9.1 Exercises 6 and 7 in Chapter 8 of Lee & Varaiya.

9.2 Two complex numbers z_1 and z_2 are described below:

$$z_1 = 1 + i\sqrt{3} \qquad\qquad z_2 = \exp\left(i\,\frac{2\pi}{3}\right) \ .$$

(a) Identify each of the following complex numbers as points (or vectors) on the complex plane, using a well-labeled sketch: z_1, z_2, z_1^*, z_2^*, $1/z_1$, $1/z_2$, $1/z_1^*$, and $1/z_2^*$.

(b) Determine each of the sums $z_1 + 2z_2$, $z_1^2 + z_2$, and $\frac{1}{2}z_1 + z_2^*$.

(c) Determine each of the magnitudes $|z_1 z_2|$, $|z_1 z_2^*|$, $|z_1/z_2|$, and $|z_2/z_1|$.

(d) Determine each of the following powers of z_1 and z_2:

(i) z_1^2 (ii) z_1^3 (iii) z_1^6 (iv) z_2^4.

(e) Determine $z_2^{1/4}$. Be mindful of how many fourth roots z_2 has and identify each of them graphically on a well-labeled sketch of the complex plane.

Express each of your answers in Cartesian form ($a + ib$), in polar form ($re^{i\theta}$, where $r > 0$), as a real number, as an imaginary number, or graphically in a well-labeled complex-plane diagram, whichever form is less cluttered and more appropriate.

9.3 With little algebraic manipulation, determine each of the following sums:

(i) $\displaystyle\sum_{n=0}^{N} \cos(n\,\theta)$

(ii) $\displaystyle\sum_{n=1}^{N} \sin(n\,\theta)$

Hint: You may find geometric series useful.

9.4 Consider the following sixth-order equation:

$$z^6 - 2\sqrt{3}z^4 + 4z^2 = 0.$$

Determine the six solutions (roots) of the equation, and express each root in both a simple rectangular and a simple polar form. Explain your work succinctly, but clearly and convincingly. Also, plot these solutions on a single, well-labeled diagram of the complex plane.

9.5 For each set defined below, provide a well-labeled diagram identifying all the points on the *complex plane* that belong to it. $\mathbb{C}$ refers to the set of complex numbers, $\mathbb{R}$ refers to the set of real numbers, and $\mathbb{Z}$ refers to the set of integers.

(a) $\{z \in \mathbb{C} \mid |z - i| = |z + i|\}$

(b) $\{z \in \mathbb{C} \mid \mathrm{Im}(z) > \mathrm{Re}(z)\}$

(c) $\{z \in \mathbb{C} \mid 0 < \angle z < \pi/4\}$

(d) $\{z \in \mathbb{C} \mid 1 < |z - 2i| < 3\}$

(e) $\{z \in \mathbb{C} \mid z + z^* = 0\}$

(f) $\{z \in \mathbb{C} \mid z = e^{i(2\pi/3)t},\, t \in \mathbb{R}\}$

(g) $\{z \in \mathbb{C} \mid z = e^{i(2\pi/3)n},\, n \in \mathbb{Z}\}$

(h) $\{z \in \mathbb{C} \mid \mathrm{Re}(z) > \mathrm{Re}(i^i)\}$