EECS20a, Quiz 3, 3/14/05

The quiz will take 15 minutes. Write your response on the sheet.

Please print your name and lab time here:

Last Name: \underline{Solution} _First _______ Lab time _______

1. [10 points] Give the zero-input state response (response if input = 0) of the state machine:

\[
\begin{align*}
    s(n+1) &= a s(n) + b x(n) \\
    y(n) &= c s(n) + d x(n) \\
    n &\in \text{Naturals}_0
\end{align*}
\]

Here \( x \) is the input, \( y \) is the output and \( s \) is the state trajectory.

Sketch the response for \( a = 1/2, b = c = d = 1 \), and for \( a = 2, b = c = d = 1 \). For the sketches assume an initial state of \( s(0) = 2 \).
2. [20 points] Consider the system given by the difference equations:

\[ s(n+1) = \beta n s(n) + x(n) \]
\[ y(n) = s(n) + x(n) \quad n \in \text{Natural}_{0} \]

with \( s(0) = 0 \). Here \( x \) is the input, \( y \) is the output and \( s \) is the state trajectory.

(a) [5 points] Assume \( \beta = 1 \). Compute and plot the first few values of the output if \( i \) the input is an impulse: \( x(n) = \delta(n) \) for all \( n \in \text{Natural}_{0} \). \( ii \) if the input is \( x(n) = \delta(n-1) \) for all \( n \in \text{Natural}_{0} \).

\[
\begin{align*}
\gamma(0) &= 1 \\
\gamma(1) &= 1 \\
\gamma(2) &= 1 \\
\gamma(3) &= 2 \\
\gamma(4) &= 6 \\
\gamma(5) &= 15 \\
\gamma(6) &= 45
\end{align*}
\]

(b) [5 points] Is the system linear? Circle the right answer below.

A) No, for all values of \( \beta \in \text{Reals} \).

[\] [\] [\] [\] [\]

B) Yes, for all values of \( \beta \in \text{Reals} \).

C) Yes, for some but not all values of \( \beta \in \text{Reals} \).

D) Depends on the input sequence.
(c) [5 points] Is the system time-invariant? Circle the right answer below.

A) No, for all values of $\beta \in \text{Reals}$.
B) Yes, for all values of $\beta \in \text{Reals}$.
C) Yes, for some but not all values of $\beta \in \text{Reals}$.
D) Depends on the input sequence.

(b) [5 points] Assume again $\beta = 1$. Compute and plot the first few values of the output if the input sequence is $x(0) = 1, x(1) = 3$ and $x(n) = 0$ for $n \geq 2$.

\[ y(n) = \begin{cases} 0 & \text{if } n \leq 1 \\ \delta(n-1) & \text{if } n \geq 2 \end{cases} \]

\[ y(n) = \begin{cases} 0 & \text{if } n \leq 1 \\ \delta(n-1) & \text{if } n \geq 2 \end{cases} \]