
Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. Write clearly and put a box around your answer, and show your work.

Print your name and SID below

Last Name ____________  First ____________  SID ____________

Problem 1:
Problem 2:
Problem 3:
Problem 4:
Problem 5:
Total:
1. 20 points

(a) Plot the Fourier Transform $X(\omega)$ of a signal $x \in \text{ContSignals}$ whose total energy is 2 and such that $X(\omega) = 0$ for $|\omega - 2\pi| > \pi$.

(b) Now find the time-domain signal $x$ by taking the inverse FT of $X$. 

2. **15 points** Fill in the blanks.

(a) The LT of \( x(t) = tu(t) \) is \[
\text{__________}
\] and its ROC is \[
\text{__________}
\].

(b) The LT of \( x(t) = e^{-t}u(t) \) is \[
\text{__________}
\] and its FT is \[
\text{__________}
\].

(c) The transfer function \( H(s) = \frac{s-1}{s+1} \) of an LTI system has a pole at \[
\text{__________}
\] and its impulse response is \( h(t) = \text{__________} \).
3. **20 points** Find the solution \( y(t), t \geq 0, \) of the differential equation

\[
\ddot{y}(t) - 3\dot{y}(t) + 2y(t) = 0,
\]

with initial condition \( y(0^-) = 1, \dot{y}(0^-) = 1. \) Check that your solution satisfies these initial conditions.
4. **20 points** In Figure 1 $K$ is a real constant. Find the closed-loop transfer function $H(s)$. Use the Routh test to determine the values of $K$ for which $H$ is stable.
5. **25 points** In Figure 2 \( m_1 \) and \( m_2 \) are real signals with real Fourier Transforms \( M_1(f) \) and \( M_2(f) \) respectively. Suppose that \( M_i(f) = 0 \), for \( |f| > 15 \) kHz. The carrier frequency \( f_c = 100 \) kHz.

(a) Determine the Fourier Transform \( X(f) \) of the modulated signal \( x \). Write an expression for \( |X(f)| \). What is the bandwidth of \( x \)?

(b) Find a scheme to demodulate \( x \) and recover both signals \( m_1 \) and \( m_2 \). Prove that your scheme works.