Test problems

Z Transform

1. Let $x(k)$ satisfy the difference equation

$$x(k) = x(k - 2) + 2^k, \quad k = 0, 1, 2, ...$$

with $x(-1) = x(-2) = 0$.

(a) Find the ZT $X(z)$ of $\{x(k)\}$.
(b) Find $\{x(k)\}$.

2. Consider the difference equation

$$y(k) + 2y(k - 1) + y(k - 2) = x(k) + 2x(k - 1), \quad k = 0, 1, 2, ...$$

(a) Find the transfer function $H(z)$. Is the system BIBO stable?
(b) Suppose $y(-1) = y(-2) = x(-1) = 1$, $x(k) = 0$ for $k \geq 0$. Find $y(k)$, $k \geq 0$.
(c) Obtain a direct form realization of the difference equation using only two delay elements.

3. Obtain a direct form realization of the transfer function

$$H(z) = \frac{1}{1 + 2z^{-1} + 2z^{-2}}$$

using only two delay elements.

4. Consider the analog transfer function $H(s) = 1/(s^2 + 5s + 6)$.

(a) Consider the scheme of Figure 1. Suppose $x(t) = u(t)$. Find $H_1(z)$ so that $\epsilon(nT) \equiv 0$, for all $n$. This is the step-invariant filter.
(b) Suppose $x(t) = \delta(t)$. Find $H_1(z)$ so that $\epsilon(nT) \equiv 0$. In this case assume that $x(nT)$ is the Kronecker delta. This is the impulse-invariant filter.
(c) Compare the frequency response $H(j\omega)$ and $H_1(e^{j\omega T})$ for $\omega \approx 0$.

5. In this problem we will consider the analog system

$$H_0(s) = \frac{1}{(1 + s)^2}$$

as a low pass filter with a pass band from 0 to 1 Hz. Note that the amplitude response at 1 Hz is 0.5 or -6 db.
(a) Draw the Bode plots of this filter (both amplitude and phase). Carefully mark the axes. In the amplitude response plot also indicate the magnitude of the slope.

(b) Suppose you have to design a low pass filter with the following specification:
    sampling frequency = 10 kHz, and pass band from 0 to 2.5 kHz with 6 db attenuation at 2.5 kHz.
    Using the filter \( H_a \) find a second order digital filter \( H_d(z) \) with this specification.

(c) Sketch the amplitude and phase response of your filter \( H_d \). Carefully mark the axes.