More Practice Problems for Midterm #2, Fall 1998.

1. Consider a continuous-time LTI system $H$. Suppose that when the input is given by
   \[ x(t) = \begin{cases} \sin(\pi t) & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \]

   then the output is given by
   \[ y(t) = \begin{cases} \sin(\pi t) & 0 \leq t < 1 \\ \sin(\pi(t-1)) & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases} \]

   for all $t \in \text{Reals}$.
   
   a) Carefully sketch these two signals.
   b) Give an expression and a sketch for the output of the same system if the input is
      \[ x'(t) = \begin{cases} \sin(\pi t) & 0 \leq t < 1 \\ -\sin(\pi(t-1)) & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases} \]

2. Suppose you are given the following building blocks:
   - An LTI system that is an ideal continuous-time lowpass filter with frequency response
     \[ H(\omega) = \begin{cases} 1 & -W < \omega < W \\ 0 & \text{otherwise} \end{cases} \]
     where $W$ is a parameter you can set.
   - A gain block, where if the input is $x$ then the output is given by
     \[ y(t) = gx(t) \]
     for all $t \in \text{Reals}$, where again $g$ is a parameter you can set.
   - An adder, which can add two continuous time signals.

   Use these building blocks to construct a system with the frequency response shown below:
3. Consider a continuous-time signal \( x \) with Fourier transform \( X \). Find expressions for the Fourier transform of the following signals in terms of the Fourier transform \( X \).
   a) \( y \) such that \( \forall t \in \text{Reals}, y(t) = x(at) \), for some real number \( a \).
   b) \( w \) such that \( \forall t \in \text{Reals}, w(t) = x(t)e^{j\alpha t} \), for some real number \( \alpha \).
   c) \( z \) such that \( \forall t \in \text{Reals}, z(t) = x(t)\cos(\alpha t) \), for some real number \( \alpha \).

4. Consider the FIR system described by the following block diagram:

   ![Block Diagram](image)

   Suppose that this system has frequency response \( H(\omega) \). Define a new system with the identical structure as above, except that each unit delay is replaced by a double delay (two cascaded unit delays). Find the frequency response of that system in terms of \( H(\omega) \).