1. The fundamental frequency is $\omega_0 = \pi/10$, in units of radians per second. To get this systematically, note that the first cosine has a period of 10 and the second has a period of $20/3$. The least common multiple of these is 20, so the fundamental frequency is $2\pi/20 = \pi/10$.

To get the Fourier series coefficients, just write the signal as a sum of complex exponentials,

$$x(t) = \frac{1}{2} e^{i(\pi/10)2t} + \frac{1}{2} e^{-i(\pi/10)2t} + \frac{1}{2} e^{i(\pi/10)3t} + \frac{1}{2} e^{-i(\pi/10)3t},$$

from which we can read off the coefficients,

$$X_{-3} = \frac{1}{2}$$
$$X_{-2} = \frac{1}{2}$$
$$X_{2} = \frac{1}{2}$$
$$X_{3} = \frac{1}{2}.$$

The rest of the coefficients are zero.

2. The Fourier series coefficients of the output will be the above Fourier series coefficients multiplied by $H(\omega)$ for the corresponding value of $\omega$. At $\omega = 2\pi/10$ and $-2\pi/10$, $H(\omega) = 1$. At $\omega = 3\pi/10$ and $-3\pi/10$, $H(\omega) = -1$. This yields

$$y(t) = \frac{1}{2} e^{i(\pi/10)2t} + \frac{1}{2} e^{-i(\pi/10)2t} - \frac{1}{2} e^{i(\pi/10)3t} - \frac{1}{2} e^{-i(\pi/10)3t},$$

so

$$y(t) = \cos(\pi t/5) - \cos(3\pi t/10).$$