1. The signal \( s : \text{Reals} \rightarrow \text{Reals} \) is given by

\[
\forall t \in \text{Reals}, \quad s(t) = 2 + \sin 2\pi t + \sin 3\pi t.
\]

(a) What is the period of \( s \) in seconds (assume \( t \) is in seconds)?

The period is 2 s. We can see this by rewriting \( s \) as

\[
\forall t, \quad s(t) = \sin 2\pi \times 1 \times t + \sin 2\pi \times 3/2 \times t,
\]

so the fundamental frequency is \( f_0 = \gcd(1, 3/2) = 1/2 \) and the period is \( 1/f_0 = 2 \) s.

(b) Write down the Fourier series expansion of \( s \) in the form

\[
\forall t, \quad s(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(2\pi k f_0 t + \phi_k),
\]

i.e. identify \( f_0 \) and the coefficients, \( A_0, A_k, \phi_k \).

Rewrite the signal as

\[
\forall t, \quad s(t) = 2 + \cos(2\pi 2 f_0 t - \pi/2) + \cos(2\pi 3 f_0 t - \pi/2).
\]

Comparing coefficients with Fourier series representation gives:

\[
f_0 = \frac{1}{2}; \quad A_0 = 2, A_2 = 1, A_3 = 1, A_k = 0, \text{otherwise}; \quad \phi(k) = -\frac{\pi}{2}, \text{ all } k.
\]

(c) In the following \( x \) is a discrete-time signal \( x : \text{Integers} \rightarrow \text{Reals} \). For each case determine whether \( x \) is periodic and if it is periodic find its period (in samples).

i.

\[
\forall n, \quad x(n) = 1 + \cos(2\pi \times 5n).
\]

The period \( p \) is the smallest integer \( p \) such that \( 2\pi 5p \) is a multiple of \( 2\pi \), which gives \( p = 1 \) sample. Indeed, \( \forall n, \quad \cos(2\pi \times 5n) = 1 \). The signal is periodic.

ii.

\[
\forall n, \quad x(n) = \sin(2\pi \times 5/7n).
\]

The period is the smallest integer \( p \) such that \( 2\pi 5/7p \) is a multiple of \( 2\pi \), which gives \( p = 7 \) samples. The signal is periodic.