Problem 9.18

When we are using the proportional controller, we have \( L(s) = \frac{K_{p}(s+3.1)}{s(s+4.4)(s+4.4)} \).

The characteristic equation is \( s^4 + (K_p - 4.4^2)s^2 - K_p 3.1^2 = 0 \).

\[
\begin{bmatrix}
1 & K_p - 4.4^2 & -K_p 3.1^2 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

Using the Routh test, we see that the system is not stable for any \( K_p \).

If we let \( H(s) = \frac{Ks^2(s+4.4)}{(s+5)^2} \), \( L(s) = \frac{K_{p}(s+3.1)(s-3.1)}{s(s+4.4)(s+4.4)} \). Below is the root locus plot, it is obviously stable for some \( K \).

Problem 9.20

Let \( F(s) = 1 + KG(s)H(s) \) and multiply \( \frac{1}{K} \) on both side of the equation, we get \( R(s) = \frac{F(s)}{K} = \frac{1}{K} + G(s)H(s) \).

Now as we traverse the contour \( C \) clockwise, instead of investigating how many time \( KG(s)H(s) \) encircles the \(-1\), we want to see how many times \( G(s)H(s) \) encircles \(-\frac{1}{K}\).

Thus, \( \# \) of times \( G(s)H(s) \) encircles \(-\frac{1}{K} \) clockwise
\[
= (\# \) of times \( KG(s)H(s) \) encircles \(-1 \) clockwise)
= (\# \) of times \( \frac{1}{K} + G(s)H(s) \) encircles \(0 \) clockwise)
= (\# of zeros of \( \frac{1}{K} + G(s)H(s) \) inside \( C \) - (\# of poles of \( \frac{1}{K} + G(s)H(s) \) inside \( C \))
= (\# of zeros of \( \frac{1}{K} + G(s)H(s) \) inside \( C \) - (\# of poles of \( G(s)H(s) \) inside \( C \))
The above equation is derived from the fact that poles of \( \frac{1}{K} + G(s)H(s) \) are poles of \( G(s)H(s) \).

Since we take the contour \( C \) to be the entire right half of \( s \)-plane except for poles at the \( j\omega \) axis, 

\[
\text{(# of zeros of } \frac{1}{K} + G(s)H(s) \text{ inside } C) \\
= \text{(# of zeros of } \frac{1}{K} + G(s)H(s) \text{ in the right half of } s \text{-plane}) \\
= \text{( # of poles of the closed loop transfer function in the right half of the } s \text{-plane)}
\]

and, \( \text{( # of poles of } G(s)H(s) \text{ inside } C) \\
= \text{( # of poles of } G(s)H(s) \text{ inside the right half of the } s \text{-plane}) \\
= \text{( # of poles of } \frac{1}{K} + G(s)H(s) \text{ inside the right half of the } s \text{-plane})
\]

Thus, \( \text{( # of times } G(s)H(s) \text{ encircles } -\frac{1}{K} \text{ clockwise}) \\
= \text{( # of poles of the closed loop transfer function in the right half of the } s \text{-plane) - ( # of poles of } G(s)H(s) \text{ inside the right half of the } s \text{-plane)}
\]

If we want a system to be stable, we always want ( # of poles of the closed loop transfer function in the right half of the \( s \)-plane) = 0.

In this problem, we have \( L(s) = \frac{K}{s(s+1)(s+2)} \Rightarrow G(s)H(s) = \frac{1}{s(s+1)(s+2)} \). Obviously, we see that \( G(s)H(s) \) has two poles at \( s = -1 \). Thus, \( \text{( # of poles of } G(s)H(s) \text{ inside the right half of the } s \text{-plane}) = 0 \). From above equation, we get \( \text{( # of times } G(s)H(s) \text{ encircles } -\frac{1}{K} \text{ clockwise) should be zero.} \)

From the figure below, we can see that for \( K > 0 \), \( -\frac{1}{K} \) will never be inside the loop. Therefore, the system is stable for all \( K > 0 \).

Problem 9.25

(a) From the problem, \( L(s) = \frac{K}{s(s+1)(s+2)} \) and \( G(s)H(s) = \frac{1}{s(s+1)(s+2)} \). We first plot the nyquist diagram (in matlab, you cannot see the loop unless you zoom in.)
We also know that the (# of poles of $G(s)H(s)$ inside the right half of the $s$-plane) = 0. So we want (# of times $G(s)H(s)$ encircles $-\frac{1}{K}$ clockwise) = 0 also. This implies that $-\frac{1}{K}$ should not be in the loop. We need to calculate the exact value of point $\alpha$ in the figure. To do that, we see $\alpha$ is a real value, i.e. $Im\{\alpha\} = 0$.

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)} = \frac{j\omega(-\omega^2-3j\omega+2)}{-\omega^2+|j\omega+1|(j\omega+2)}$$

Then, $Im\{G(j\omega)H(j\omega)\} = \frac{2\omega^2-\omega^3}{-\omega^2+|j\omega+1|(j\omega+2)} = 0 \Rightarrow 2\omega = \omega^3 \Rightarrow \omega = \sqrt{2}, -\sqrt{2}$.

\[ \alpha = Re\{G(j\omega)H(j\omega)\}|_{\omega=\sqrt{2}} = -\frac{6}{2}\sqrt{2} = -\frac{1}{\sqrt{2}}. \]

Obviously, for $0 < K < 6$, $-\frac{1}{K}$ will not be in the loop; thus, the system is stable.

Using Routh Criteria, the closed loop characteristic equation is $s^3 + 3s^2 + 2s + K = 0$.

\[
\begin{array}{cccc}
1 & 2 & \frac{6-K}{3} & 0 \\
3 & K & \frac{6-K}{3} & 0 \\
K & 0 & \frac{6-K}{3} & 0 \\
\end{array}
\]

For stability, we need $\frac{6-K}{3} > 0$ and $K > 0$. That is $0 < K < 6$.

(b) For $K = 2$, $L(s) = \frac{2j\omega(j\omega+1)(j\omega+2)}{j\omega(j\omega+1)(j\omega+2)}$. To find the gain margin, we need to find $K$ such that the following equation holds true.

$$j\omega_1(j\omega_1+1)(j\omega_1+2) = 1 \text{ where } \omega_1 \text{ satisfies the equation } -\frac{1}{2} + tan^{-1}(\omega_1) + tan^{-1}(\frac{2}{\omega_1}) = -\pi. \text{ We get } \omega_1 = 1.414 \Rightarrow \text{gain margin } = \frac{1}{\frac{1}{|L(j\omega_1)|}} = 3 = 9.5dB.$$

To find the phase margin, we want $e^{-j\phi}L(j\omega_2) = -1$ where $\omega_2$ satisfies the equation $|L(j\omega_2)| = 1$. We get $\omega_2 = 0.749$. So $tan^{-1}(0.749) + tan^{-1}(\frac{2.749}{\omega_2}) + \frac{\pi}{2} = 147.36^\circ$. phase margin = $180 - 147.36 = 32.6^\circ$.

(c) Here the phase margin = $20^\circ = 0.349$. That is $-0.349 + angle \{L(j\omega)\} = -\pi \Rightarrow angle \{L(j\omega)\} = -2.792 rad \Rightarrow \frac{\pi}{2} + tan^{-1}(\omega) + tan^{-1}(\frac{2.792}{\omega}) = 2.792 \Rightarrow \omega = 1$. Since we want $\frac{K}{|L(j\omega+1)(j\omega+2)|} = 1$, we get $K = |j\omega(j\omega+1)(j\omega+2)| = 3$ in which $\omega = 1$.

Problem 9.29

% Problem 9.29(a)

\[
\text{sys = tf([50],[1 3 2]);} \\
\text{bode(sys);} \\
\text{title('Prob 9.29 (a)');} \\
\]

\[3\]
% Problem 9.29(b)

sys = tf([10],[1 8 17 10]);
bode(sys);
title('Prob 9.29 (b)');

% Problem 9.29(c)

sys = tf([5],[1 3 3 1]);
bode(sys);
title('Prob 9.29 (c)');
% Problem 9.29(d)

sys = tf([10 5],[1 8 17 10]);
bode(sys);

title('Prob 9.29 (d)');