

# Fundamentals of Computer Systems

## Caches

Stephen A. Edwards

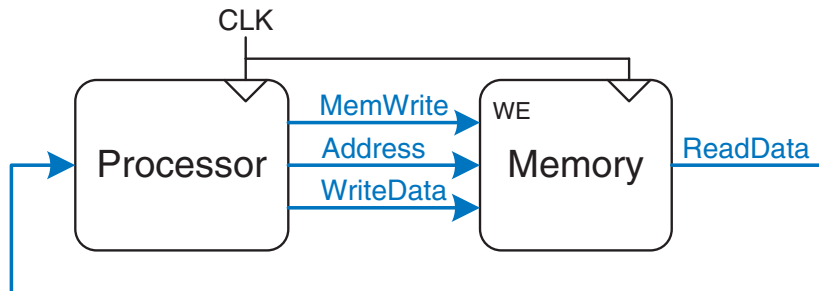
Columbia University

Summer 2016

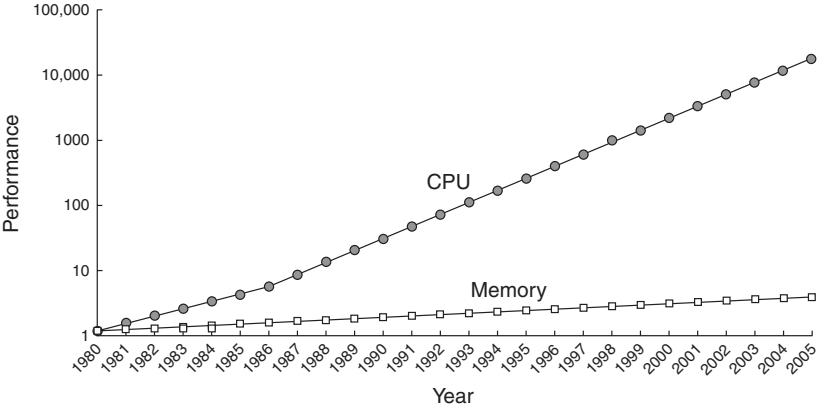
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# Computer Systems

Performance depends on which is slowest:  
the processor or the memory system



# Memory Speeds Haven't Kept Up



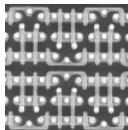
Our single-cycle memory assumption has been wrong since 1980.

Hennessy and Patterson. *Computer Architecture: A Quantitative Approach*. 3rd ed., Morgan Kaufmann, 2003.

# Your Choice of Memories

Fast    Cheap    Large

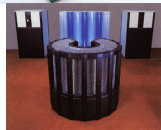
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On-Chip SRAM



Commodity DRAM



Supercomputer



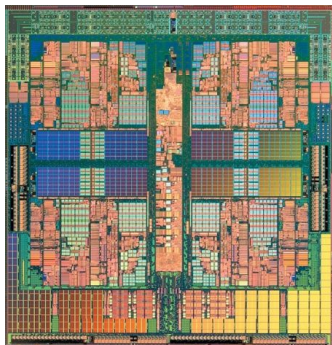
# Memory Hierarchy

Fundamental trick to making a big memory appear fast

<b>Technology</b>	<b>Cost (\$/Gb)</b>	<b>Access Time (ns)</b>	<b>Density (Gb/cm<sup>2</sup>)</b>
SRAM	30 000	0.5	0.00025
DRAM	10	100	1 – 16
Flash	2	300*	8 – 32
Hard Disk	0.1	10 000 000	500 – 2000

\* Read speed; writing much, much slower

# A Modern Memory Hierarchy



AMD Phenom 9600

Quad-core

2.3 GHz

1.1–1.25 V

95 W

65 nm

My desktop machine:

Level	Size	Tech.
L1 Instruction*	64 K	SRAM
L1 Data*	64 K	SRAM
L2*	512 K	SRAM
L3	2 MB	SRAM
Memory	4 GB	DRAM
Disk	500 GB	Magnetic

\*per core

# Temporal Locality

## FIRST BOOK



### DEFINITIONS.

1. A point is that which is without parts.
2. A line is length without breadth.
3. The extremities of a line are points.
4. A right line, is that which lies evenly between its extremities.
5. A superficies, is that which has only length and breadth.
6. The boundings of a superficies are lines.
7. A plane superficies, is that which lies evenly between its extreme right lines.
8. A rectilinear angle, is the inclination of two right lines to each other, which touch, but do not form one straight line.  
An angle is designated either by one letter at the vertex; or three, of which the middle one is at the vertex, the remaining two any place on the legs.
9. The legs of an angle, are the lines which make the angle.
10. The vertex of an angle is the point in which the legs mutually touch each other.



What path do your eyes take when you read this?

Did you look at the drawings more than once?

# Spatial Locality



If you need something, you may also need something nearby



# Memory Performance

Hit: Data is found in the level of memory hierarchy

Miss: Data not found; will look in next level

$$\text{Hit Rate} = \frac{\text{Number of hits}}{\text{Number of accesses}}$$

$$\text{Miss Rate} = \frac{\text{Number of misses}}{\text{Number of accesses}}$$

$$\text{Hit Rate} + \text{Miss Rate} = 1$$

The expected access time  $E_L$  for a memory level  $L$  with latency  $t_L$  and miss rate  $M_L$ :

$$E_L = t_L + M_L \cdot E_{L+1}$$



## Memory Performance Example

Two-level hierarchy: Cache and main memory

Program executes 1000 loads & stores

750 of these are found in the cache

*What's the cache hit and miss rate?*

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Two-level hierarchy: Cache and main memory

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$$\text{Hit Rate} = \frac{750}{1000} = 75\%$$

$$\text{Miss Rate} = 1 - 0.75 = 25\%$$

---

If the cache takes 1 cycle and the main memory 100,

*What's the expected access time?*

## Memory Performance Example

Two-level hierarchy: Cache and main memory

Program executes 1000 loads & stores

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*What's the cache hit and miss rate?*

$$\text{Hit Rate} = \frac{750}{1000} = 75\%$$

$$\text{Miss Rate} = 1 - 0.75 = 25\%$$

---

If the cache takes 1 cycle and the main memory 100,

*What's the expected access time?*

Expected access time of main memory:  $E_1 = 100$  cycles

Access time for the cache:  $t_0 = 1$  cycle

Cache miss rate:  $M_0 = 0.25$

$$E_0 = t_0 + M_0 \cdot E_1 = 1 + 0.25 \cdot 100 = 26 \text{ cycles}$$

# Cache

Highest levels of memory hierarchy

Fast: level 1 typically 1 cycle access time

With luck, supplies most data

Cache design questions:

What data does it hold? *Recently accessed*

How is data found? *Simple address hash*

What data is replaced? *Often the oldest*



# What Data is Held in the Cache?

Ideal cache: always correctly guesses what you want before you want it.

Real cache: never that smart

## Caches Exploit

### Temporal Locality

Copy newly accessed data into cache, replacing oldest if necessary

### Spatial Locality

Copy nearby data into the cache at the same time  
Specifically, always read and write a block at a time (e.g., 64 bytes), never a single byte.

# A Direct-Mapped Cache

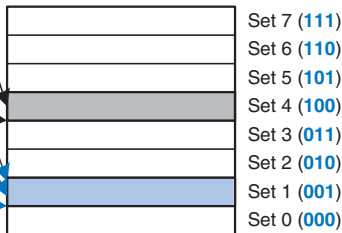
Address	Data
11...111 <b>111</b> 00	mem[0xFFFFFFFFC]
11...111 <b>110</b> 00	mem[0xFFFFFFFF8]
11...111 <b>101</b> 00	mem[0xFFFFFFFF4]
11...111 <b>100</b> 00	mem[0xFFFFFFFF0]
11...111 <b>011</b> 00	mem[0xFFFFFEC]
11...111 <b>010</b> 00	mem[0xFFFFFE8]
11...111 <b>001</b> 00	mem[0xFFFFFE4]
11...111 <b>000</b> 00	mem[0xFFFFFE0]
⋮	⋮
00...001 <b>001</b> 00	mem[0x00000024]
00...001 <b>000</b> 00	mem[0x00000020]
00...000 <b>111</b> 00	mem[0x0000001C]
00...000 <b>110</b> 00	mem[0x00000018]
00...000 <b>101</b> 00	mem[0x00000014]
00...000 <b>100</b> 00	mem[0x00000010]
00...000 <b>011</b> 00	mem[0x0000000C]
00...000 <b>010</b> 00	mem[0x00000008]
00...000 <b>001</b> 00	mem[0x00000004]
00...000 <b>000</b> 00	mem[0x00000000]

$2^{30}$ -Word Main Memory

This simple cache has

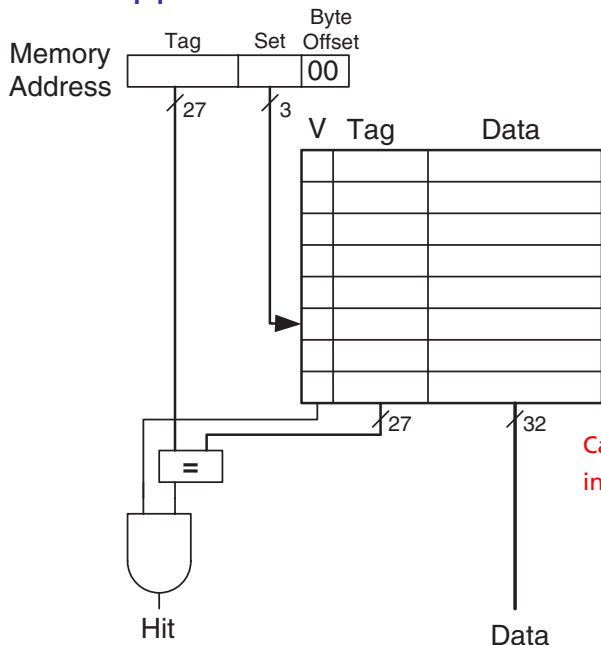
- ▶ 8 sets
- ▶ 1 *block* per set
- ▶ 4 bytes per block

To simplify answering “is this memory in the cache?,” each byte is mapped to exactly one set.



$2^3$ -Word Cache

# Direct-Mapped Cache Hardware



Address bits:

0–1: byte within block

2–4: set number

5–31: block "tag"

Set 7

Set 6

Set 5

Set 4

Set 3

Set 2

Set 1

Set 0

8-entry x  
(1+27+32)-bit  
SRAM

Cache hit if  
in the set of the address,

- ▶ block is valid (V=1)
- ▶ tag (address bits 5–31) matches



# Direct-Mapped Cache Behavior

A dumb loop:

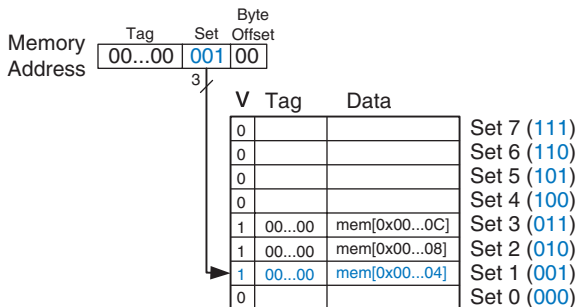
repeat 5 times

load from 0x4;

load from 0xC;

load from 0x8.

```
li    $t0, 5
l1:  beq    $t0, $0, done
     lw     $t1, 0x4($0)
     lw     $t2, 0xC($0)
     lw     $t3, 0x8($0)
     addiu  $t0, $t0, -1
     j     l1
done:
```



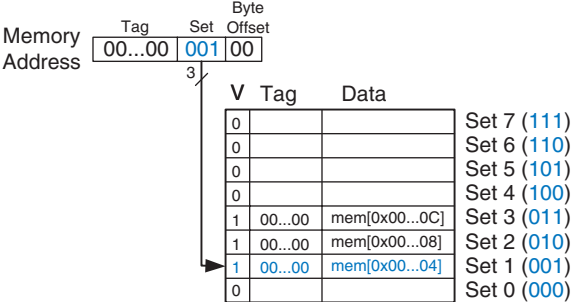
Assuming the cache starts empty, what's the miss rate?

# Direct-Mapped Cache Behavior

A dumb loop:  
 repeat 5 times  
 load from 0x4;  
 load from 0xC;  
 load from 0x8.

```

li    $t0, 5
l1:  beq    $t0, $0, done
     lw     $t1, 0x4($0)
     lw     $t2, 0xC($0)
     lw     $t3, 0x8($0)
     addiu $t0, $t0, -1
     j     l1
done:
  
```



Cache when reading 0x4 last time

Assuming the cache starts empty, what's the miss rate?

4 C 8 4 C 8 4 C 8 4 C 8 4 C 8

---

M M M H H H H H H H H H H H H

3/15 = 0.2 = 20%

# Direct-Mapped Cache: Conflict

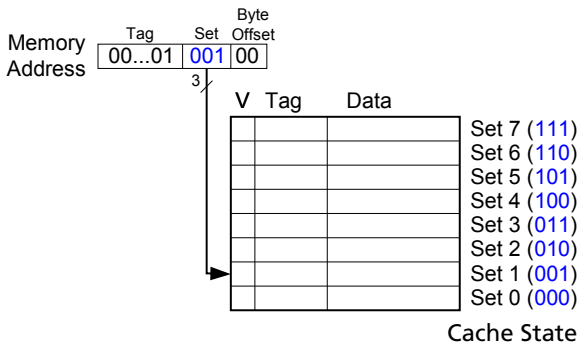
A dumber loop:

repeat 5 times

load from 0x4;

load from 0x24

```
li    $t0, 5
l1:  beq    $t0, $0, done
     lw     $t1, 0x4($0)
     lw     $t2, 0x24($0)
     addiu  $t0, $t0, -1
     j     l1
done:
```



Assuming the cache starts empty, what's the miss rate?

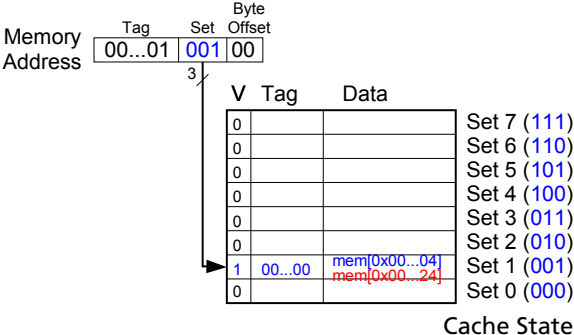
These are *conflict misses*

# Direct-Mapped Cache: Conflict

A dumber loop:  
 repeat 5 times  
 load from 0x4;  
 load from 0x24

```

    li    $t0, 5
l1: beq  $t0, $0, done
    lw    $t1, 0x4($0)
    lw    $t2, 0x24($0)
    addiu $t0, $t0, -1
    j     l1
done:
  
```



Assuming the cache starts empty, what's the miss rate?

4 24 4 24 4 24 4 24 4 24

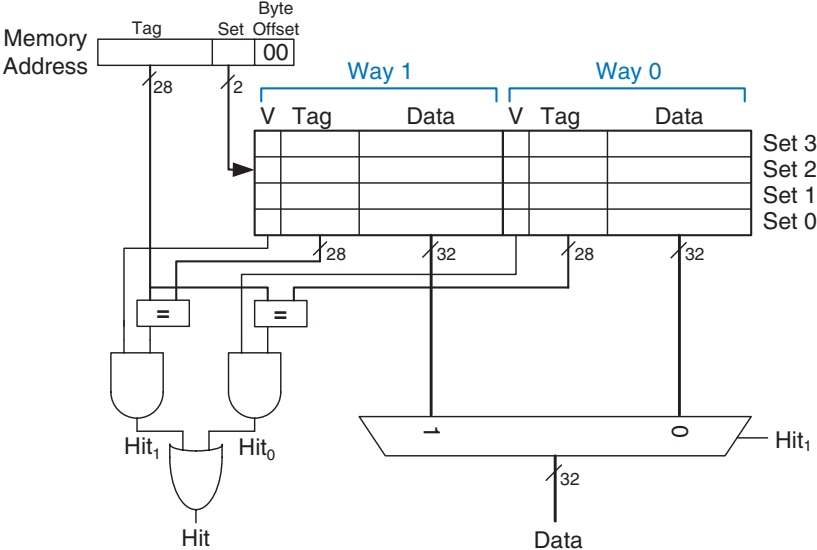
---

M M M M M M M M M M

10/10 = 1 = 100% Oops

These are *conflict misses*

# No Way! Yes Way! 2-Way Set Associative Cache



## 2-Way Set Associative Behavior

```
li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0x24($0)
      addiu  $t0, $t0, -1
      j     l1
done:
```

Assuming the cache starts empty, what's the miss rate?

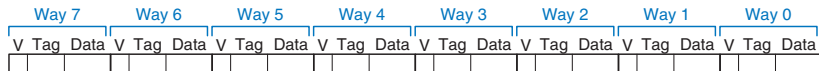
4 24 4 24 4 24 4 24 4 24  
M M H H H H H H H H

$$2/10 = 0.2 = 20\%$$

*Associativity reduces conflict misses*

Way 1			Way 0			
V	Tag	Data	V	Tag	Data	
0			0			Set 3
0			0			Set 2
1	00...00	mem[0x00...24]	1	00...10	mem[0x00...04]	Set 1
0			0			Set 0

# An Eight-way Fully Associative Cache



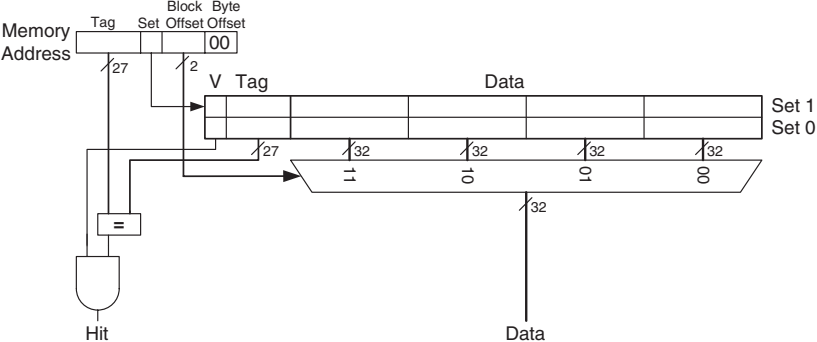
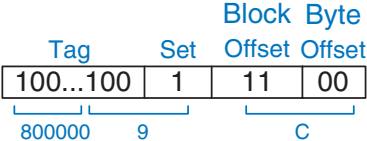
No conflict misses: only compulsory or capacity misses

Either very expensive or slow because of all the associativity

# Exploiting Spatial Locality: Larger Blocks

0x8000 0009C:

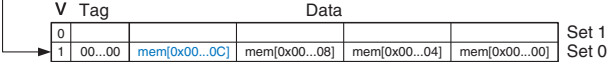
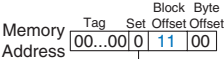
Memory Address



- 2 sets
- 1 block per set (Direct Mapped)
- 4 words per block



# Direct-Mapped Cache Behavior w/ 4-word block



Cache when reading 0xC

The dumb loop:

repeat 5 times

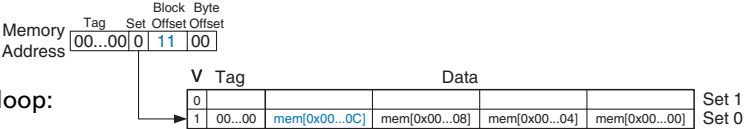
- load from 0x4;
- load from 0xC;
- load from 0x8.

```

li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0xC($0)
      lw     $t3, 0x8($0)
      addiu $t0, $t0, -1
      j     l1
done:
    
```

Assuming the cache starts empty, what's the miss rate?

# Direct-Mapped Cache Behavior w/ 4-word block



The dumb loop:

repeat 5 times

- load from 0x4;
- load from 0xC;
- load from 0x8.

Cache when reading 0xC

```

li    $t0, 5
l1:  beq    $t0, $0, done
      lw     $t1, 0x4($0)
      lw     $t2, 0xC($0)
      lw     $t3, 0x8($0)
      addiu $t0, $t0, -1
      j     l1
done:
    
```

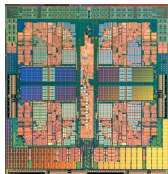
Assuming the cache starts empty, what's the miss rate?

4	C	8	4	C	8	4	C	8	4	C	8	4	C	8
M	H	H	H	H	H	H	H	H	H	H	H	H	H	H

$1/15 = 0.0666 = 6.7\%$

*Larger blocks reduce compulsory misses by exploiting spatial locality*

# Stephen's Desktop Machine Revisited












AMD Phenom  
9600  
Quad-core  
2.3 GHz  
1.1–1.25 V  
95 W  
65 nm

On-chip caches:

Cache	Size	Sets	Ways	Block
L1I*	64 K	512	2-way	64-byte
L1D*	64 K	512	2-way	64-byte
L2*	512 K	512	16-way	64-byte
L3	2 MB	1024	32-way	64-byte

\* per core

# Intel On-Chip Caches

Chip	Year	Freq. (MHz)	L1		L2	
			Data	Instr		
	80386	1985	16–25	off-chip		none
	80486	1989	25–100	8K unified		off-chip
	Pentium	1993	60–300	8K	8K	off-chip
	Pentium Pro	1995	150–200	8K	8K	256K–1M (MCM)
	Pentium II	1997	233–450	16K	16K	256K–512K (Cartridge)
	Pentium III	1999	450–1400	16K	16K	256K–512K
	Pentium 4	2001	1400–3730	8–16K	12k op trace cache	256K–2M
	Pentium M	2003	900–2130	32K	32K	1M–2M
	Core 2 Duo	2005	1500–3000	32K per core	32K per core	2M–6M