EECS 20. Final Exam
May 22, 2002.

Please use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and show your work for full credit.** Please check that you have 12 numbered pages.

Print your name below

Name: ________________________________

Problem 1 (20):

Problem 2 (15):

Problem 3 (15):

Problem 4 (20):

Problem 5 (10):

Problem 6 (10):

Problem 7 (10):

Total:

1. **20 points.** Consider a continuous-time signal $x : Reals \to Reals$ de£ned by

$$\forall t \in Reals, \;\; x(t) = \cos(\pi t/2) + \cos(\pi t) + \cos(3\pi t) + \cos(5\pi t).$$

(a) Obtain the Fourier series coef£cients of $x(t)$, i.e., £nd the coef£cients $A_0, A_1, A_2, \ldots$ and $\phi_1, \phi_2, \ldots$ and w_0 such that

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k w_0 t + \phi_k).$$

(b) Obtain the Fourier series expansion for $x(t)$, i.e., £nd the coef£cients X_k for all $k \in Integers$ such that

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{i k w_0 t} \quad .$$

(c) Consider a continuous-time LTI system $Filter : [Reals \to Reals] \to [Reals \to Reals]$ with the following frequency response

$$\mathrm{H}(\omega) = \begin{cases} e^{i\omega} & \text{if } 2 < |\omega| < 4 \text{ radians per second} \\ 0 & \text{otherwise} \end{cases}$$

For the $x(t)$ given above, £nd an expression for the output $y(t)$ of the system, where $y = Filter(x)$.

(d) For the output $y(t)$ calculated above, £nd the fundamental frequency in radians per second, i.e., £nd the largest $\tilde{\omega}_0$ such that $\forall t \in Reals$

$$y(t) = y(t + 2\pi/\tilde{\omega}_0).$$

2. **15 points.** Consider a system whose input and output are related by

$$\forall n \in Integers, \;\; y(n) + \alpha^2 y(n-2) = x(n) + 2\,x(n-1).$$

(a) Construct a state-space model for the system. It is suf£cient to give the state de£nition, the A matrix, vectors b and c, and scalar d.

(b) Give an expression for the zero-state impulse response.

(c) Recall that a system is stable if a bounded input always produces a bounded output. For what values of α is this system stable?

3. **15 points.** Consider the example of a system called *CodeRecognizer* where the input signals are sequences of 0 and 1 (with arbitrarily inserted stuttering symbols, which have no effect). The system outputs *recognize* at the end of every subsequence 1101 or 0010, and otherwise it outputs *absent*. In other words, if the input x is given by a sequence

$$(x(0), x(1), \ldots),$$

and the output y is given by the sequence

$$(y(0), y(1), \ldots),$$

then, if none of the input symbols is *absent*, the output is

$$y(n) = \begin{cases} \textit{recognize} & \text{if } (x(n-3), x(n-2), x(n-1), x(n)) = (1,1,0,1) \\ \textit{recognize} & \text{if } (x(n-3), x(n-2), x(n-1), x(n)) = (0,0,1,0) \\ \textit{absent} & \text{otherwise} \end{cases}$$

Construct a machine that implements *CodeRecognizer*. It is suf£cient to provide a state transition diagram.

4. **10 points.** Let x be a discrete-time, real-valued signal. The DTFT of x is a function $X : Reals \to Reals$ given by

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n}.$$

(a) What is the period p and fundamental frequency ω_0 of $X(\omega)$?

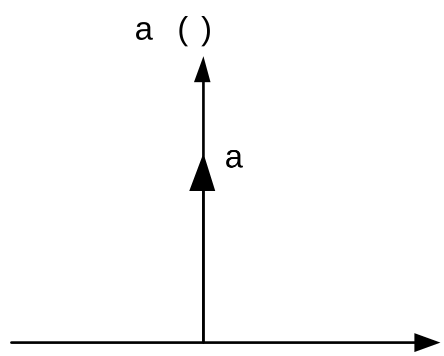
(b) Since X is periodic, it has a Fourier series expansion

$$X(\omega) = \sum_{n=-\infty}^{\infty} \tilde{X}_n e^{i\omega n}.$$

Find a simple expression for the Fourier coef£cients $\tilde{X}_n$ in terms of $x(n)$?

(c) Since $X(\omega)$ is a continuous-time function, it has a CTFT (continuous-time fourier transform) y. Denote by t the argument of y. Express $y(\tau)$ in terms of $x(n)$.

(d) Plot $y(\tau)$ as a function of τ.
Use the following notation for plotting $a\delta(\tau)$:



5. **10 points.** Consider the discrete-time signals $h_1(n)$ and $h_2(n)$ given by

$$\mathrm{h}_1(n) = \begin{cases} 1 & \text{if } 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{h}_2(n) = \begin{cases} (-1)^n & \text{if } 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $H_1(\omega)$ and $H_2(\omega)$, the discrete time fourier transforms (DTFT) of $h_1(n)$ and $h_2(n)$, respectively.

(b) Find a value of ϕ for which $H_1(\omega) = H_2(\omega + \phi)$.

6. **10 points.** Consider state machines A and B, described below by their state space, input alphabet, output alphabet, and state transition diagram.

Let

$$\textit{Set1} = \{T, F, \textit{absent}\}$$

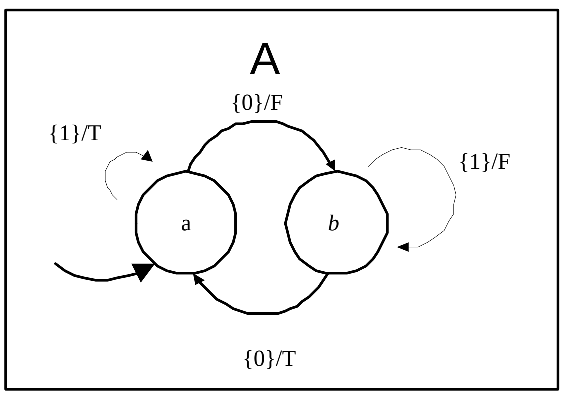
$$\textit{Set2} = \{1, 0, \textit{absent}\}$$

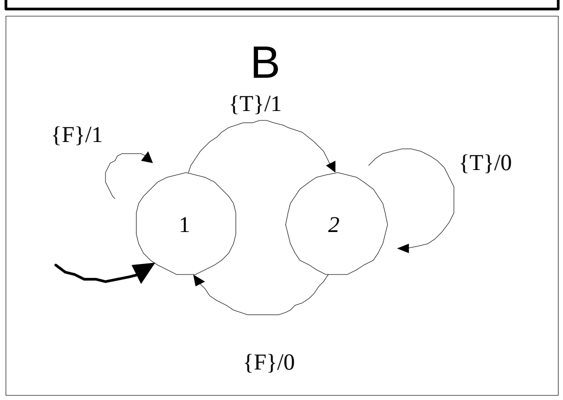
State machine A:

$$\textit{Inputs} = \textit{Set2},\ \textit{Outputs} = \textit{Set1},\ \textit{States} = \{a, b\}$$

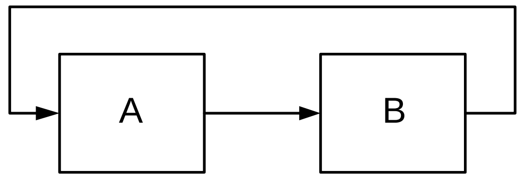
State machine B:

$$\textit{Inputs} = \textit{Set1},\ \textit{Outputs} = \textit{Set2},\ \textit{States} = \{1, 2\}$$

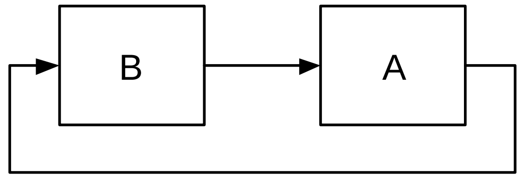




(a) Is the following composition well-formed? Explain your answer.



(b) Is the following composition well-formed? If yes, draw a state transition diagram for the composite machine. Otherwise, explain why this is the case and what could be done to make the composite machine well-formed.



7. **10 points.** Consider continuous-time systems with input $x : Reals \to Reals$ and output $y : Reals \to Reals$. Each of the following de£nes such a system. For each of the following, indicate whether it is linear only (L), time-invariant only (TI), both (LTI), or neither (N). Also indicate whether the system is causal (C) or non-causal (NC).

   (a) $\forall t \in Reals, \;\; y(t) = x(|t|)$

   (b) $\forall t \in Reals, \;\; y(t) = |x(t)|$

   (c) $\forall t \in Reals, \;\; y(t) = x(2t)$

Use this page for over¤ow