EECS 20. Midterm No. 2. November 12, 1999.

Carefully read the questions. Use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. **Write clearly and put a box around your answer, and show your work.**

Print your name below

Last Name ______________________ First ______________________

Name of your Lab TA ____

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. **20 points** Let $x : Reals \to Comps$ be a continuous-time signal with Fourier Transform X. The **bandwidth** of x is defined to be the smallest number Ω_x (rads/sec) such that $|X(\omega)| = 0$ for $|\omega| > \Omega_x$. If there is no such finite number, we say that the signal is not bandlimited and let $\Omega_x = \infty$.

   Answer the following and give a brief justification for your answer.

   (a) If $\forall t, x(t) = 1$, what is X and what is the bandwidth of x?

   (b) If $\forall t, x(t) = \delta(t)$ (Dirac delta), what is X and what is the bandwidth of x?

   (c) If $\forall t, x(t) = \cos(t)$, what is X and what is the bandwidth of x?

   (d) If x has bandwidth Ω_x what is the bandwidth of the signal $2x$?

   (e) If x has bandwidth Ω_x and y has bandwidth $\Omega_y > \Omega_x$, what is the bandwidth of the signal $x + y$?

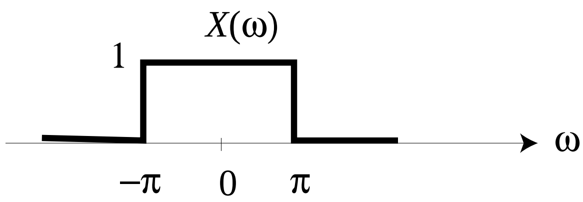

Figure 1: X for problem 2

2. **30 points** Suppose x is a continuous-time signal, with Fourier Transform X.

   (a) What are the units of ω in $X(\omega)$?

   (b) Write down the definition of $y = Sampler_T(x)$.

   (c) Let $Y(\hat{\omega})$ be the DTFT of y. What are the units of $\hat{\omega}$?

   (d) What is Y in terms of X?

   (e) Suppose X is as shown in Figure 1. For what values of T will there be no aliasing?

   (f) Sketch $Y(\hat{\omega})$ when $T = 1/2$ and when $T = 3/4$?

3. **30 points** Let $x : Ints \to Reals$ be a discrete-time signal with DTFT X. Let $h : Ints \to Reals$ be another discrete-time signal with DTFT H. Let $y = h * x$, the convolution sum of h and x.

(a) Give an explicit expression for y in terms of h and x.

(b) Let X, H, Y be the DTFT of x, h, and y, respectively. Express Y in terms of X, H.

(c) Suppose

$$X(\omega) = \begin{cases} 1, & |\omega| \leq \pi/4 \\ 0, & \pi/4 < |\omega| \leq \pi \end{cases}$$

Find the signal x.

(d) Suppose

$$H(\omega) = \begin{cases} 0, & |\omega| \leq \pi/4 \\ 1, & \pi/4 < |\omega| \leq \pi \end{cases}$$

Find $y = h * x$?

4. **20 points** Construct a state machine with

$$Inputs = \{0, 1\}, \quad Outputs = \{Yes, No, absent\},$$

such that for any input signal, the machine outputs Yes if the most recent three input values are 111, outputs No if the most recent three input values are 000, and in all other cases it outputs $absent$. In other words, if the input signal is

$$u(0), u(1), \cdots,$$

then the output signal

$$y(0), y(1), \cdots,$$

where

$$y(n) = \begin{cases} yes, & \text{if } (u(n-2), u(n-1), u(n)) = 111 \\ no, & \text{if } (u(n-2), u(n-1), u(n)) = 000 \\ absent, & \text{otherwise} \end{cases}$$