rounded, yielding the nearest value representable with the specified precision. If the value to represent is out of range, then it is saturated, meaning that the maximum or minimum fixed point value is returned, depending on the sign of the specified value. For example,

```
fix(5.375, 8, 3)
```

will yield 3.968758 , the maximum value possible with the $(8 / 3)$ precision.
In addition to the fix() function, the expression language offers a quantize() function. The arguments are the same as those of the fix() function, but the return type is a DoubleToken or DoubleMatrixToken instead of a FixToken or FixMatrixToken. This function can therefore be used to quantize double-precision values without ever explicitly working with the fixed-point representation.
To make the FixToken accessible within the expression language, the following functions are available:

- To create a single FixPoint Token using the expression language:

```
fix(5.34, 10, 4)
```

This will create a FixToken. In this case, we try to fit the number 5.34 into a 10 bit representation with 4 bits used in the integer part. This may lead to quantization errors. By default the round quantizer is used.

- To create a Matrix with FixPoint values using the expression language:

```
fix([ -.040609, -.001628, .17853 ], 10, 2)
```

This will create a FixMatrixToken with 1 row and 3 columns, in which each element is a FixPoint value with precision(10/2). The resulting FixMatrixToken will try to fit each element of the given double matrix into a 10 bit representation with 2 bits used for the integer part. By default the round quantizer is used.

- To create a single DoubleToken, which is the quantized version of the double value given, using the expression language:

```
quantize(5.34, 10, 4)
```

This will create a DoubleToken. The resulting DoubleToken contains the double value obtained by fitting the number 5.34 into a 10 bit representation with 4 bits used in the integer part. This may lead to quantization errors. By default the round quantizer is used.

- To create a Matrix with doubles quantized to a particular precision using the expression language:

```
quantize([ -.040609, -.001628, .17853 ], 10, 2)
```

This will create a DoubleMatrixToken with 1 row and 3 columns. The elements of the token are obtained by fitting the given matrix elements into a 10 bit representation with 2 bits used for the integer part. Instead of being a fixed point value, the values are converted back to their double representation and by default the round quantizer is used.

## 12 Units

Ptolemy II supports units systems, which are built on top of the expression language. Units systems allow parameter values to be expressed with units, such as " $1.0 * \mathrm{~cm}$ ", which is equal to " $0.01 *$ meters". These are expressed this way (with the * for multiplication) because "cm" and "meters" are actually variables that become in scope when a units system icon is dragged in to a model. A few simple units systems are provided (mainly as examples) in the utilities library.

A model using one of the simple provided units systems is shown in figure 8 This unit system is called BasicUnits; the units it defines can be examined by double clicking on its icon, or by invoking Configure, as shown in figure 9. In that figure, we see that "meters", "meter", and "m" are defined, and are all synonymous. Moreover, "cm" is defined, and given value " 0.01 *meters", and "in", "inch" and "inches" are defined, all with value " $2.54 * \mathrm{~cm}$ ".

In the example in figure 8 , a constant with value " 1.0 * meter" is fed into a Scale actor with scale factor equal to " $2.0 / \mathrm{ms}$ ". This produces a result with dimensions of length over time. If we feed this result directly into a Display actor, then it is displayed as " 2000.0 meters/seconds", as shown in figure 10, top display. The canonical units for length are meters, and for time are seconds.


Figure 8: Example of a model that includes a unit system.


Figure 9: Units defined in a units system can be examined by invoking Configure on its icon.


Figure 10: Result of running the model in figure 8


Figure 11: Example of an exception resulting from a units mismatch.

In figure 8 , we also take the result and feed it to the InUnitsOf actor, which divides its input by its argument, and checks to make sure that the result is unitless. This tells us that 2 meters $/ \mathrm{ms}$ is equal to about 78,740 inches/second.

The InUnitsOf actor can be used to ensure that numbers are interpreted correctly in a model, which can be effective in catching certain kinds of critical errors. For example, if in figure 8, we had entered "seconds/inch" instead of "inches/second" in the InUnitsOf actor, we would have gotten the exception in figure 11 instead of the execution in figure 10.

Units systems are built entirely on the expression language infrastructure in Ptolemy II. The units system icons actually represent instances of scope-extending attributes, which are attributes whose parameters are in scope as if those parameters were directly contained by the container of the scopeextending attribute. That is, scope-extending attributes can define a collection of variables and constants that can be manipulated as a unit. In version 2.0 of Ptolemy II, two fairly extensive units systems are provided, CGSUnitBase and ElectronicUnitBase. Nonetheless, these are intended as examples only, and can no doubt be significantly improved and extended.

## 13 Functions

In this appendix, we tabulate the functions available in the expression language. Further explanation of many of these functions is given in section section 8 above.

The argument and return types are the widest type that can be used. For example, acos() will take any argument that can be losslessly cast to a double, such as unsigned byte, short, integer, float. Long cannot be cast losslessly cast to double, so acos(1L) will fail.

## 13.A Trigonometric Functions

Table 4: Trigonometric functions.

| function | argument type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| acos | double in the range [-1.0, 1.0] or complex | double in the range [0.0, pi] or NaN if out of range or complex | arc cosine <br> complex case: $\operatorname{acos}(z)=-i \log (z+$ $\left.i \sqrt{i-z^{2}}\right)$ |
| asin | double in the range [-1.0, 1.0] or complex | double in the range [-pi/2, pi/2] or NaN if out of range or complex | arc sine complex case: $\operatorname{asin}(z)=-i \log (i z+$ $\left.\sqrt{i-z^{2}}\right)$ |
| atan | double or complex | double in the range [-pi/2, pi/2] or complex | arc tangent complex case: $\operatorname{atan}(z)=-\frac{i}{2} \log \frac{i-z}{i+z}$ |
| $\operatorname{atan} 2$ | double, double | double in the range [-pi, pi] | angle of a vector (note: the arguments are ( $\mathrm{y}, \mathrm{x}$ ), not $(\mathrm{x}, \mathrm{y})$ as one might expect). |
| acosh | double greater than 1 or complex | double or complex | hyperbolic arc cosine, defined for both double and complex case by: $\operatorname{acosh}(z)=\log z+\sqrt{z^{2}-1}$ |
| asinh | double or complex | double or complex | hyperbolic arc sine complex case: $\operatorname{asinh}(z)=\log z+\sqrt{z^{2}+1}$ |
| $\cos$ | double or complex | double in the range , or complex | cosine complex case: $\cos (z)=\frac{\exp (i z)+\exp (-i z)}{2}$ |
| cosh | double or complex | double or complex | hyperbolic cosine, defined for double or complex by: $\cosh (z)=$ $\frac{\exp (z)+\exp (-z)}{2}$ |
| $\sin$ | double or complex | double or complex | sine function complex case: $\sin (z)=$ $\frac{\exp (i z)-\exp (-i z)}{2 i}$ |
| sinh | double or complex | double or complex | hyperbolic sine, defined for double or complex by: $\sinh (z)=\frac{\exp (z)-\exp (-z)}{2}$ |
| $\tan$ | double or complex | double or complex | tangent function, defined for double or complex by: $\tan (z)=\frac{\sin (z)}{\cos (z)}$ |
| tanh | double or complex | double or complex | hyperbolic tangent, defined for double or complex by: $\tanh (z)=\frac{\sinh (z)}{\cosh (z)}$ |

## 13.B Basic Mathematical Functions

Table 5: Basic mathematical functions, part 1.

| function | argument type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| abs | double or complex | double or int or long (complex returns double) | absolute value complex case: $a b s(a+i b)=$ $\|z\|=\sqrt{a^{2}+b^{2}}$ |
| angle | complex | double in the range [-pi, pi] | angle or argument of the complex number: $\angle z$ |
| ceil | double or float | double | ceiling function, which returns the smallest (closest to negative infinity) double value that is not less than the argument and is an integer. |
| compare | double, double | int | compare two numbers, returning $-1,0$, or 1 if the first argument is less than, equal to, or greater than the second. |
| conjugate | complex | complex | complex conjugate |
| exp | double or complex | double in the range [0.0, infinity] or complex | exponential function  <br> (eârgument) complex case: <br> $e^{a+i b}=e^{a}(\cos b+i \sin b)$   |
| floor | double | double | floor function, which is the largest (closest to positive infinity) value not greater than the argument that is an integer. |
| gaussian | double, double or double, double, int, or double, double, int, int | double or double or [double] | one or more Gaussian random variables with the specified mean and standard deviation (see 8.0.2). |
| imag | complex | double | imaginary part |
| isInfinite | double | boolean | return true if the argument is infinite |
| isNaN | double | boolean | return true if the argument is "not a number" |
| $\log$ | double or complex | double or complex | natural logarithm complex case: $\log (z)=\log (a b s(z)+i \angle z)$ |
| $\log 10$ | double | double | $\log$ base 10 |
| $\log 2$ | double | double | $\log$ base 2 |

Table 6: Basic mathematical functions, part 2.

| function | argument type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| max | double, double or double | a scalar of the same type as the arguments | maximum |
| min | double, double or double | a scalar of the same type as the arguments | minimum |
| pow | double, double or complex, complex | double or complex | first argument to the power of the second |
| random | no arguments or int or int, int | double or double or [double] | one or more random numbers between 0.0 and 1.0 (see 8.0.2) |
| real | complex | double | real part |
| remainder | double, double | double | remainder after division, according to the IEEE 754 floatingpoint standard (see 8.0.4. |
| round | double | long | round to the nearest long, choosing the next greater integer when exactly in between, and throwing an exception if out of range. If the argument is NaN , the result is 0 L . If the argument is out of range, the result is either MaxLong or MinLong, depending on the sign. |
| roundToInt | double | int | round to the nearest int, choosing the next greater integer when exactly in between, and throwing an exception if out of range. If the argument is NaN , the result is 0 . If the argument is out of range, the result is either MaxInt or MinInt, depending on the sign. |
| sgn | double | int | -1 if the argument is negative, 1 otherwise |
| sqrt | double or complex | double or complex | square root. If the argument is double with value less than zero, then the result is NaN . complex case: $\operatorname{sqrt}(z)=\sqrt{\|z\|}\left(\cos \frac{\frac{L z}{2}}{2}+\right.$ $\left.i \sin \frac{\angle z}{2}\right)$ |

Table 7: Basic mathematical functions, part 3.

| function | argument type(s) | return type | description |
| :--- | :--- | :--- | :--- |
| toDegrees | double | double | convert radians to degrees |
| toRadians | double | double | convert degrees to radians |
| within | type, type, double | boolean | return true if the first argument <br> is in the neighborhood of the <br> second, meaning that the dis- <br> tance is less than or equal to <br> the third argument. The first <br> two arguments can be any type <br> for which such a distance is de- <br> fined. For composite types, ar- <br> rays, records, and matrices, then <br> return true if the first two argu- <br> ments have the same structure, <br> and each corresponding element <br> is in the neighborhood. |

## 13.C Matrix, Array, and Record Function.

Table 8: Functions that take or return matrices, arrays, or records, part 1.

| function | argument <br> type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| arrayToMatrix | type, int, int | [type] | Create a matrix from the specified array with the specified number of rows and columns |
| concatenate | type, type | type | Concatenate two arrays. |
| concatenate | type | type | Concatenate arrays in an array of arrays. |
| conjugateTranspose | [complex] | [complex] | Return the conjugate transpose of the specified matrix. |
| createSequence | type, type, int | type | Create an array with values starting with the first argument, incremented by the second argument, of length given by the third argument. |
| crop | [int], int, int, int, int or [double], int, int, int, int or [complex], int, int, int, int or [long], int, int, int, int or | [int] or [double] or [complex] or [long] or | Given a matrix of any type, return a submatrix starting at the specified row and column with the specified number of rows and columns. |
| determinant | [double] or [complex] [complex] | double or complex | Return the determinant of the specified matrix. |
| diag | type | [type] | Return a diagonal matrix with the values along the diagonal given by the specified array. |
| divideElements | [type], [type] | [type] | Return the element-by-element division of two matrices |
| emptyArray | type | type | Return an empty array whose element type matches the specified token. |
| emptyRecord |  | record | Return an empty record. |
| find | type, type | int | Return an array of the indices where elements of the specified array match the specified token. |

## 13.D Functions for Evaluating Expressions

Table 9: Functions that take or return matrices, arrays, or records, part 2.

| function | argument type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| find | boolean | int | Return an array of the indices where elements of the specified array have value true. |
| hilbert | int | [double] | Return a square Hilbert matrix, where $A_{i j}=\frac{1}{i+j+1}$. A Hilbert matrix is nearly, but not quite singular. |
| identityMatrixComplex | int | [complex] | Return an identity matrix with the specified dimension. |
| identityMatrixDouble | int | [double] | Return an identity matrix with the specified dimension. |
| identityMatrixInt | int | [int] | Return an identity matrix with the specified dimension. |
| identityMatrixLong | int | [long] | Return an identity matrix with the specified dimension. |
| intersect | record, record | record | Return a record that contains only fields that are present in both arguments, where the value of the field is taken from the first record. |
| inverse | [double] or [complex] | [double] or [complex] | Return the inverse of the specified matrix, or throw an exception if it is singular. |
| matrixToArray | [type] | type | Create an array containing the values in the matrix |
| merge | record, record | record | Merge two records, giving priority to the first one when they have matching record labels. |
| multiplyElements | [type], [type] | [type] | Multiply element wise the two specified matrices. |
| orthogonalizeColumns | [double] or [complex] | [double] or [complex] | Return a similar matrix with orthogonal columns. |
| orthogonalizeRows | [double] or [complex] | [double] or [complex] | Return a similar matrix with orthogonal rows. |
| orthonormalizeColumns | [double] or [complex] | [double] or [complex] | Return a similar matrix with orthonormal columns. |
| orthonormalizeRows | [double] or [complex] | [double] or [complex] | Return a similar matrix with orthonormal rows. |
| repeat | int, type | type | Create an array by repeating the specified token the specified number of times. |

Table 10: Functions that take or return matrices, arrays, or records, part 3.

| function | argument <br> type(s) | return type | description |
| :--- | :--- | :--- | :--- |
| sort | stringor <br> realScalar | string or re- <br> alScalar | Return the specified array, but sorted <br> in ascending order. realScalar is any <br> scalar token except complex. |
| sortAscending | stringor <br> realScalar | string or re- <br> alScalar | Return the specified array, but sorted <br> in ascending order. realScalar is any <br> scalar token except complex. |
| sortDescending | stringor <br> realScalar | string or re- <br> alScalar | Return the specified array, but sorted <br> in descending order. realScalar is any <br> scalar token except complex. |
| subarray | type, int, int | type | Extract a subarray starting at the speci- <br> fied index with the specified length. |
| sum | type] | type | Sum the elements of the specified array <br> or matrix. This throws an exception if <br> the elements do not support addition or <br> if the array is empty (an empty matrix <br> will return zero). |
| [type] | type | [type] | Return the trace of the specified matrix. <br> Return the transpose of the specified <br> matrix. |
| trace | [type] | [complex] | Return a zero matrix with the specified <br> number of rows and columns. |
| transpose | [double] | Return a zero matrix with the specified <br> number of rows and columns. |  |
| zeroMatrixComplex | int, int | Return a zero matrix with the specified <br> number of rows and columns. |  |
| zeroMatrixInt | int, int | [int] | Return a zero matrix with the specified <br> number of rows and columns. |
| zeroMatrixLong | int, int | [long] |  |

Table 11: Utility functions for evaluating expressions

| function | argument <br> type(s) | return type | description |
| :--- | :--- | :--- | :--- |
| eval | string | any type | evaluate the specified expression (see <br> $8.0 .1)$. |
| parseInt | string or string, <br> int | int | return an int read from a string, using <br> the given radix if a second argument is <br> provided. |
| parseLong | string or string, <br> int | int | return a long read from a string, using <br> the given radix if a second argument is <br> provided. |
| toBinaryString | int or long | string | return a binary representation of the ar- <br> gument |
| toOctalString | int or long | string | return an octal representation of the ar- <br> gument |
| toString | double or int or <br> int, int or long <br> or long, int | string | return a string representation of the ar- <br> gument, using the given radix if a sec- <br> ond argument is provided. |
| traceEvaluation | string | string | evaluate the specified expression and <br> report details on how it was evaluated <br> (see 8.0.1). |

## 13.E Signal Processing Functions

Table 12: Functions performing signal processing operations, part 1.
$\left.\left.\begin{array}{|l|l|l|l|}\hline \text { function } & \text { argument type(s) } & \text { return type } & \text { description } \\ \hline \hline \text { close } & \text { double, double } & \text { boolean? } & \begin{array}{l}\text { Return true if the first argument } \\ \text { is close to the second (within } \\ \text { EPSILON, where EPSILON is } \\ \text { a static public variable of this } \\ \text { class). }\end{array} \\ \hline \text { convolve } & \begin{array}{l}\text { double, double or } \\ \text { complex, complex }\end{array} & \begin{array}{l}\text { double or } \\ \text { complex }\end{array} & \begin{array}{l}\text { Convolve two arrays and return } \\ \text { an array whose length is sum of } \\ \text { the lengths of the two arguments } \\ \text { minus one. Convolution of two } \\ \text { arrays is the same as polynomial } \\ \text { multiplication. }\end{array} \\ \hline \text { DCT } & \begin{array}{l}\text { double or double, } \\ \text { int or double, int, } \\ \text { int }\end{array} & \text { double } & \begin{array}{l}\text { Return the discrete cosine trans- } \\ \text { form of the specified array, using } \\ \text { the specified (optional) length } \\ \text { and normalization strategy (see } \\ 8.0 .5) .\end{array} \\ \hline \text { downsample } & \begin{array}{l}\text { double, int or dou- } \\ \text { ble, int, int }\end{array} & \text { double } & \begin{array}{l}\text { Return a new array with every } n- \\ \text { th element of the argument array, } \\ \text { where } n \text { is the second argument. }\end{array} \\ \text { If a third argument is given, then } \\ \text { it must be between 0 and } n-1, \\ \text { and it specifies an offset into the } \\ \text { array (by giving the index of the } \\ \text { first output). }\end{array} \right\rvert\, \begin{array}{l}\text { Return the fast Fourier transform } \\ \text { of the specified array. If the sec- } \\ \text { ond argument is given with value } \\ n, \text { then the length of the trans- } \\ \text { form is 2 }{ }^{n} \text {. Otherwise, the length } \\ \text { is the next power of two greater } \\ \text { than or equal to the length of the } \\ \text { input array. If the input length } \\ \text { does not match this length, then } \\ \text { input is padded with zeros. }\end{array}\right\}$

Table 13: Functions performing signal processing operations, part 2.

| function | argument type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| generateBartlettWindow | int | double | Bartlett ( rectangular) window with the specified length. The end points have value 0.0 , and if the length is odd, the center point has value 1.0. For length $\mathrm{M}+1$, the formula is: $w(n)=$ $\begin{cases}2 \frac{n}{M} ; & \text { if } 0 \leq n \leq \frac{M}{2} \\ 2-2 \frac{n}{M} ; & \text { if } \frac{M}{2} \leq n \leq M\end{cases}$ |
| generateBlackmanWindow | int | double | Return a Blackman window with the specified length. For length $\mathrm{M}+1$, the formula is: $w(n)=$ $0.42+0.5 \cos \frac{2 \pi n}{M}+0.08 \cos \frac{4 \pi n}{M}$ |
| generateBlackmanHarrisWindow | int | double | Return a Blackman-Harris window with the specified length. For length $\mathrm{M}+1$, the formula is: $w(n)=0.35875+$ $0.48829 \cos$ frac $2 \pi n M+$ $0.14128 \cos \frac{4 \pi n}{M}$ $0.01168 \cos \frac{6 \pi n}{M}$ |
| generateGaussianCurve | double, double, int | double | Return a Gaussian curve with the specified standard deviation, extent, and length. The extent is a multiple of the standard deviation. For instance, to get 100 samples of a Gaussian curve with standard deviation 1.0 out to four standard deviations, use generateGaussianCurve(1.0, 4.0, 100). |

Table 14: Functions performing signal processing operations, part 3.

| function | argument <br> type(s) | return <br> type | description |
| :--- | :--- | :--- | :--- |
| generateHammingWindow | int | double | Return a Hamming window with <br> the specified length. For length <br> M + 1, the formula is: $w(n)=$ <br> $0.54-0.46 \cos \frac{2 \pi n}{M}$ |
| generateHanningWindow | int | double | Return a Hanning window with <br> the specified length. For length <br> M + 1, the formula is: $w(n)=$ <br> $0.5-0.5$ cos $\frac{2 \pi n}{l} M$ |
| generatePolynomialCurve | double, dou- <br> ble, double, <br> int | double | Return samples of a curve spec- <br> ified by a polynomial. The first <br> argument is an array with the <br> polynomial coefficients, begin- <br> ning with the constant term, the <br> linear term, the squared term, <br> etc. The second argument is <br> the value of the polynomial vari- <br> able at which to begin, and the <br> third argument is the increment <br> on this variable for each succes- <br> sive sample. The final argument <br> is the length of the returned ar- <br> ray. |

Table 15: Functions performing signal processing operations, part 4.

| function | argument <br> type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| generateRaisedCosinePulse | double, double, int | double | Return an array containing a symmetric raised-cosine pulse. This pulse is widely used in communication systems, and is called a "raised cosine pulse" because the magnitude its Fourier transform has a shape that ranges from rectangular (if the excess bandwidth is zero) to a cosine curved that has been raised to be non-negative (for excess bandwidth of 1.0). The elements of the returned array are samples of the function: $h(t)=\frac{\sin \frac{\pi t}{T} \frac{\pi t}{T}}{x} \frac{\cos \operatorname{fracx} x t T}{1-\left(\frac{2 x t}{T}\right)^{2}}$, where <br> $x$ is the excess bandwidth (the first argument) and T is the number of samples from the center of the pulse to the first zero crossing (the second argument). The samples are taken with a sampling interval of 1.0 , and the returned array is symmetric and has a length equal to the third argument. With an excessBandwidth of 0.0 , this pulse is a sinc pulse. |
| generateRectangularWindow | int | double | Return an array filled with 1.0 of the specified length. This is a rectangular window. |

Table 16: Functions performing signal processing operations, part 5.

| function | argument type(s) | return type | description |
| :---: | :---: | :---: | :---: |
| IDCT | double or double, int or double, int, int | double | Return the inverse discrete cosine transform of the specified array, using the specified (optional) length and normalization strategy (see 8.0.5). |
| IFFT | double or complex or double, int complex, int | complex | inverse fast Fourier transform of the specified array. If the second argument is given with value $n$, then the length of the transform is $2^{n}$. Otherwise, the length is the next power of two greater than or equal to the length of the input array. If the input length does not match this length, then input is padded with zeros. |
| nextPowerOfTwo | double | int | Return the next power of two larger than or equal to the argument. |
| poleZeroToFrequency | complex, complex, complex, int | complex | Given an array of pole locations, an array of zero locations, a gain term, and a size, return an array of the specified size representing the frequency response specified by these poles, zeros, and gain. This is calculated by walking around the unit circle and forming the product of the distances to the zeros, dividing by the product of the distances to the poles, and multiplying by the gain. |

Table 17: Functions performing signal processing operations, part 4.

| function | argument type(s) | return <br> type | description |
| :---: | :---: | :---: | :---: |
| sinc | double | double | Return the sinc function, $\sin (x) / x$, where special care is taken to ensure that 1.0 is returned if the argument is 0.0 . |
| toDecibels | double | double | Return $20 \times \log _{10} z$, where $z$ is the argument. |
| unwrap | double | double | Modify the specified array to unwrap the angles. That is, if the difference between successive values is greater than $\pi$ in magnitude, then the second value is modified by multiples of $2 \pi$ until the difference is less than or equal to $\pi$. In addition, the first element is modified so that its difference from zero is less than or equal to $\pi$ in magnitude. |
| upsample | double, int | double | Return a new array that is the result of inserting $n-1$ zeroes between each successive sample in the input array, where $n$ is the second argument. The returned array has length $n L$, where $L$ is the length of the argument array. It is required that $n>0$. |

## 13.F I/O Functions and Other Miscellaneous Functions

Table 18: Miscellaneous functions, part 1.

| function | argument type(s) | return type | description |
| :--- | :--- | :--- | :--- |
| asURL | string | string | Return a URL representation of <br> the argument. |
| cast | type1, type2 | type1 | Return the second argument <br> converted to the type of the first, <br> or throw an exception if the con- <br> version is invalid. |
| constants | none | record | Return a record identifying all <br> the globally defined constants in <br> the expression language. |
| findFile | string | string | Given a file name relative to the <br> user directory, current directory, <br> or classpath, return the absolute <br> file name of the first match, or <br> return the name unchanged if no <br> match is found. |
| filter | function, type | type | Extract a sub-array consisting of <br> all of the elements of an array for <br> which the given predicate func- <br> tion returns true. |
| filter | function, type, int | type | Extract a sub-array with a lim- <br> ited size consisting of all of the <br> elements of an array for which <br> the given predicate function re- <br> turns true. |
| freeMemory | none | long | Return the approximate num- <br> ber of bytes available for future <br> memory allocation. |
| iterate | function, int, type | type | Return an array that results from <br> first applying the specified func- <br> tion to the third argument, then <br> applying it to the result of that <br> application, and repeating to get <br> an array whose length is given <br> by the second argument. |

Table 19: Miscellaneous functions, part 2.

| function | argument type(s) | return type | description |
| :--- | :--- | :--- | :--- |
| map | function, type | type | Return an array that results from <br> applying the specified function <br> to the elements of the specified <br> array. |
| property | string | string | Return a system property <br> with the specified name <br> from the environment, or <br> an empty string if there is none. <br> Some useful properties are <br> java.version, ptolemy.ptII.dir, <br> ptolemy.ptII.dirAsURL, and <br> user.dir. |
| readFile | string | string | Get the string text in the speci- <br> fied file, or throw an exception <br> if the file cannot be found. The <br> file can be absolute, or relative <br> to the current working directory <br> (user.dir), the user's home direc- <br> tory (user.home), or the class- <br> path. |
| readResource | string | string | Get the string text in the spec- <br> ified resource (which is a file <br> found relative to the classpath), <br> or throw an exception if the file <br> cannot be found. |
| totalMemory | none | long | Return the approximate number <br> of bytes used by current objects <br> plus those available for future <br> object allocation. |


[^0]:    ${ }^{0}$ The exceptions are parameters that are strictly string parameters, in which case the value of the parameter is the literal string, not the string interpreted as an expression, as for example the function parameter of the TrigFunction actor, which can take on only "sin," "cos," "tan", "asin", "acos", and "atan" as values.

[^1]:    ${ }^{0}$ Moreover, the set of available can easily be extended if you are writing Java code by registering another class that includes static methods (see the PtParser class in the ptolemy.data.expr package).

