EECS 20. Midterm No. 2
November 8, 2002.

Please use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab day and time below

Name: _____

Lab time: _____

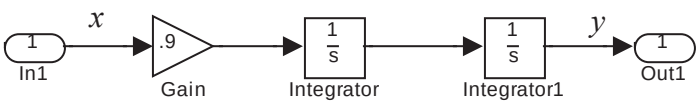
Problem 1:

Problem 2:

Problem 3:

Total:

1. **40 points (10 points each part).** Consider the Simulink diagram shown below:



This shows an LTI system with one input and one output, both of which are continuous-time signals. The input and output are indicated by the rounded boxes, and are labeled x and y. The gain is 0.9, and the integrators both have initial condition equal to 0.

(a) Write a differential equation (with no integrals, just derivatives) that relates the input x and the output y.

(b) Give the $[A, b, c, d]$ representation of this system.

(c) Find the frequency response $H: \textit{Reals} \rightarrow \textit{Reals}$ of this system.

(d) Find the output of the system if the input x is given by

$$\forall\, t \in \textit{Reals}, \quad x(t) = \cos(2t).$$

2. **30 points (5 points each part).** Consider continuous-time systems with input x: *Reals* $\rightarrow$ *Reals* and output y: *Reals* $\rightarrow$ *Reals*. Each of the following defines such a system. For each, indicate whether it is linear (L), time-invariant (TI), both (LTI), or neither (N). Note that no partial credit will be given for these questions.

   (a) $\forall\, t \in$ *Reals*, $\quad \dot{y}(t) = x(t) + 0.9y(t)$

   (b) $\forall\, t \in$ *Reals*, $\quad y(t) = \cos(2\pi t)x(t)$

   (c) $\forall\, t \in$ *Reals*, $\quad y(t) = x(t-1)$

   (d) $\forall\, t \in$ *Reals*, $\quad y(t) = x(t) + 0.1(x(t))^2$

   (e) $\forall\, t \in$ *Reals*, $\quad y(t) = x(t) + 0.1(x(t-1))^2$

   (f) $\forall\, t \in$ *Reals*, $\quad y(t) = 0$

3. **40 points (10 points each part).** Consider a discrete-time signal x: *Integers* $\rightarrow$ *Reals* defined by

$$\forall\, n \in \textit{Integers}, \quad x(n) = 1 - \cos(3\pi n/4).$$

Assume this signal is sampled at 8,000 samples/second.

(a) Give the frequency of the cosine term in Hz (cycles/second).

(b) Give period of x.

(c) Give the fundamental frequency (in any units, but be sure to give the units).

(d) Give the coefficients $A_0, A_1, A_2, \cdots, A_K$ and $\phi_1, \phi_2, \cdots, \phi_K$ of the Fourier series expansion for x,

$$x(n) = A_0 + \sum_{k=1}^{K} A_k \cos(k\omega_0 n + \phi_k)$$

where

$$K = \begin{cases} (p-1)/2 & \text{if } p \text{ is odd} \\ p/2 & \text{if } p \text{ is even} \end{cases}$$