User-Defined Types

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Algebraic Data Types

```
data Bool = False | True
```

`Bool`: Type Constructor   `False` and `True`: Data Constructors

```
Prelude> data MyBool = MyFalse | MyTrue

Prelude> :t MyFalse
MyFalse :: MyBool      -- A literal
Prelude> :t MyTrue
MyTrue :: MyBool

Prelude> :t MyBool
<interactive>:1:1: error: Data constructor not in scope: MyBool
Prelude> :k MyBool
MyBool :: *            -- A concrete type (no parameters)
```

Algebraic Types and Pattern Matching

```
data Bool = False | True
```

Type constructors may appear in type signatures;
data constructors in expressions and patterns

```
Prelude> :{
Prelude| myAnd :: Bool -> Bool -> Bool
Prelude| myAnd False _ = False
Prelude| myAnd True  x = x
Prelude| :}

Prelude> [ (a,b,myAnd a b) | a <- [False, True], b <- [False, True] ]
[(False,False,False),(False,True,False),
 (True,False,False),(True,True,True)]
```

An Algebraic Type: A Sum of Products

```
data Shape = Circle Float Float Float
           | Rectangle Float Float Float Float
```

Sum = one of A or B or C...

Product = each of D and E and F...

A.k.a. tagged unions, sum-product types

Mathematically,

$$Shape = Circle \cup Rectangle$$

$$Circle = Float \times Float \times Float$$

$$Rectangle = Float \times Float \times Float \times Float$$

An Algebraic Type: A Sum of Products

```haskell
data Shape = Circle Float Float Float
           | Rectangle Float Float Float Float

area                          :: Shape -> Float
area (Circle _ _ r)           = pi * r ^ 2
area (Rectangle x1 y1 x2 y2) = (abs $ x2 - x1) * (abs $ y2 - y1)
```

```
*Main> :t Circle
Circle :: Float -> Float -> Float -> Shape
*Main> :t Rectangle
Rectangle :: Float -> Float -> Float -> Float -> Shape
*Main> :k Shape
Shape :: *

*Main> area $ Circle 10 20 10
314.15927
*Main> area $ Rectangle 10 10 20 30
200.0
```

Printing User-Defined Types: Deriving Show

```
*Main> Circle 10 20 30

<interactive>:9:1: error:
    * No instance for (Show Shape) arising from a use of 'print'
    * In a stmt of an interactive GHCi command: print it
```

Add `deriving (Show)` to make the compiler generate a default *show*:

```
data Shape = Circle Float Float Float
           | Rectangle Float Float Float Float
           deriving Show
```

```
*Main> Circle 10 20 30
Circle 10.0 20.0 30.0
*Main> show $ Circle 10 20 30
"Circle 10.0 20.0 30.0"
```

Every Possible Automatic Derivation

```
data Bool = False | True      -- Standard Prelude definition
            deriving (Eq, Ord, Enum, Read, Show, Bounded)
```

```
Prelude> True == True
True                                -- Eq
Prelude> False < False
False                               -- Ord
Prelude> succ False
True                                -- Enum
Prelude> succ True
*** Exception: Prelude.Enum.Bool.succ: bad argument
Prelude> read "True" :: Bool
True                                -- Read
Prelude> show False
"False"                             -- Show
Prelude> minBound :: Bool
False                               -- Bounded
```

Types as Documentation

When in doubt, add another type

```
data Point = Point Float Float deriving Show
data Shape = Circle Point Float
           | Rectangle Point Point
           deriving Show

area :: Shape -> Float
area (Circle _ r) = pi * r ^ 2
area (Rectangle (Point x1 y1) (Point x2 y2)) =
    (abs $ x2 - x1) * (abs $ y2 - y1)
```

```
*Main> area $ Rectangle (Point 10 20) (Point 30 40)
400.0
*Main> area $ Circle (Point 0 0) 100
31415.928
```

```haskell
moveTo :: Point -> Shape -> Shape
moveTo p (Circle _ r) = Circle p r
moveTo p@(Point x0 y0) (Rectangle (Point x1 y1) (Point x2 y2)) =
    Rectangle p $ Point (x0 + x2 - x1) (y0 + y2 - y1)

origin :: Point
origin = Point 0 0

originCircle :: Float -> Shape
originCircle = Circle origin -- function in "point-free style"originRect ::
Float -> Float -> Shape
originRect x y = Rectangle origin (Point x y)
```

```
Prelude> :l Shapes
[1 of 1] Compiling Shapes           ( Shapes.hs, interpreted )
Ok, one module loaded.
*Shapes> moveTo (Point 10 20) $ originCircle 5
Circle (Point 10.0 20.0) 5.0
*Shapes> moveTo (Point 10 20) $ Rectangle (Point 5 15) (Point 25 35)
Rectangle (Point 10.0 20.0) (Point 30.0 40.0)
```

Shapes.hs

```haskell
module Shapes
( Point(..)   -- Export the Point constructor
, Shape(..)   -- Export Circle and Rectangle constructors
, area
, moveTo
, origin
, originCircle
, originRect
) where

data Point = Point Float Float deriving Show
-- etc.
```

Records: Naming Product Type Fields

```
data Person = Person { firstName :: String
                     , lastName :: String
                     , age :: Int
                     , height :: Float
                     , phoneNumber :: String
                     , flavor :: String
                     } deriving Show

hbc = Person { lastName = "Curry", firstName = "Haskell",
               age = 42, height = 6.0, phoneNumber = "555-1212",
               flavor = "Curry" }
```

```
*Main> :t lastName
lastName :: Person -> String
*Main> lastName hbc
"Curry"
```

Updating and Pattern-Matching Records

```
*Main> hbc
Person {firstName = "Haskell", lastName = "Curry", age = 42,
        height = 6.0, phoneNumber = "555-1212", flavor = "Curry"}

*Main> hbc { age = 43, flavor = "Vanilla" }
Person {firstName = "Haskell", lastName = "Curry", age = 43,
        height = 6.0, phoneNumber = "555-1212", flavor = "Vanilla"}

*Main> sae = Person "Stephen" "Edwards" 49 6.0 "555-1234" "Durian"
```

```
fullName :: Person -> String
fullName (Person { firstName = f, lastName = l }) = f ++ " " ++ l
```

```
*Main> map fullName [hbc, sae]
["Haskell Curry","Stephen Edwards"]
```

Record Named Field Puns In Patterns

`:set -XNamedFieldPuns` in GHCi or put a pragma at the beginning of the file

```
{-# LANGUAGE NamedFieldPuns #-}
```

```
favorite :: Person -> String
favorite (Person { firstName, flavor } ) =
    firstName ++ " loves " ++ flavor
```

```
*Main> favorite hbc
"Haskell loves Curry"
```

Omitting a field when constructing a record is a compile-time error unless you `:set -Wno-missing-fields`, which allows uninitialized fields. Evaluating an unititialized field throws an exception.

Parameterized Types: Maybe

A safe replacement for null pointers

```haskell
data Maybe a = Nothing | Just a
```

The *Maybe* type constructor is a function with a type parameter (*a*) that returns a type (*Maybe a*).

```
Prelude> :k Maybe
Maybe :: * -> *

Prelude> Just "your luck"
Just "your luck"
Prelude> :t Just "your luck"
Just "your luck" :: Maybe [Char]
Prelude> :t Nothing
Nothing :: Maybe a
Prelude> :t Just (10 :: Int)
Just (10 :: Int) :: Maybe Int
```

Maybe In Action

Useful when a function may "fail" and you don't want to throw an exception

```
Prelude> :m + Data.List
Prelude Data.List> :t uncons
uncons :: [a] -> Maybe (a, [a])
Prelude Data.List> uncons [1,2,3]
Just (1,[2,3])
Prelude Data.List> uncons []
Nothing

Prelude Data.List> :t lookup
lookup :: Eq a => a -> [(a, b)] -> Maybe b
Prelude Data.List> lookup 5 [(1,2),(5,10)]
Just 10
Prelude Data.List> lookup 6 [(1,2),(5,10)]
Nothing
```

Data.Map: Multiple Type Parameters

```
Prelude Data.Map> :k Map
Map :: * -> * -> *

Prelude Data.Map> :t empty
empty :: Map k a

Prelude Data.Map> :t singleton (1::Int) "one"
singleton (1::Int) "one" :: Map Int [Char]
```

Note: while you can add type class constraints to type constructors, e.g.,

```
data Ord k => Map k v = ...
```

it's bad form to do so. By convention, to reduce verbosity, only functions that actually rely on the type classes are given such constraints.

The `type` Keyword: Introduce an Alias

```
Prelude> type AssocList k v = [(k, v)]
Prelude> :k AssocList
AssocList :: * -> * -> *
Prelude> :{
Prelude| lookup :: Eq k => k -> AssocList k v -> Maybe v
Prelude| lookup _ [] = Nothing
Prelude| lookup k ((x,v):xs) | x == k = Just v
Prelude|                     | otherwise = lookup k xs
Prelude| :}
Prelude> :t lookup
lookup :: Eq k => k -> AssocList k v -> Maybe v
Prelude> lookup 2 [(1,"one"),(2,"two")]
Just "two"
Prelude> lookup 0 [(1,"one"),(2,"two")]
Nothing
Prelude> :t [(1,"one"),(2,"two")]
[(1,"one"),(2,"two")] :: Num a => [(a, [Char])]
```

Either: Funky Type Constructor Fun

```
data Either a b = Left a | Right b
                  deriving (Eq, Ord, Read, Show)
```

```
Prelude> :k Either
Either :: * -> * -> *
Prelude> Right 20
Right 20
Prelude> Left "Stephen"
Left "Stephen"
Prelude> :t Right "Stephen"
Right "Stephen" :: Either a [Char]    -- Only second type inferred
Prelude> :t Left True
Left True :: Either Bool b
Prelude> :k Either Bool
Either Bool :: * -> *
```

Either: Often a more verbose Maybe

By convention, Left = “failure,” Right = “success”

```
Prelude> type AssocList k v = [(k,v)]
Prelude> :{
Prelude| lookup :: String -> AssocList String a -> Either String a
Prelude| lookup k [] = Left $ "Could not find " ++ k
Prelude| lookup k ((x,v):xs) | x == k = Right v
Prelude|                     | otherwise = lookup k xs
Prelude| :}
Prelude> lookup "Stephen" [("Douglas",42),("Don",0)]
Left "Could not find Stephen"
Prelude> lookup "Douglas" [("Douglas",42),("Don",0)]
Right 42
```

```haskell
data List a = Cons a (List a)         -- A recursive type
            | Nil
            deriving (Eq, Ord, Show, Read)
```

```
*Main> :t Nil
Nil :: List a                      -- Nil is polymorphic
*Main> :t Cons
Cons :: a -> List a -> List a      -- Cons is polymorphic
*Main> :k List
List :: * -> *                     -- Type constructor takes an argument
*Main> Nil
Nil
*Main> 5 `Cons` Nil
Cons 5 Nil
*Main> 4 `Cons` (5 `Cons` Nil)
Cons 4 (Cons 5 Nil)
*Main> :t 'a' `Cons` Nil
'a' `Cons` Nil :: List Char         -- Proper type inferred
```

Lists of Our Own with User-Defined Operators

```haskell
infixr 5 :.
data List a = a :. List a
            | Nil
            deriving (Eq, Ord, Show, Read)
```

Haskell symbols are `! # $ % & * + . / < = > ? @ \ ^ | - ~`

A (user-defined) operator is a symbol followed by zero or more symbols or `:`

A (user-defined) constructor is a `:` followed by one or more symbols or `:`

```
*Main> (1 :. 2 :. 3 :. Nil) :: List Int
1 :. (2 :. (3 :. Nil))
*Main> :t (:.)
(:.) :: a -> List a -> List a
```

Fixity of Standard Prelude Operators

```
infixr 9  ., !!                                  -- Highest precedence
infixr 8  ^, ^^, **                              -- Right-associative
infixl 7  *, /, `quot`, `rem`, `div`, `mod`
infixl 6  +, -                                   -- Left-associative
infixr 5  :, ++                                  -- : is the only builtin
infix  4  ==, /=, <, <=, >=, >, `elem`           -- Non-associative
infixr 3  &&
infixr 2  ||
infixl 1  >>, >>=
infixr 1  =<<
infixr 0  $, $!, `seq`                           -- Lowest precedence
```

```
*Main> (1::Int) == 2 == 3
<interactive>:9:1: error:
  Precedence parsing error
    cannot mix '==' [infix 4] and '==' [infix 4] in the
    same infix expression
```

The List Concatenation Operator

```
infixr 5 ++.        -- Define operator precedence & associativity
(++.)               :: List a -> List a -> List a
Nil        ++. ys = ys
(x :. xs) ++. ys = x :. (xs ++. ys)
```

```
*Main> (1 :. 2 :. 3 :. Nil ++. 4 :. 5:. Nil) :: List Int
1 :. (2 :. (3 :. (4 :. (5 :. Nil))))
```

The only thing special about lists in Haskell is the [,] syntax

```
*Main> :k List
List :: * -> *
*Main> :k []
[] :: * -> *
```

Our *List* type constructor has the same kind as the built-in list constructor []

```haskell
data Tree a = Node a (Tree a) (Tree a)  -- Unbalanced binary tree
            | Nil
            deriving (Eq, Show, Read)

singleton :: a -> Tree a
singleton x = Node x Nil Nil

insert :: Ord a => a -> Tree a -> Tree a
insert x Nil = singleton x
insert x n@(Node a left right) = case compare x a of
  LT -> Node a (insert x left) right
  GT -> Node a left (insert x right)
  EQ -> n

fromList :: Ord a => [a] -> Tree a
fromList = foldr insert Nil

toList :: Tree a -> [a]
toList Nil = []
toList (Node a l r) = toList l ++ [a] ++ toList r
```

```haskell
member :: Ord a => a -> Tree a -> Bool
member _ Nil = False
member x (Node a left right) = case compare x a of
  LT -> member x left
  GT -> member x right
  EQ -> True
```

```
*Main> t = fromList ([8,6,4,1,7,3,5] :: [Int])
*Main> t
Node 5 (Node 3 (Node 1 Nil Nil) (Node 4 Nil Nil))
       (Node 7 (Node 6 Nil Nil) (Node 8 Nil Nil))
*Main> toList t
[1,3,4,5,6,7,8]
*Main> 1 `member` t
True
*Main> 42 `member` t
False
```

Specifying and Implementing Type Classes

```
class  Eq a  where                  -- Standard Prelude definition of Eq
    (==), (/=) :: a -> a -> Bool  -- The class: names & signatures
    x /= y     =  not (x == y)    -- Default implementations
    x == y     =  not (x /= y)

data TrafficLight = Red | Yellow | Green

instance Eq TrafficLight where
  Red    == Red    = True   -- Suffices to only supply
  Green  == Green  = True   -- an implementation of ==
  Yellow == Yellow = True
  _      == _      = False  -- "deriving Eq" would have been easier
```

```
*Main> Red == Red
True                        -- Uses TrafficLight defintion of ==
*Main> Red /= Yellow
True                        -- Relies on default implementation
```

Implementing Show

```
instance Show TrafficLight where
  show Red    = "Red Light"
  show Green  = "Green Light"
  show Yellow = "Yellow Light"
```

```
*Main> show Yellow
"Yellow Light"
*Main> [Red, Yellow, Green]
[Red Light,Yellow Light,Green Light]     -- GHCi uses show

*Main> :k Maybe
Maybe :: * -> *                -- A polymorphic type constructor
*Main> :k Eq
Eq :: * -> Constraint          -- Like a polymorphic type constructor
*Main> :k Eq TrafficLight
Eq TrafficLight :: Constraint -- Give it a type to make it happy
```

The MINIMAL Pragma: Controlling Compiler Warnings

```haskell
infix 4 ==., /=.

class MyEq a where
  {-# MINIMAL (==.) | (/=.) #-}
  (==.), (/=.) :: a -> a -> Bool
  x /=. y     =  not (x ==. y)
  x ==. y     =  not (x /=. y)

instance MyEq Int where

instance MyEq Integer where
  x ==. y = (x `compare` y) == EQ
```

The MINIMAL pragma tells the compiler what to check for. Operators are , (and) and | (or). Parentheses are allowed.

```
Prelude> :load myeq
[1 of 1] Compiling Main

myeq.hs:9:10: warning:
  [-Wmissing-methods]
 * No explicit implementation for
     either '==.' or '/=.'
 * In the instance declaration
     for 'MyEq Int'
  |
9 | instance MyEq Int where
  |          ^^^^^^^^
```

Eq (Maybe t)

```
data Maybe t = Just t | Nothing

instance Eq t => Eq (Maybe t) where
  Just x == Just y   = x == y    -- This comparison requires Eq t
  Nothing == Nothing = True
  _ == _             = False
```

The Standard Prelude includes this by just `deriving Eq`

```
*Main> :info Eq
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool
  {-# MINIMAL (==) | (/=) #-}
instance [safe] Eq TrafficLight
instance (Eq a, Eq b) => Eq (Either a b)
instance Eq a => Eq (Maybe a)
instance Eq a => Eq [a]
instance Eq Ordering
instance Eq Int
instance Eq Float
instance Eq Double
instance Eq Char
instance Eq Bool
instance (Eq a, Eq b) => Eq (a, b)
instance (Eq a, Eq b, Eq c) => Eq (a, b, c)
instance (Eq a, Eq b, Eq c, Eq d) => Eq (a, b, c, d)
```

ToBool: Treat Other Things as Booleans

```
class ToBool a where
  toBool :: a -> Bool

instance ToBool Bool where
  toBool = id                    -- Identity function

instance ToBool Int where
  toBool 0 = False               -- C-like semantics
  toBool _ = True

instance ToBool [a] where
  toBool [] = False              -- JavaScript, python semantics
  toBool _  = True

instance ToBool (Maybe a) where
  toBool (Just _) = True
  toBool Nothing  = False
```

Now We Can toBool Bools, Ints, Lists, and Maybes

```
*Main> :t toBool
toBool :: ToBool a => a -> Bool
*Main> toBool True
True
*Main> toBool (1 :: Int)
True
*Main> toBool "dumb"
True
*Main> toBool []
False
*Main> toBool [False]
True
*Main> toBool $ Just False
True
*Main> toBool Nothing
False
```

The Functor Type Class: Should be "Mappable"†

```
class Functor f where
  fmap    :: (a -> b) -> f a -> f b
  (<$)    :: b -> f a -> f b
  m <$ b = fmap (\_ -> b)
```

If `f :: a -> b`,

```
bs = fmap f as
```

applies *f* to every *a* in *as* to give *bs*; `bs = as <$ x` replaces every *a* in *as* with *x*.

Here, *f* is a type constructor that takes an argument, like Maybe or List

```
Prelude> :k Functor
Functor :: (* -> *) -> Constraint
```

† "Functor" is from Category Theory

```
class Functor (f :: * -> *) where
  fmap :: (a -> b) -> f a -> f b
  (<$) :: a -> f b -> f a
  {-# MINIMAL fmap #-}
instance Functor (Either a)
instance Functor []
instance Functor Maybe
instance Functor IO
instance Functor ((->) r)
instance Functor ((,) a)
-- Many others; these are
-- just the Prelude's
```

Functor Instances for * -> * Kinds

```
data [] a = [] | a : [a]           -- The List type: not legal syntax

instance Functor [] where          -- Prelude definition
  fmap = map                       -- The canonical example
```

```
data Maybe t = Nothing | Just t -- Prelude definition

instance Functor Maybe where
  fmap _ Nothing  = Nothing        -- No object a here
  fmap f (Just a) = Just (f a)     -- Apply f to the object in Just a
```

```
data Tree a = Node a (Tree a) (Tree a) | Nil -- Our binary tree

instance Functor Tree where
  fmap f Nil          = Nil
  fmap f (Node a lt rt) = Node (f a) (fmap f lt) (fmap f rt)
```

Functor Either a

```
data Either a b = Left a | Right b
```

`instance Either` does not type check because `Either :: * -> * -> *`

The Prelude definition of `fmap` only modifies `Right`

```
instance Functor (Either a) where
  fmap _ (Left x)  = Left x
  fmap f (Right y) = Right (f y)
```

This works because `Either a :: * -> *` has the right kind

Kinds: The Types of Types

```
Prelude> :k Int
Int :: *                 -- A concrete type
Prelude> :k [Int]
[Int] :: *               -- A specific type of list: also concrete
Prelude> :k []
[] :: * -> *             -- The list type constructor takes a parameter
Prelude> :k Maybe
Maybe :: * -> *          -- Maybe also takes a type as a parameter
Prelude> :k Maybe Int
Maybe Int :: *           -- Specifying the parameter makes it concrete
Prelude> :k Either
Either :: * -> * -> *          -- Either takes two type parameters
Prelude> :k Either String
Either String :: * -> *        -- Partially applying Either is OK
Prelude> :k (,)
(,) :: * -> * -> *       -- The pair (tuple) constructor takes two
```

Crazy Kinds

```
Prelude> class Tofu t where tofu :: j a -> t a j
```

Type class *Tofu* expects a single type argument *t*

j must take an argument *a* and produce a concrete type, so `j :: * -> *`

t must take arguments *a* and *j*, so `t :: * -> (* -> *) -> *`

```
Prelude> :k Tofu
Tofu :: (* -> (* -> *) -> *) -> Constraint
```

Let's invent a type constructor of kind `* -> (* -> *) -> *`. It has to take two type arguments; the second needs to be a function of one argument

```
data What a b = What (b a) deriving Show
```

```
Prelude> :k What
What :: * -> (* -> *) -> *    -- Success
```

What?

```
data What a b = What (b a) deriving Show
```

```
Prelude> :t What "Hello"
What "Hello" :: What Char []
Prelude> :t What (Just "Ever")
What (Just "Ever") :: What [Char] Maybe
```

What holds any type that is a “parameterized container,” what *Tofu* wants:

```
Prelude> :k What
What :: * -> (* -> *) -> *
Prelude> :k Tofu
Tofu :: (* -> (* -> *) -> *) -> Constraint
Prelude> instance Tofu What where tofu x = What x
Prelude> tofu (Just 'a') :: What Char Maybe
What (Just 'a')
Prelude> tofu "Hello" :: What Char []
What "Hello"
```

```
Prelude> data Barry t k a = Barry a (t k)
Prelude> :k Barry
Barry :: (* -> *) -> * -> * -> *   -- Bizarre kind, by design
Prelude> :t Barry (5::Int) "Hello"
Barry (5::Int) "Hello" :: Barry [] Char Int
```

A *Barry* is two objects: any type and one built from a type constructor

```
Prelude> :k Functor
Functor :: (* -> *) -> Constraint    -- Takes a one-arg constructor
```

```
instance Functor (Barry t k) where    -- Partially applying Barry
  fmap f (Barry x y) = Barry (f x) y -- Applying f to first object
```

```
Prelude> fmap (+1) (Barry 5 "Hello")
Barry 6 "Hello"                       -- It works!
Prelude> fmap show (Barry 42 "Hello")
Barry "42" "Hello"
Prelude> :t fmap show (Barry 42 "Hello")
fmap show (Barry 42 "Hello") :: Barry [] Char String
```