We know your quiz score is important to you, and we take it very seriously. We want our evaluation process to be generous to each of you, yet fair to all of you. Grading inconsistencies can occur despite our best efforts. Accordingly, you can appeal your quiz score by adhering to the following mandatory guidelines.

- As a first step, study the official quiz solutions carefully. To address any lingering concern, arrange a meeting with your lab GSI or one of the lecturers (respectively, your first and second lines of defense) to discuss how a particular aspect of your quiz performance was assessed.

- We can accept only written requests for quiz regrades, however mundane the issue at stake may seem. Without exception, no member of the teaching staff will accommodate an oral or any approximation to a real-time regrade request.

- Quiz 1 regrade request materials must be physically handed to a member of the teaching staff no later than the end of lecture on Friday, 7 October, 2005; no regrade request will be accepted after this deadline.

- Please do not add to, delete from, or in any other way modify the papers bearing your original, scored, handwritten responses to the quiz problems. It is a serious breach of the Code of Student Conduct to submit quiz papers for regrade after any portion thereof has been modified. A violation of this clause will be prosecuted vigorously.

- On paper distinct from your original, scored, handwritten quiz submission, draft an “appeal brief” by identifying the portions of your quiz solutions that you want us to reconsider and explaining therein the basis of your appeal for each identified portion. Please follow this procedure even if you want to point out a possible arithmetic error in the computation of your quiz score.

- Only quiz papers submitted in entirety and bearing your original, scored, handwritten responses to the quiz problems are acceptable for regrade consideration, and they must be submitted with the appeal brief described above. A facsimile representation (e.g., a photocopy) of your original quiz submission is not acceptable unless it is submitted in addition to your original quiz papers; is part, or in lieu, of the appeal brief; and is clearly annotated.

- Any regrade request automatically triggers a thorough reexamination of the entire quiz, not merely the portion under appeal. Accordingly, reexamination may have a favorable, adverse, or neutral effect on your overall quiz score.
• (5 Points) Print your name and lab time in legible, block lettering in the appropriate spaces provided above.

• This quiz should take you up to 15 minutes to complete. You will be given at least 15 minutes—up to a maximum of 20 minutes—to work on the quiz.

• **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct.

• **The quiz printout consists of pages numbered 1 through 6.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the six numbered pages. If you find a defect in your copy, notify the staff immediately.

• Please write neatly and legibly, because if we can’t read it, we can’t grade it.

• For each problem, limit your work to the space provided specifically for that problem. **No other work will be considered in grading your quiz. No exceptions.**

• Unless explicitly waived by the specific wording of a problem, you will receive full credit for the problem only if you justify your answer and explain your work clearly.

• We hope you do a **fantastic** job on this quiz.

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You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.
Problem 1 (20 Points) Consider the set

\[ X = \{a \to \{1, 2, 3\} \}. \]

(a) (5 Points) How many elements are in the set \(X\)? Explicitly specify one element in \(X\).

There are three elements in \(X\), each of which is a function whose graph is in the set \(\{(a, 1)\}, \{(a, 2)\}, \{(a, 3)\}\}. The set \(X\) specifies three functions \(f_i : \{a\} \to \{1, 2, 3\}\), \(i = 1, 2, 3\), where \(f_1(a) = 1\), \(f_2(a) = 2\), and \(f_3(a) = 3\).

(b) (5 Points) How many elements are in the set \(Y = [X \to X]\)? Explicitly specify one element in \(Y\).

There are 27 elements in \(Y\), each of which is a function whose graph has the form \(\{(a, 1), \alpha\}, \{(a, 2), \beta\}, \{(a, 3), \gamma\}\), where each of \(\alpha\), \(\beta\), and \(\gamma\) is an element of \(\{(a, 1), (a, 2), (a, 3)\}\), thereby offering three possibilities each, for a total of \(3^3\) choices of functions in \(Y\).

The set \(Y\) consists of 27 functions \(g_1, \ldots, g_{27}\), where \(g_j : \{f_1, f_2, f_3\} \to \{f_1, f_2, f_3\}\) for \(j = 1, \ldots, 27\). For example, the function \(g_1\) may be characterized by: \(g_1(f_1) = f_1, g_1(f_2) = f_1,\) and \(g_1(f_3) = f_1.\)

(c) (10 Points) By circling an appropriate answer choice, state whether each of the following assertions, considered independently of the other, is true or false. No explanation is necessary:

i. \(X \subset \{\{a, b\} \to \{1, 2, 3\}\}\).

\[ \boxed{\text{True}} \quad \boxed{\text{False}} \]

One element in \(\{\{a, b\} \to \{1, 2, 3\}\}\) is a function having a graph \(\{(a, 1), (b, 1)\}\). This graph is not in the set \(\{(a, 1)\}, \{(a, 2)\}, \{(a, 3)\}\) given in part (a).

ii. \(X \subset \{a \to \{1, 2, 3, 4\}\}\).

\[ \boxed{\text{True}} \quad \boxed{\text{False}} \]

The set \(\{a \to \{1, 2, 3, 4\}\}\) consists of four functions. Each function has a graph in the set \(\{(a, 1)\}, \{(a, 2)\}, \{(a, 3)\}, \{(a, 4)\}\}, clearly a superset of the set of graphs of the functions in \(X\)—given by \(\{(a, 1)\}, \{(a, 2)\}, \{(a, 3)\}\) in part (a).
Problem 2 (20 points) Let $X = [N_0 \rightarrow \{0, 1\}]$, where $N_0 = \{0, 1, 2, 3, \ldots \}$.

Consider a system

$$F : X \rightarrow X$$

$$\forall x \in X, \forall n \in N_0, \ (F(x))(n) = \begin{cases} 
0 & \text{if } n \mod 3 = 0 \\
1 & \text{otherwise}
\end{cases}.$$

(Note: The numbers 0, 3, 6, 9, \ldots are divisible by 3; therefore, they have mod 3 equal to zero. Modulo 3 numbers work as follows. For any $n \in N_0$, $n \mod 3 = k$, where $0 \leq k \leq 2$ is the unique number in $N_0$ such that $3$ divides $n - k$. Thus, there are only three distinct modulo 3 numbers, namely, 0, 1, 2.)

(a) (6 Points) Is the system $F$ memoryless? Justify your answer.

No. For example, consider the input sequence

$$x(0) = 0, x(1) = 0, \ldots$$

Assume the system is memoryless. Then the output values $(F(x))(0)$ and $(F(x))(1)$ must be equal. Clearly, this is not the case, because according to the description given, $(F(x))(0) = 0$ whereas $(F(x))(1) = 1$. We have a contradiction. Therefore, the system cannot be memoryless.

(b) (6 Points) Is the system $F$ causal? Justify your answer.

Yes. Any two input signals produce the same output signals $\forall n \in N_0$. Therefore, if two distinct input signals $x_1$ and $x_2$ are identical up to $n \leq n_0$, $\exists n_0 \in N_0$, it must be true that their corresponding outputs must be equal for $n \leq n_0$; in fact, in this case, their corresponding outputs will be identical $\forall n \in N_0$.

Note: The output function can be written in more compact form as follows:

$$(F(x))(n) = \min(1, n \mod 3).$$
(c) (8 Points) Determine the input alphabet and the output alphabet for, and draw the state transition diagram of, a state machine that implements the system $F$. Ignore stuttering symbols, but clearly specify the initial state on the state transition diagram that you provide.

The input alphabet and the output alphabet are identical, and each is given by $\{0, 1\}$. The state transition diagram, including the initial state, is shown below:

![State Transition Diagram]

- The input and output alphabet are both $\{0, 1\}$.
- The state transitions are as follows:
  - From state 0 to state 1 on input 0.
  - From state 1 to state 2 on input 1.
  - From state 2 to state 0 on input 1.
  - From state 0 to state 0 on input 1.

The initial state is clearly specified on the diagram.
You may use this page for scratch work only. Without exception, subject matter on this page will *not* be graded.