1. Signals & systems

Signals = [Time or Space → Symbols, Reals, Complex],
Systems: InputSignals → OutputSignals

Examples

Signals = [Indexes → Symbols]

Images = [Hspace × Vspace → {0, 1, ⋮, 255}]

DiscSignals = [Ints → Complex]

ContSignals = [Reals → Complex]

2. State machines

System is represented by a state machine M

Design problem. Suppose input-output function is ∀x ∈ [Ints⁺ → {0, 1, absent}]

∀n, y(n) = \begin{cases} 
0, & \text{if number of 0’s equals} \\
\text{number of 1’s in } x(0), ⋯, x(n) \\
1, & \text{else}
\end{cases}

Find a state machine M that realizes this input-output function.

Analysis problem. Given M₁, M₂, M₃, find M.
3. Linear state machine \( [A, b, c^T, d], \text{SISO} \)

\[
\begin{align*}
s(n + 1) &= As(n) + bx(n) \\
y(n) &= c^T s(n) + dx(n)
\end{align*}
\]

\( A \) is \( N \times N; \) \( b \) is \( N \times 1; \) \( c^T \) is \( 1 \times N; \) \( d \) is \( 1 \times 1; \)

\( s(n) \in \mathbb{R}^N; \ x(n), y(n) \in \mathbb{R}. \)

\( \forall x \in [\mathbb{Ints} \rightarrow \mathbb{R}], \) the response \( y \in [\mathbb{Ints} \rightarrow \mathbb{R}] \) is the zero-input + zero-state response.

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4. Hybrid system

Design problem. If "open" button is pressed
door remains open for 30 s and then it closes;
if button is pressed again while door is open, it
stays open for 30 s after the last time button
is pressed.

Design a hybrid system that implements the
design using a single clock \( \dot{s}(t) = 1 \).
5. LTI and frequency response

\[ S : [\text{Ints or } R \to R] \to [\text{Ints or } R \to R] \text{ is LTI if} \]

1. \( \forall x, u, a, b, \quad S(ax + bu) = aS(x) + bS(u) \); and
2. \( \forall T, \quad D_T \circ S = S \circ D_T \)

If \( S \) is LTI, there is a function \( H : R \to \text{Complex} \), such that the response to

\[ \forall n, x(n) = e^{j\omega n} \mid y(n) = H(\omega)e^{j\omega n} \]
\[ \forall t, x(t) = e^{j\omega t} \mid y(t) = H(\omega)e^{j\omega t} \]

6. Convolution

If \( h \) is impulse response of discrete-time LTI system, its response to \( x : \text{Ints} \to R \) is

\[ \forall n, \quad y(n) = (h * x)(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \]

If \( h \) is impulse response of continuous-time LTI system, its response to \( x : R \to R \) is

\[ \forall t, \quad y(t) = (h * x)(t) = \int_{-\infty}^{\infty} h(s)x(t-s)ds \]

7. The four Fourier Transforms

<table>
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Analysis problem. Given a periodic signal \( x \in \text{ContPeriodic}_p \), find its FS representation

\[
\forall t, x(t) = \sum X_k e^{j2\pi k t}
\]

Suppose \( x \) is input to LTI system with frequency response \( H \). What is the FS representation of the output \( y \)?

**Example**

Find the appropriate FT or DTFT

- \( x(t) = \cos(\omega_0 t) \)
  - \( X(\omega) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] \)
  - \( p(t) \)
  - \( P(\omega) = \frac{2 \sin(\omega T)}{\omega T} \)

- \( y(t) = x(t) p(t) \)
  - \( Y(\omega) = \frac{1}{T} (X * P)(\omega) \)
  - \( w(k) = y(kT_s) \)
  - \( W(\omega) = \frac{1}{T} \sum Y(\omega - \frac{2\pi k}{T_s}) \)