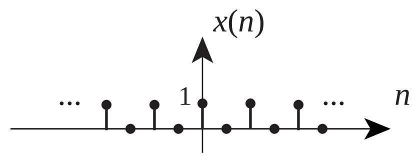
EECS20n, Quiz 4 Solution

1. Consider a discrete-time signal x given by
$$\forall\, n \in \textit{Integers}, \quad x(n) = \sum_{k=-\infty}^{\infty} \delta(n - 2k),$$
where δ is the Kronecker delta function. Sketch this signal.

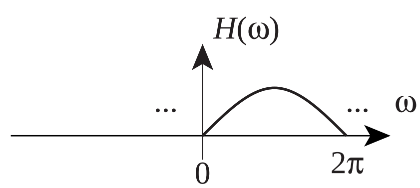
Solution:



2. For the same signal as in the previous problem, find the Fourier series coefficients X_k in
$$x(n) = \sum_{k=0}^{p-1} X_k e^{i\omega_0 kn}.$$
Solution: Note that $p = 2$ so $\omega_0 = \pi$. By inspection, therefore, $X_0 = X_1 = 1/2$.

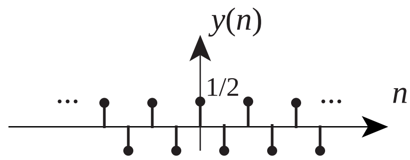
3. Consider a discrete-time LTI system with frequency response H given by
$$\forall\, \omega \in \textit{Reals}, \quad H(\omega) = |sin(\omega/2)|.$$
Sketch this over one period.

Solution:



4. Suppose the signal in problem 1 is the input to the system in problem 3. Find the output y and sketch it. ("Find" means give an expression for $y(n)$ that is valid for all integers n).

Solution: Since we have the Fourier series for the input, we can just scale each term by the frequency response, as follows:
$$\forall\, n \in \textit{Integers}, \quad y(n) = \sum_{k=0}^{1} X_k H(k\omega_0) e^{i\omega_0 kn}$$
This becomes
$$y(n) = (1/2)H(0) + (1/2)H(\pi)e^{i\pi n} = (1/2)(-1)^n.$$
Here is a sketch:



y(n)
1/2
...
...
n