Practice Problems for Midterm #2, Fall 1998.

1. Consider a continuous-time signal x where for all \( t \in \) Reals,
\[
x(t) = \sum_{k=-\infty}^{\infty} r(t-k)
\]
where
\[
r(t) = \begin{cases} 
1 & 0 \leq t < 0.5 \\
0 & \text{otherwise}
\end{cases}
\]
Define \( \text{Sampler}_T : \text{ContSignals} \to \text{DiscSignals} \) in the usual way to be a sampler with sampling interval \( T \), where if \( y = \text{Sampler}_T(x) \), then for all integers \( n \), \( y(n) = x(nT) \). Define \( \text{IdealDiscToCont} : \text{DiscSignals} \to \text{ContSignals} \) to be an ideal reconstruction system.

a) Is \( x(t) \) periodic? If so, what is the period?
b) Suppose that \( T = 1 \). Give a simple expression for \( y = \text{Sampler}_T(x) \).
c) For the same \( T = 0.5 \), give a simple expression for \( \text{IdealDiscToCont} (\text{Sampler}_T(x)) \).
d) Find an upper bound for \( T \) (in seconds) such that \( x = \text{IdealDiscToCont} (\text{Sampler}_T(x)) \), or argue that no value of \( T \) makes this assertion true.

2. Consider an LTI discrete-time system Filter with impulse response
\[
h(n) = \delta(n) + \delta(n-2)
\]
where \( \delta \) is the Kronecker delta function.

a) Sketch \( h(n) \).
b) Suppose \( x(n) = \cos(\omega n) \), where \( \omega = \pi/2 \) radians/sample. Give a simple expression for \( y = \text{Filter}(x) \).
c) Give an expression for \( H(\omega) \) that is valid for all \( \omega \), where \( H = \text{DTFT}(h) \).

3. Consider a system \( \text{Abs} : \text{ContSignals} \to \text{ContSignals} \) where if \( y = \text{Abs}(x) \) then \( y(t) = |x(t)| \)
(the absolute value).

a) Show that this system is not linear.
b) Show that this system is time-invariant.

4. Suppose that the frequency response of a discrete-time LTI system \( \text{Filter} \) is given by
\[
H(\omega) = \cos(\omega)
\]
where \( \omega \) has units of radians/sample.

a) Suppose the input is \( x(n) = e^{jn} \). Given an expression for the output \( y = \text{Filter}(x) \).
b) Find \( h(n) \), the impulse response.
c) Is \( \text{Filter} \) causal?