Sample problems for Fall 99 MT1

1. Consider the following continuous-time systems: $\forall t$,

$$\begin{aligned} H_1(x)(t) &= x(t-1), \\ H_2(x)(t) &= x(-t), \\ H_3(x)(t) &= x(2t), \\ H_4(x)(t) &= 2tx(t), \\ H_5(x)(t) &= x(t)+1. \end{aligned}$$

   Which of these systems are (1) linear, (2) time-invariant, (3) causal, (4) memoryless? Why?

2. Consider an LTI system with impulse response h given by

$$\forall\, n \in \mathit{Ints}, \quad h(n) = \delta(n) + 2\delta(n-1),$$

   where δ is the Kronecker delta function.

   (a) Plot the impulse response.

   (b) What is the output y of the system when the input x is the unit step,

$$\forall n, \quad x(n) = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0 \end{cases}$$

   Plot the output y.

   (c) What is the output when the input x is the ramp,

$$\forall n, \quad x(n) = \begin{cases} n, & n \geq 0, \\ 0, & n < 0 \end{cases}$$

   Plot the output y.

   (d) Find an expression for the frequency response $\hat{H}(\omega)$ of the system.

   (e) From this expression show that the frequency response is periodic with period 2π, i.e. verify that

$$\forall \omega, \quad \hat{H}(\omega) = \hat{H}(\omega + 2\pi).$$

   (f) Check that

$$\forall \omega \in \mathit{Reals}, \quad \hat{H}(-\omega) = [\hat{H}(\omega)]^*, \text{ and } \hat{H}(\omega) = [\hat{H}(-\omega)]^*$$

   (g) Find expressions for the amplitude response $|\hat{H}(\omega)|$ and the phase response $\angle \hat{H}(\omega)$.

   (h) From these expressions check that the amplitude response is an even function of ω, i.e.

$$\forall \omega, \quad |\hat{H}(\omega)| = |\hat{H}(-\omega)|$$

   and the phase response $\angle \hat{H}(\omega)$ is an odd function, i.e.

$$\angle \hat{H}(-\omega) = -\angle \hat{H}(-\omega).$$

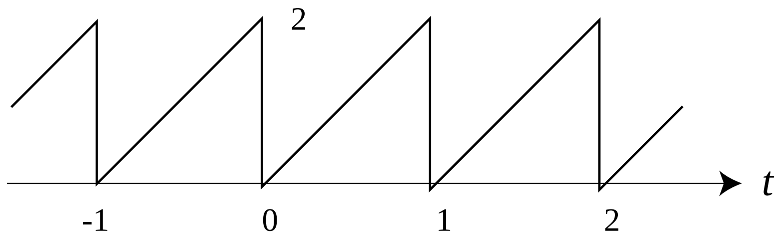


Figure 1: A periodic function

(i) Plot the amplitude and phase response for $0 \leq \omega \leq \pi$.

(j) Use these plots to obtain the plots for $-\infty < \omega < \infty$.

(k) Suppose the input signal is

$$\forall n, \quad x(n) = \cos(25n + \pi/6) + \sin(26n + \pi/3).$$

Obtain an expression for the output signal y?

3. Consider a continuous-time LTI system H. Suppose the response of H to the unit step input x:

$$x(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

is the signal y:

$$y(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Express y in terms of sums and differences of x and $D_1(x)$.

(b) Using the unit delay system D_1, constant gain blocks g, and adders, build a system which gives y as an output when the input is x. Call your system G.

(c) What is the output of H when the input is

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

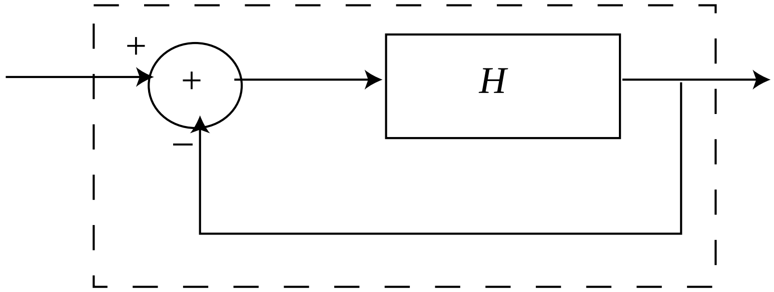
(d) What is the output of your system G for this input?

(e) What is the frequency response $\hat{G}(\omega)$ of G?

4. Obtain the exponential Fourier series respresentation of the periodic function shown in Figure 4. Suppose this periodic signal is input to an LTI system H whose frequency response is

$$\hat{H}(\omega) = \begin{cases} 1, |\omega| \leq 2.5 \text{ rad/sec} \\ 0, \text{otherwise} \end{cases}$$

What is the corresponding output?



5. A discrete-time LTI system H has the impulse response

$$h(n) = \begin{cases} 0, & n < 0 \\ \frac{1}{2^n}, & n \geq 1 \end{cases}$$

(a) What is its response to a unit step input?

(b) What is its frequence response? Plot the amplitude and phase response.

(c) Suppose H is put in cascade with another system G with the same frequency response as H. What is the frequency response of the cascade system?

(d) Suppose H is put in a feedback configuration as in Figure 5d: What is the frequency response of the closed-loop system?