EECS20n, Quiz 5, 4/27/05

The quiz will take 15 minutes. Write your response on the sheet.

Please print your name and lab time here:

Last Name __________________ First __________________ Lab time __________________

Recall:

The (complex) Fourier series representation of a discrete-time periodic signal x with period p is given by:

$$x(n) = \sum_{k=0}^{p-1} X_k e^{\frac{i2\pi kn}{p}}$$

where X_k's are the Fourier coefficients, given by:

$$X_k = \frac{1}{p} \sum_{n=0}^{p-1} x(n) e^{\frac{-i2\pi kn}{p}}.$$

The discrete-time Fourier transform (DTFT) X of a discrete-time signal x is related to x by:

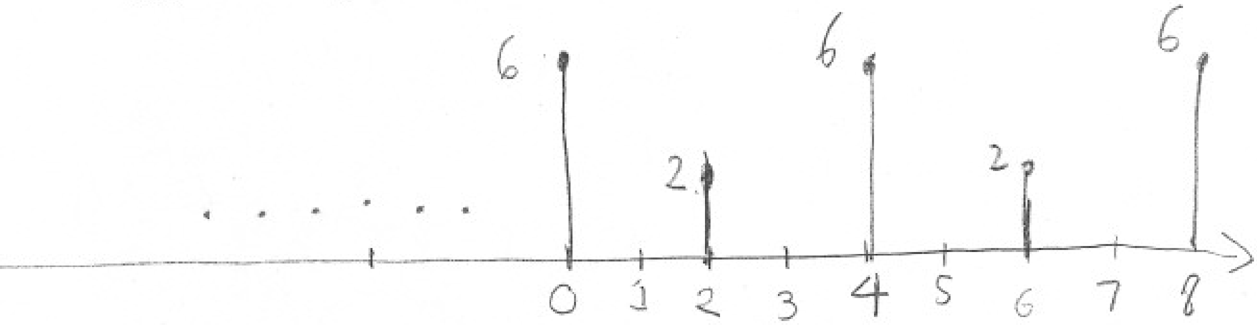
$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{i\omega n} d\omega$$

1. [20 points total] Consider the discrete-time signal x, given by

$$x(n) = 2 \sum_{m=-\infty}^{\infty} \delta(n-2m) + 4 \sum_{m=-\infty}^{\infty} \delta(n-4m), \qquad \text{for all integers } n$$

[5](a) Sketch the signal.



[8] (b) Is x periodic? If so, determine its period p and the Fourier coefficients in its (complex) Fourier series representation.

yes.

$p=4$.

$X_0 = 2, \quad X_1 = \frac{1}{4}(6+2e^{-i\pi}) = 1.$

$X_2 = \frac{1}{4}(6+2e^{-i2\pi}) = 2$

$X_3 = \frac{1}{4}(6+2e^{-i3\pi}) = 1.$

[7] c) Determine X, the DTFT of x. Is X periodic? If so, what is the period?

$X(\omega) = 2\pi\left(2\delta(\omega) + \delta(\omega - \frac{\pi}{2}) + 2\delta(\omega-\pi) + \delta(\omega - \frac{3\pi}{2})\right)$ for $\omega \in [0, 2\pi)$.

and is periodic with period 2π

fundamental frequency $\omega_0 = \frac{2\pi}{p} = \frac{\pi}{2}$

2. [15 points total] Consider a system where the discrete-time input x is related to the output y by:

$$y(n) = (-1)^n x(n) \qquad \text{for all integers } n.$$

[3] (a) Is the system linear?

Yes.

[3] (b) Is the system time-invariant?

No.

[9] (c) Determine Y, the DTFT of the signal y in terms of X, the DTFT of the signal x.

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y(n) e^{-i\omega n}$$

$$= \sum_{n=-\infty}^{\infty} (-1)^n x(n) e^{-i\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left(-e^{-i\omega}\right)^n$$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-i(\omega+\pi)n}$$

$$= X(\omega+\pi)$$