EECS20n, Quiz 8 Solution

Let the continuous-time signal \( c \) given by

\[
\forall \ t \in \text{Reals}, \quad c(t) = 2 \cos(\omega_c t)
\]

be a carrier wave for a radio signal. Let \( x \) given by

\[
\forall \ t \in \text{Reals}, \quad x(t) = 2 \cos(\omega_x t)
\]

be the signal to be carried by that radio signal (that it, it is a highly simplified stand-in for, say, a voice signal). To be concrete, let \( \omega_c = 2\pi \cdot 8000 \) radians/second, and \( \omega_x = 2\pi \cdot 400 \) radians/second.

1. Find and sketch the CTFT \( Y \) of \( y \) where

\[
\forall \ t \in \text{Reals}, \quad y(t) = c(t)x(t).
\]

Label your sketch carefully. **Hint:** The CTFT of \( e^{i\omega_0 t} \) is \( 2\pi \delta(\omega - \omega_0) \).

**Answer** Using \( 2 \cos(\omega t) = e^{i\omega t} + e^{-i\omega t} \),

\[
y(t) = [e^{i\omega_c t} + e^{-i\omega_c t}][e^{i\omega_x t} + e^{-i\omega_x t}]
\]

\[
= e^{i(\omega_c + \omega_x) t} + e^{i(\omega_c - \omega_x) t} + e^{i(-\omega_c + \omega_x) t} + e^{i(-\omega_c - \omega_x) t}.
\]

Using the Hint and the fact that the CTFT is linear, we have

\[
\forall \omega, \ Y(\omega) = 2\pi \left[ \delta(\omega - (\omega_c + \omega_x)) + \delta(\omega - (\omega_c - \omega_x)) + \delta(\omega - (-\omega_c + \omega_x)) + \delta(\omega - (-\omega_c - \omega_x)) \right]
\]

2. Let \( y \) from part 1 be the input to an LTI system with frequency response \( H \) where

\[
\forall \omega \in \text{Reals}, \quad H(\omega) = \begin{cases} 0 & \text{if } \omega \leq 0 \\ 1 & \text{if } \omega > 0 \end{cases}
\]

Find the output \( u \) as a function of \( t \).

**Answer** Since

\[
\forall \omega, \ U(\omega) = H(\omega) Y(\omega)
\]

\[
= 2\pi \left[ \delta(\omega - (\omega_c + \omega_x)) + \delta(\omega - (\omega_c - \omega_x)) \right],
\]

\[
\forall t, \ u(t) = e^{i(\omega_c + \omega_x) t} + e^{i(\omega_c - \omega_x) t}.
\]