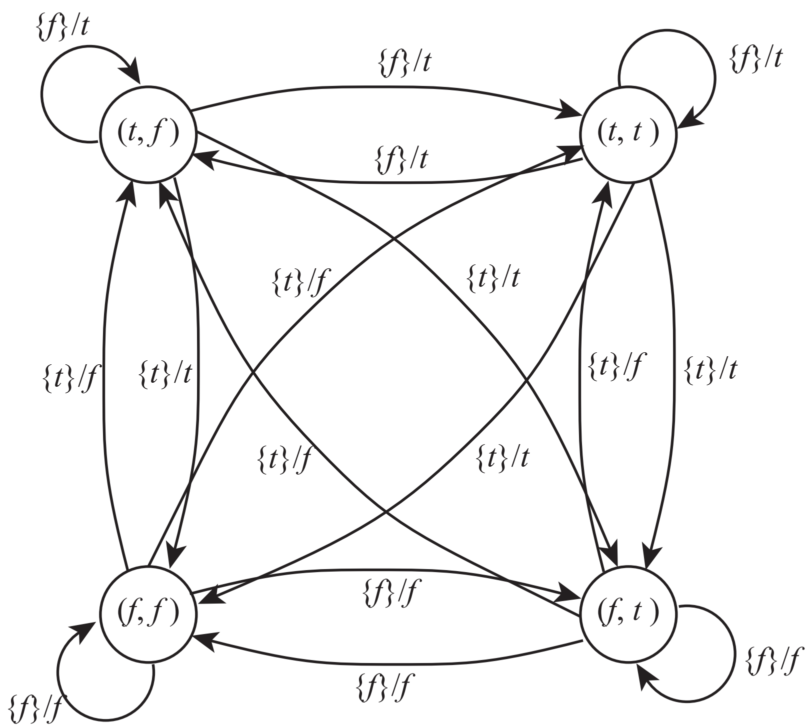
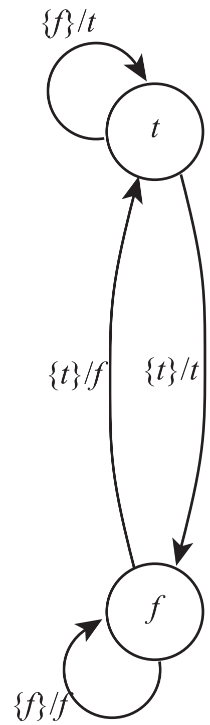
EECS 20. Final Exam Solutions
December 19, 2002.

1. (a) d
   (b) a
   (c) none
   (d) c
   (e) e
   (f) b

2. (a) The solution is:



   (b) $(f, f, f, t, f, f, \cdots)$.
   (c) The solution is:



The bisimilarity relation is

$$\{((t,f),t),((t,t),t),((f,t),f),((f,f),f)\}.$$

3. (a) false
   (b) true
   (c) true
   (d) false
   (e) true
   (f) true
   (g) true

4. (a) $p = 8$ and $\omega_0 = \pi/4$ radians/sample.
   (b) $K = 4$, $A_0 = A_1 = A_2 = 1$, $A_3 = A_4 = 0$, $\phi_1 = 0$, $\phi_2 = -\pi/2$, and the values of ϕ_3 and ϕ_4 do not matter.
   (c) $X_0 = 1$, $X_1 = 1/2$, $X_2 = 1/2i = -i/2$, $X_3 = X_4 = X_5 = 0$, $X_6 = -1/2i = i/2$, and $X_7 = 1/2$.
   (d) The output is

   $$y(n) = 1 + \sin(\pi n/4) - \cos(\pi n/2).$$

   (e) The frequency response of the composite is

   $$\forall\, \omega \in \textit{Reals}, \quad H'(\omega) = 1 + iH(\omega),$$

by linearity, or

$$H'(\omega) = \begin{cases} 2 & \text{if } 0 < \omega < \pi \\ 1 & \text{if } \omega = 0 \text{ or } \omega = \pi \\ 0 & \text{if } -\pi < \omega < 0 \end{cases}$$

For other values of ω, the value of $H'(\omega)$ is determined by the fact that it is periodic with period 2π.

(f) The output is

$$y(n) = 1 + e^{i\pi n/4} - ie^{i\pi n/2}$$

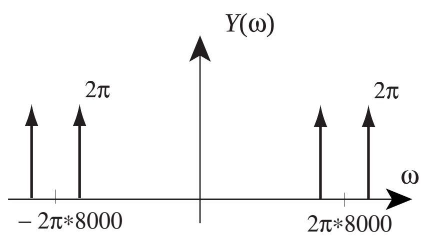
or equivalently

$$y(n) = 1 + e^{i\pi n/4} + e^{i(\pi n/2 - \pi/2)}.$$

5. (a) The CTFT is, by linearity,

$$\forall\, \omega \in \textit{Reals}, \quad Y(\omega) = 2\pi(\delta(\omega - (\omega_c + \omega_x)) + \delta(\omega + (\omega_c + \omega_x)) + \delta(\omega - (\omega_c - \omega_x)) + \delta(\omega + (\omega_c -$$

The sketch is:



(b) For all $t \in \textit{Reals}$,

$$u(t) = e^{i(\omega_c + \omega_x)t} + e^{i(\omega_c - \omega_x)t}.$$

(c) For all $n \in \textit{Integers}$,

$$u'(n) = u(nT) = 2\cos(2\pi 400 n/8000) = 2\cos(\pi n/10).$$

(d) For all $t \in \textit{Reals}$,

$$z(t) = 2\cos(2\pi 400 t).$$