Some Developments in the Tagged Signal Model

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The Tagged Signal Model

- A set of tags $T$, e.g. $T = [0, \infty)$
- A set of values $V$, e.g. $V = \mathbb{N}$
- An event $e$ is a pair of a tag and a value: $e = (t, v)$
- A signal $s$ is a set of events, e.g. $\text{clock}_1 = \{(0.0, 1), (1.0, 1), (2.0, 1), \ldots\}$
- A process $P$ is a relation on signals

\[
P = \{(s_1, s_2, s_3) | s_1 + s_2 - s_3 = 0 \}
\]


Xiaojun Liu, Berkeley 2
Signals and Processes

<table>
<thead>
<tr>
<th>Signals</th>
<th>Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics</td>
<td>Newton’s Laws</td>
</tr>
<tr>
<td>Velocities,</td>
<td></td>
</tr>
<tr>
<td>Accelerations, and</td>
<td></td>
</tr>
<tr>
<td>Forces</td>
<td></td>
</tr>
<tr>
<td>Electrical Engineering</td>
<td>Resistors and Capacitors,</td>
</tr>
<tr>
<td>Voltages and Currents</td>
<td>Kirchhoff’s Laws</td>
</tr>
<tr>
<td>Computer Science</td>
<td>Streams</td>
</tr>
<tr>
<td>Streams</td>
<td>Dataflow Processes</td>
</tr>
</tbody>
</table>

Approach

- Study the mathematical structure of signal sets
  - Partial order/CPO, topological/metric space, algebra
- Study the properties of processes as relations/functions on signals
  - Continuity
  - Causality
  - Composition
- From the declarative to the imperative
Signals

- Let $T$, a poset, be the set of all tags. Let $\mathcal{D}(T)$ be the set of down-sets of $T$.

- A signal is a function from a down-set $D \in \mathcal{D}(T)$ to some value set $V$,
  
  $$\text{signal: } D \rightarrow V$$

- Let $S(T, V)$ be the set of all signals from down-sets of $T$ to $V$.

Prefix Order on Signals

- A signal $s_1: D_1 \rightarrow V$ is a prefix of $s_2: D_2 \rightarrow V$, denoted $s_1 \leq s_2$, if and only if
  
  $$D_1 \subseteq D_2, \text{ and } s_1(t) = s_2(t), \forall t \in D_1$$
Prefix Order – Properties

- For any poset $T$ of tags and set $V$ of values, $S(T, V)$ with the prefix order is
  - a poset
  - a CPO
  - a complete lower semilattice (i.e. any subset of signals have a "longest" common prefix)

Tagged Process Networks

- A direct generalization of Kahn process networks

$$\begin{align*}
(x, y, z) & = F(x) \\
(y, z) & = P(x, z) \\
z & = Q(y)
\end{align*}$$

- If processes $P$ and $Q$ are Scott-continuous, then $F$ is Scott-continuous.
Timed Signals

- Let $T = [0, \infty)$, and $V_\varepsilon = V \cup \{\varepsilon\}$, where $\varepsilon$ represents the absence of value, $\mathcal{S}(T, V_\varepsilon)$ is the set of timed signals.

$$s(t) = 1$$

$$s\left(\frac{1}{1/k}\right) = 1, \ k = 1, 2, \ldots$$

$$s(k) = 1, \ k = 0, 1, 2, \ldots$$

Timed Processes

- $s_1: D_1 \rightarrow V_\varepsilon$
- $s_2: D_2 \rightarrow V_\varepsilon$
- $s: D \rightarrow V_\varepsilon$

**add**

$$D = D_1 \cap D_2$$

$$s(t) = s_1(t) + \varepsilon \cdot s_2(t)$$

**delay by 1**

$$D_2 = D_1 \oplus \{1\} \cup [0, 1)$$

$$s_2(t) = s_1(t-1), \ \text{when } t \geq 1$$

$$s_2(t) = \varepsilon, \ \text{when } t \in [0, 1)$$

**biased merge**

$$D = D_1 \cap D_2$$

$$s(t) = s_1(t), \ \text{when } s_1(t) \in V$$

$$s_2(t), \ \text{otherwise}$$
A Timed Process Network

A Non-Causal Process in the Network
Causality

- A timed process $P$ is causal if
  - It is monotonic, i.e. for all $s_1, s_2$
    $$s_1 \leq s_2 \Rightarrow P(s_1) \leq P(s_2)$$
  - For all $s$: $D_1 \rightarrow V_1$, $P(s): D_2 \rightarrow V_2$
    $$D_1 \subseteq D_2$$
- A timed process $P$ is strictly causal if it is monotonic, and
  - For all $s$: $D_1 \rightarrow V_1$, $P(s): D_2 \rightarrow V_2$
    $$D_1 \subset D_2 \text{ or } D_2 = [0, \infty)$$

Causality and Continuity

- Neither implies the other.
- A process may be continuous but not causal, e.g. “lookahead by 1”.
- A process may be causal but not continuous, e.g. one that produces an output event after counting an infinite number of input events.
If processes $P$ and $Q$ are causal and continuous, and at least one of them is strictly causal, then $F$ is causal and continuous.

\[(y, z) = F(x)\]

where $(y, z)$ is the least solution of the equations

\[y = P(x, z)\]
\[z = Q(y)\]
Discrete Event Signals - Properties

- For $T = [0, \infty)$ and any set $V$ of values, the set of all discrete event signals with the prefix order is
  - a poset
  - a CPO
  - a complete lower semilattice (i.e. any subset of signals have a “longest” common prefix)

A Discrete Event Process Network

Diagram:
- Delay by 1
- Biased merge

Xiaojun Liu, Berkeley 17

Xiaojun Liu, Berkeley 18
A Sufficient Condition for Non-Zeno Composition

If processes $P$ and $Q$ are discrete, causal and continuous, and at least one of them is strictly causal, then $F$ is discrete, causal and continuous.

- $F$ is non-Zeno in the sense that if $x$ is non-Zeno, $F(x)$ is non-Zeno.

Conclusions

- Progress in developing the foundation of the tagged signal model
- Extend Kahn process networks to tagged process networks
- Develop discrete event semantics as a special case of tagged process networks
- Develop a sufficient condition for the non-Zeno composition of discrete event processes