CSEE W3827

Fundamentals of Computer Systems

Homework Assignment 1

Solutions

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Due February 6, 2012 at 1:10 PM

Write your name **and UNI** on your solutions

Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.

1. (5 pts.) What are the values, in decimal, of the bytes

$$10011100$$

and

$$01111000,$$

if they are interpreted as 8-bit

(a) binary numbers;

$10011100_2 = 128 + 16 + 8 + 4 = 156;$
$01111000_2 = 64 + 32 + 16 + 8 = 120$

(b) one's complement numbers; and

$-(1100011_2) = -(64 + 32 + 2 + 1) = -99;$
$01111000_2 = 64 + 32 + 16 + 8 = 120$

(c) two's complement numbers?

$10011100_2 = -128 + 16 + 8 + 4 = -100$ or
$01100011 + 1 = 01100100 = 64 + 32 + 4 = -100;$
$01111000_2 = 64 + 32 + 16 + 8 = 120$

2. (10 pts.) Show how to compute $6 + -14$ using 5-bit

(a) signed-magnitude numbers;

$00110 + 11110 = -(1110 - 0110) = -(1000) = 11000$

Make sure you strip off the sign bits

(b) one's complement numbers; and

$00110 + 10001 = 10111 = -(1000)$ (normal binary addition)

(c) two's complement numbers.

$00110 + 100010 = 11000 = -(1000)$ (normal binary addition)

3. (10 pts.) Show how to compute $45 + 57$ in BCD.

	0100	0101	
+	0101	0111	
	1001	1100	The result of normal binary addition
	+	0110	Add 6 since the first digit exceeded 9
	1010	0010	
+	0110		Add 6 since the second digit exceeded 9
0001	0000	0010	$=102_{10}$

4. (10 pts.) Complete the truth table for the following Boolean functions:

$$a = X\overline{Y}\,\overline{Z} + \overline{X}\,\overline{Y}Z + \overline{X}\,\overline{Z}$$
$$b = (X + \overline{Y})(Y + \overline{Z})(X + \overline{Z})$$

X	**Y**	**Z**	**a**	**b**
0	0	0	1	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	0	1

5. (10 pts.) Consider the function F, whose truth table is below.

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

(a) Write F as a sum of minterms and draw the corresponding circuit.

$$\overline{X}\,\overline{Y}Z + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z}$$

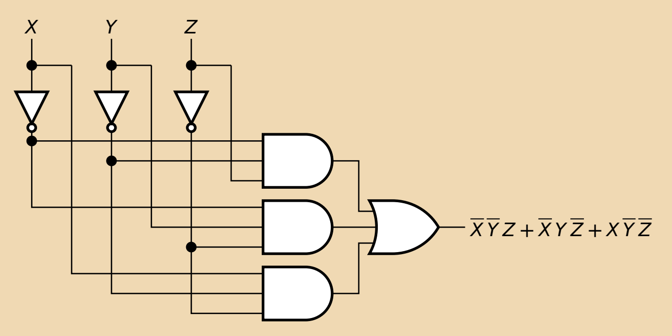
(b) Write F as a product of maxterms and draw the corresponding circuit.

$$(X+Y+Z)(X+\overline{Y}+\overline{Z})(\overline{X}+Y+\overline{Z})$$
$$(\overline{X}+\overline{Y}+Z)(\overline{X}+\overline{Y}+\overline{Z})$$

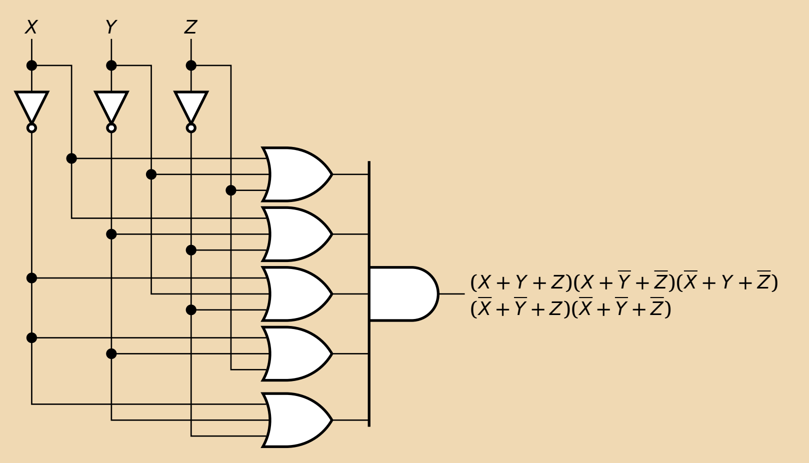
(c) Complete the Karnaugh map for F as shown below. You do not have to simplify it.

		Z	Z	
	0	1	0	1
X	1	0	0	0
			Y	Y

5. (a)


X
Y
Z
$\overline{X}\,\overline{Y}Z + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z}$

5. (b)



X
Y
Z
$(X+Y+Z)(X+\overline{Y}+\overline{Z})(\overline{X}+Y+\overline{Z})$
$(\overline{X}+\overline{Y}+Z)(\overline{X}+\overline{Y}+\overline{Z})$

6. (10 pts.) Consider the function F whose truth table is shown below

W	X	Y	Z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

(a) Write the function F in sum-of-minterms form

$$F = \overline{W}\,\overline{X}\,Y\,Z + \overline{W}\,X\,\overline{Y}\,Z + \overline{W}\,X\,Y\,Z + W\,X\,\overline{Y}\,\overline{Z} + W\,X\,\overline{Y}\,Z$$

(b) Minimize the sum-of-minterms expression, justifying each step

see next page

$$
\begin{aligned}
F &= \overline{W}\,\overline{X}\,Y\,Z + \overline{W}\,X\,\overline{Y}\,Z + \overline{W}\,X\,Y\,Z + W\,X\,\overline{Y}\,\overline{Z} + W\,X\,\overline{Y}\,Z \\
&= \overline{W}(\overline{X}YZ + X\overline{Y}Z + XYZ) + WX\overline{Y}(\overline{Z} + Z) \text{ (Factoring)} \\
&= \overline{W}((\overline{X} + X)YZ + X\overline{Y}Z) + WX\overline{Y} \text{ (Factoring, } A + \overline{A} = 1\text{)} \\
&= \overline{W}YZ + \overline{W}X\overline{Y}Z + WX\overline{Y}
\end{aligned}
$$

or

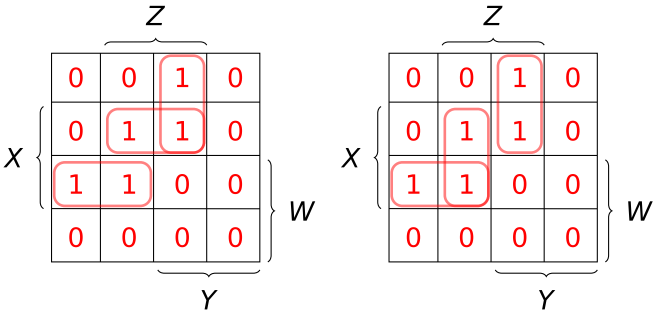
$$
\begin{aligned}
F &= \overline{W}\,\overline{X}\,Y\,Z + \overline{W}\,X\,\overline{Y}\,Z + \overline{W}\,X\,Y\,Z + W\,X\,\overline{Y}\,\overline{Z} + W\,X\,\overline{Y}\,Z \\
&= \overline{W}(\overline{X}YZ + X\overline{Y}Z + XYZ) + WX\overline{Y}(\overline{Z} + Z) \text{ (Factoring)} \\
&= \overline{W}((\overline{Y} + Y)XZ + \overline{X}YZ) + WX\overline{Y} \text{ (Factoring, } A + \overline{A} = 1\text{)} \\
&= \overline{W}XZ + \overline{W}\,\overline{X}YZ + WX\overline{Y}
\end{aligned}
$$

or

$$
\begin{aligned}
F &= \overline{W}\,\overline{X}\,Y\,Z + \overline{W}\,X\,\overline{Y}\,Z + \overline{W}\,X\,Y\,Z + W\,X\,\overline{Y}\,\overline{Z} + W\,X\,\overline{Y}\,Z \\
&= \overline{W}(\overline{X}YZ + X\overline{Y}Z + XYZ + XYZ) + WX\overline{Y}(\overline{Z} + Z) \text{ (Factoring, } A + A = 1\text{)} \\
&= \overline{W}(YZ(X + \overline{X}) + XZ(\overline{Y} + Y)) + WX\overline{Y} \text{ (factoring)} \\
&= \overline{W}YZ + \overline{W}XZ + WX\overline{Y} \text{ (} A + \overline{A} = 1\text{)}
\end{aligned}
$$

7. (10 pts.) Consider the function F from problem 6.

(a) Fill in and minimize the following Karnaugh map for F



(b) Express your minimized Karnaugh map as a Boolean expression

$\overline{W}YZ + \overline{W}XZ + WX\overline{Y}$ or $\overline{W}YZ + X\overline{Y}Z + WX\overline{Y}$

(c) Are your minimized expressions in problem 6 and problem 7 the same? Why or why not?

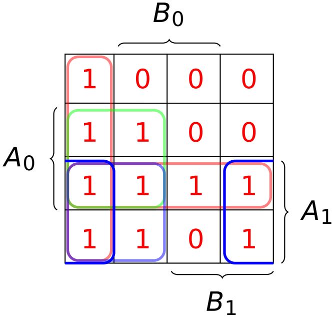
Not necessarily; it's easy to get to a "local minimum" where to simplify the expression further, you have to make it messier first.

8. (20 pts.) Design a circuit that takes two two-bit binary numbers (A_1 and A_0, B_1 and B_0) and produces a true output when, in binary, A is greater than or equal to B.

(a) Fill in the truth table

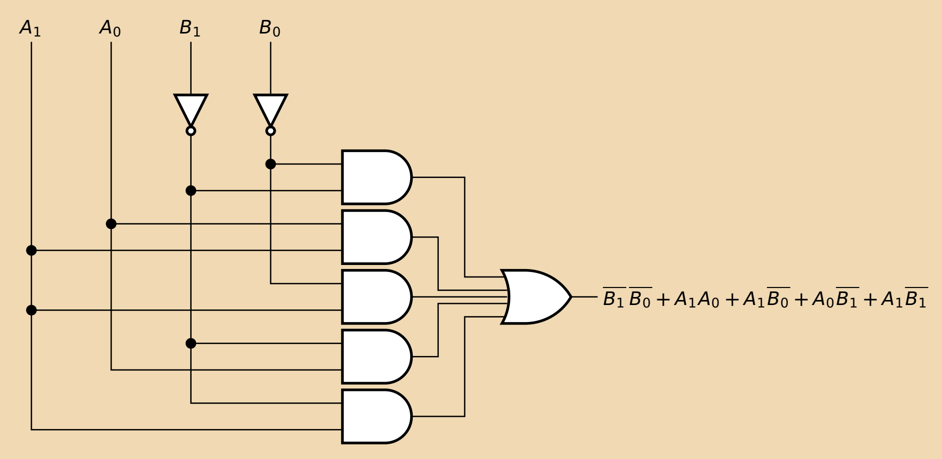
A_1	A_0	B_1	B_0	$A \geq B$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

(b) Fill in the Karnaugh map and use it to minimize



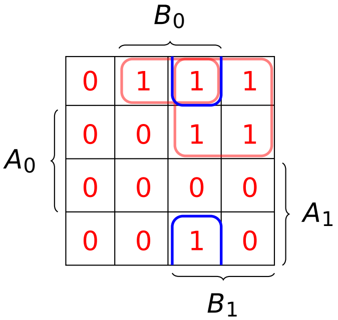
$\overline{B_1}\,\overline{B_0} + A_1 A_0 + A_1 \overline{B_0} + A_0 \overline{B_1} + A_1 \overline{B_1}$

(c) Draw the corresponding circuit.

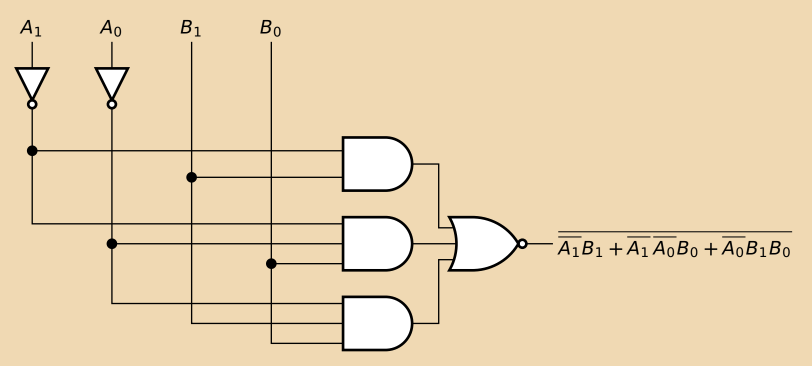

A_1
A_0
B_1
B_0
$\overline{B_1}\,\overline{B_0} + A_1 A_0 + A_1\overline{B_0} + A_0\overline{B_1} + A_1\overline{B_1}$

9. (15 pts.)

(a) Minimize the Karnaugh map for the *complement* of the $A \geq B$ function from problem 8.



(b) Use this to draw a circuit for $A \geq B$ (i.e., *not the complement*).


A_1
A_0
B_1
B_0
$\overline{\overline{A_1}B_1 + \overline{A_1}\,\overline{A_0}B_0 + \overline{A_0}B_1B_0}$