1. An audio signal $x$ has Fourier Transform $X$ such that $X(\omega) = 0, |\omega| > 2\pi \times 10,000 \text{ rad/sec}$. The transmitted signal is $y(t) = \cos(2\pi \times f_c t) \times x(t)$, in which the carrier frequency is $f_c = 100,000 \text{ Hz}$.

1. **5 points** For what values of $\omega$ is $Y(\omega) = 0$?

2. **5 points** An AM receiver constructs the signal $z(t) = y(t) \times \cos(2\pi f_c t)$. Express $z$ in terms of $x$.

**Answer 1.** Let $\omega_0 = 20,000 \times \pi$. Then,

\[
Y(\omega) = \frac{1}{2} [X(\omega - \omega_0) + X(\omega + \omega_0)],
\]

so

\[
Y(\omega) = 0, \text{ unless } |\omega - \omega_0| \leq 10,000\pi \text{ or } |\omega + \omega_0| \leq 10,000\pi.
\]

2. Directly from $z(t) = x(t)[\cos(\omega_0 t)]^2$ or from

\[
Z(\omega) = \frac{1}{2} [Y(\omega - \omega_0) + Y(\omega + \omega_0)]
\]

\[
= \frac{1}{2} X(\omega) + \frac{1}{4} [X(\omega - 2\omega_0) + X(\omega + 2\omega_0)],
\]

one gets

\[
z(t) = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos(2\omega_0 t).
\]

2. **10 points** Consider the feedback system below. First find the frequency response $H$ and then the impulse response $h$ for $k = 1, 10, 100$. [Hint Recall $e^{-t}u(t) \leftrightarrow \frac{1}{1+i\omega}$ and the time change formula $x(at) \leftrightarrow \frac{1}{|a|} X(\frac{\omega}{a})$.

![Feedback System Diagram]

**Answer** The frequency response is

\[
\forall \omega, \quad H(\omega) = \frac{k}{(k + 1) + i\omega},
\]

and so the impulse response is

\[
\forall t, \quad h(t) = ke^{-(k+1)t}u(t).
\]