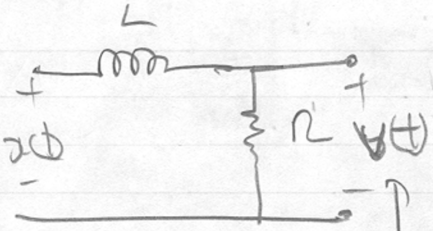
(15)



should be $y(t)$!

$$\frac{L}{R}\dot{y}(t) + y(t) = x(t)$$

(a) Let $x(t) = e^{i\omega t}$

$$\Rightarrow y(t) = \hat{H}(\omega)e^{i\omega t}$$

(By definition)

$$\therefore \frac{L}{R}\left[\hat{H}(\omega)\, i\omega\, \cancel{e^{i\omega t}}\right] + \hat{H}(\omega)\cancel{e^{i\omega t}} = \cancel{e^{i\omega t}}$$

$$\hat{H}(\omega)\left[\frac{L}{R}i\omega + 1\right] = 1$$

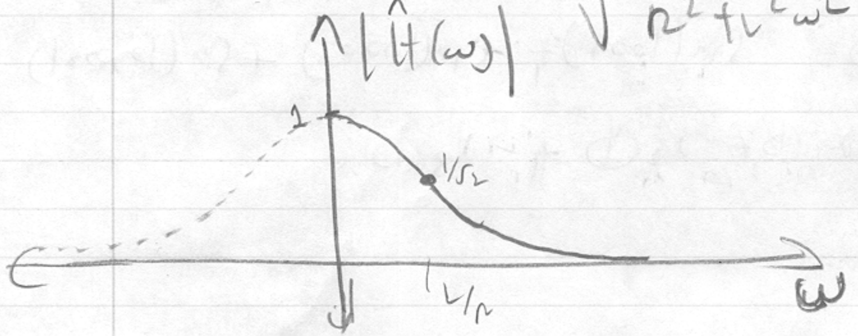
$$\Rightarrow \hat{H}(\omega) = \frac{1}{\frac{L}{R}i\omega + 1} \Rightarrow \hat{H}(\omega) = \frac{1\, e^{i0}}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}\, e^{+i\tan^{-1}\left(\frac{\omega L}{R}\right)}}$$

if $\tau = L/R$

$$\hat{H}(\omega) = \frac{1}{1+i\omega\tau}$$

$$\Rightarrow \boxed{\hat{H}(\omega) = \frac{R}{\sqrt{R^2+L^2\omega^2}}\, e^{-i\tan^{-1}\left(\frac{L\omega}{R}\right)}}$$

(b) Now, $$|\hat{H}(\omega)| = \frac{R}{\sqrt{R^2+L^2\omega^2}}$$



@ $\omega = \frac{1}{L/R} = R/L$

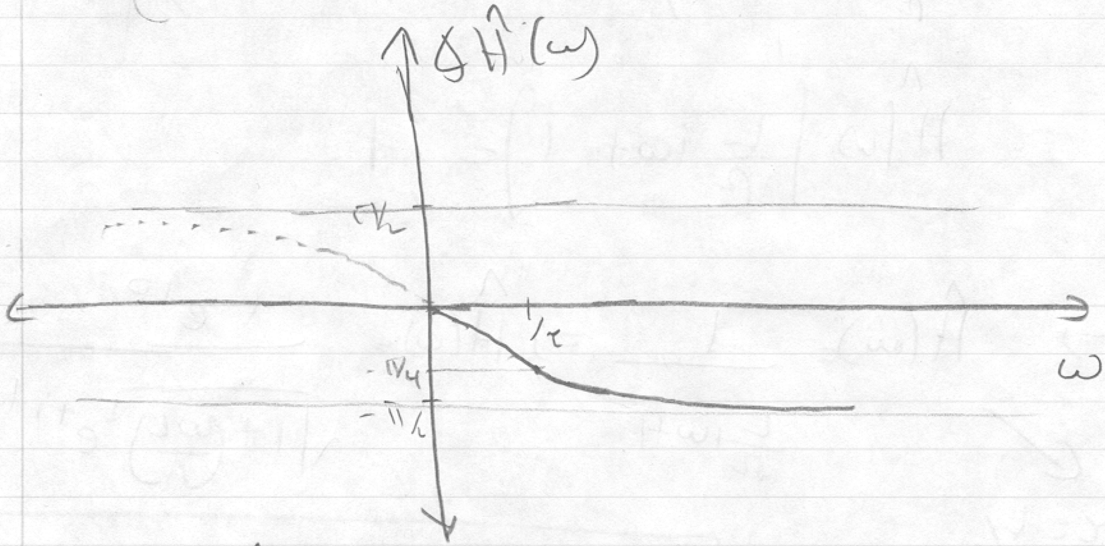
$$|\hat{H}(R/L)| = \frac{R}{\sqrt{R^2+R^2}} = 1/\sqrt{2}$$

$$\measuredangle \hat{H}(\omega) = -\tan^{-1}\left(\frac{L\omega}{R}\right)$$

@ $\omega = 0$, $\measuredangle \hat{H}(\omega) = 0$

& $\measuredangle \hat{H}\left(\frac{1}{\tau}\right) = -\tan^{-1}\left(\frac{L}{R}\frac{R}{L}\right) = -\pi/4$

as $\omega \to \infty$, $\measuredangle \hat{H}(\omega) \to -\pi/2$



Notice that the magnitude response is an even function and the phase response is an odd function.

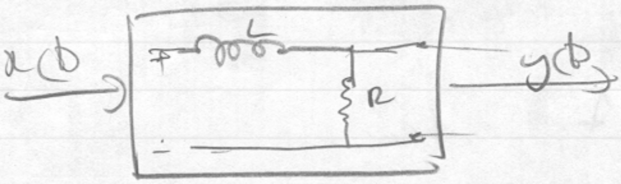
(c) Now, $\hat{H}(\omega) = \dfrac{1}{1+i\omega}$.

We have: $x(t) = \sin(100t) + \sin(1000t) + \sin(10{,}000t)$

$= x_1(t) + x_2(t) + x_3(t)$.

Suppose:

H



we want $H(x)(t)$

$$= H(x_1 + x_2 + x_3)(t)$$

$$= H(x_1)(t) + H(x_2)(t) + H(x_3)(t) \quad \text{System is Linear!!!}$$

$$= |\hat{H}(100)| \sin(100t + \measuredangle \hat{H}(100))$$

$$+ |\hat{H}(1000)| \sin(1000t + \measuredangle \hat{H}(1000))$$

$$+ |\hat{H}(10000)| \sin(1000t + \measuredangle \hat{H}(10000))$$

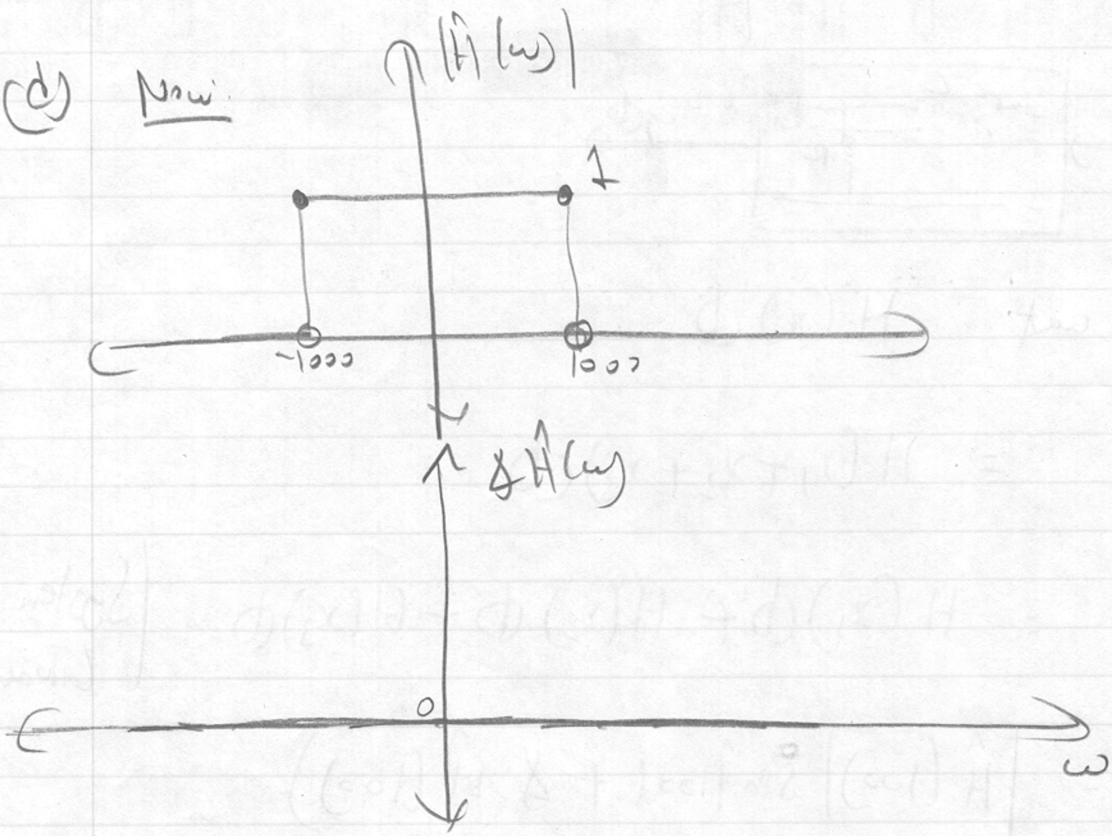
(By definition)

$$= 0.01 \sin(100t - 1.561) + 1 \times 10^{-3}(\sin(1000t + -1.569))$$

$$+ 1 \times 10^{-4}(\sin(1000t + -1.57))$$

Notice that as $\omega \to \infty$, the signal gets attenuated more and more. It also suffers a phase reversal of $-\pi/2$ (~ -1.57 radians)

(d) Now:



In this case, only the first two signals pass through, since:

$$|\hat{H}(100)| = |\hat{H}(1000)| = 1$$

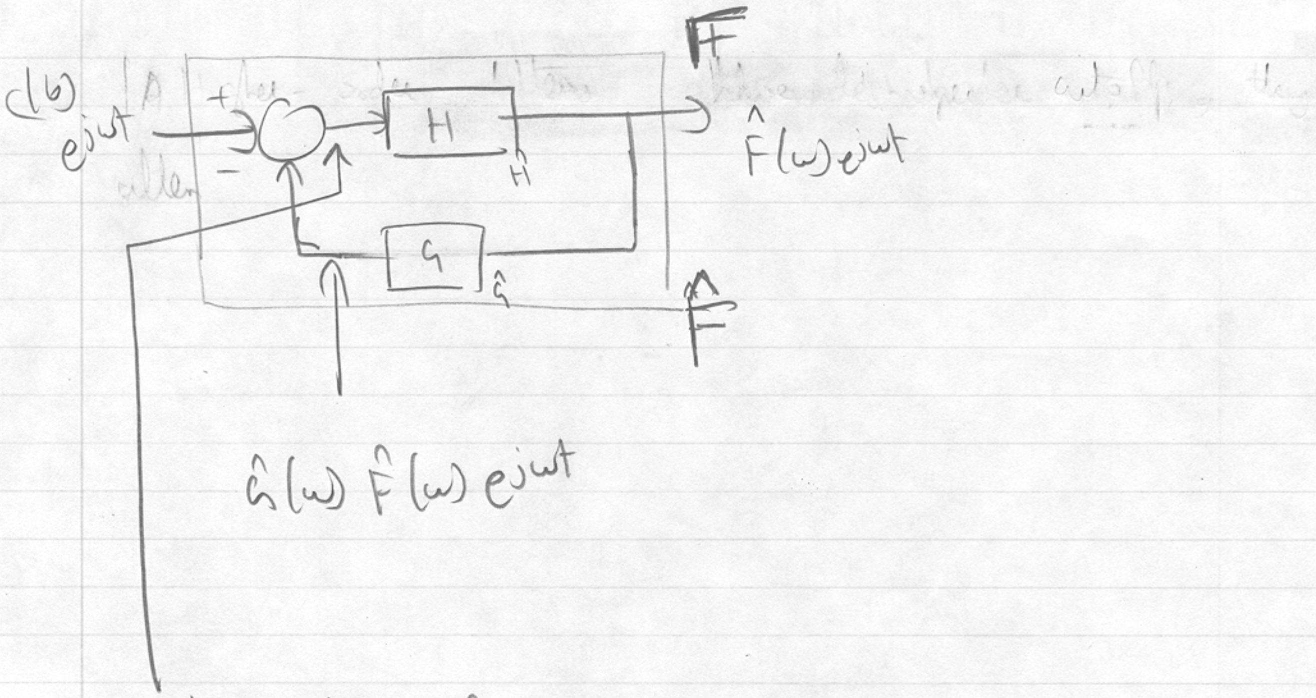
& $$\measuredangle \hat{H}(100) = \measuredangle \hat{H}(1000) = 0$$

However $|\hat{H}(10000)| = 0$, $\measuredangle \hat{H}(10000) = 0$

$$\boxed{y(t) = \sin(100t) + \sin(1000t)}$$

(e) The circuit is called low-pass because it allows only low values of frequencies to pass through with little attenuation. It is called first-order because $\hat{H}(\omega) \propto \frac{1}{\omega}$

(b)



$e^{j\omega t} - \hat F(\omega)\hat G(\omega)e^{j\omega t}$

Therefore, $\hat H(\omega)\cancel{e^{j\omega t}} - \hat H(\omega)\hat F(\omega)\hat G(\omega)\cancel{e^{j\omega t}} = \hat F(\omega)\cancel{e^{j\omega t}}$

$$\Rightarrow \frac{\hat H(\omega)}{1+\hat H(\omega)\hat G(\omega)} = \underline{\underline{\hat F(\omega)}}$$

(a) Now $\hat F(\omega) = \dfrac{\frac{1}{1+j\omega\tau}}{1+\frac{1}{1+j\omega\tau}} = \underline{\underline{\dfrac{1}{2+j\omega\tau}}}$

(b) $\hat F(\omega) = \dfrac{1}{1+\frac{1}{1+j\omega\tau}} = \underline{\underline{\dfrac{1+j\omega\tau}{2+j\omega\tau}}}$

Practice problems (cont'd)

(1) Now $H(\omega) = \sum_{k=-\infty}^{\infty} h(k) e^{-i\omega k}$ [By def.]

$$H(\omega) = \sum_{k=0}^{4} e^{-i\omega k} \quad \left[\text{From the given } h(k)\right]$$

$$= 1 + e^{-i\omega} + e^{-i\omega 2} + e^{-i\omega 3} + e^{-i\omega 4}$$

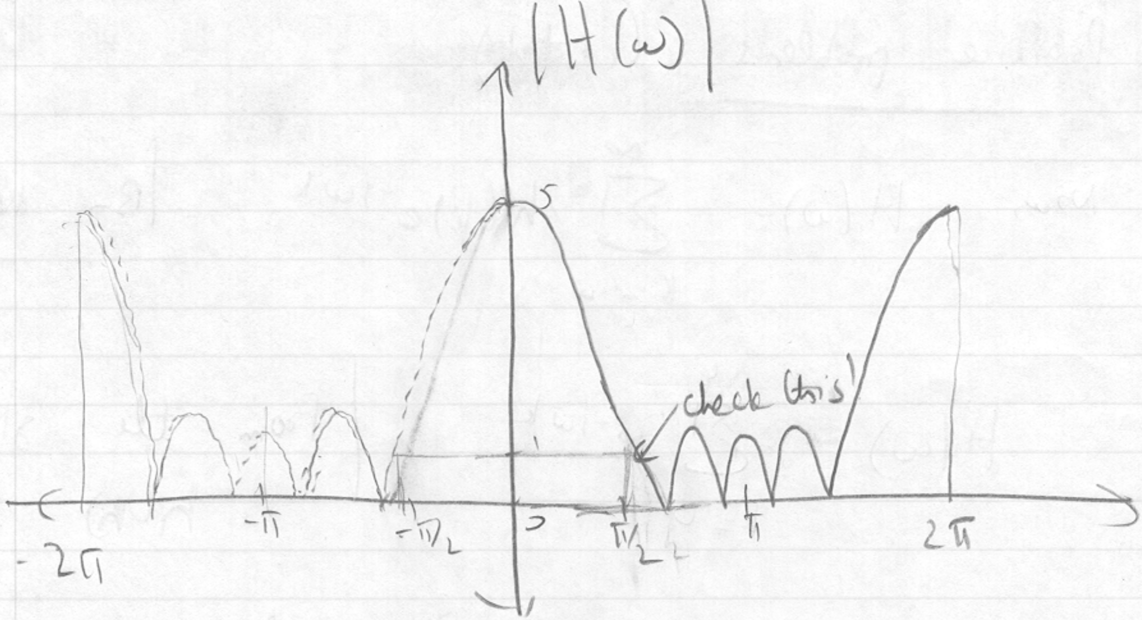
$$= 1 + \cos(\omega) + \cos(2\omega) + \cos(3\omega) + \cos(4\omega)$$

$$- i\left[\sin(\omega) + \sin(2\omega) + \sin(3\omega) + \sin(4\omega)\right]$$

$$|H(\omega)| = \sqrt{\left[1 + \cos(\omega) + \cos(2\omega) + \cos(3\omega) + \cos(4\omega)\right]^2 + \left[\sin(\omega) + \sin(2\omega) + \sin(3\omega) + \sin(4\omega)\right]^2}$$

$$\phi\, H(\omega) = -\tan^{-1}\left[\frac{\sum_{k=1}^{4} \sin(k\omega)}{1 + \sum_{k=1}^{4} \cos(k\omega)}\right]$$

This is a 5-point MA filter, ~~of order~~



Why is $|H(\omega)|$ 2π periodic?

(2) Let $x(n) = e^{i\omega n}$

Do this in matlab!

$\Rightarrow y(n) = H(\omega) e^{i\omega n}$

$H(\omega) e^{i\omega(n-1)} = e^{i\omega n} + e^{i\omega(n-2)}$

$\Rightarrow H(\omega) \cancel{e^{i\omega n}} \cdot e^{-i\omega} = \cancel{e^{i\omega n}} + \cancel{e^{i\omega n}} \cdot e^{-i\omega 2}$

$\Rightarrow \boxed{H(\omega) = \dfrac{1 + e^{-i\omega 2}}{e^{-i\omega}}}$

(3) Check q. 16 of the first set of questions.

(i) $\text{Response}(\omega) = \dfrac{H(\omega)}{1 - H(\omega)}$

(ii) $\text{Response}(\omega) = \dfrac{1}{1 - G(\omega)}$

(iii) $\text{Response}(\omega) = H(\omega) / [1 - H(\omega) G(\omega)]$