1. Basis for periodic signals

A. Continuous time

If \( f : \text{Reals} \to \text{Reals} \) is a periodic signal with period \( p > 0 \ (p \in \text{Reals}) \), and \( \omega_0 = \frac{2\pi}{p} \) (rad/sec) is the fundamental frequency, \( f \) has a unique representation (if \( A_k \geq 0, 0 \leq \phi_k < 2\pi \))

\[
\forall t \in \text{Reals}, \quad f(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \phi_k)
\]

B. Discrete time

If \( f : \text{Ints} \to \text{Ints} \) is a periodic signal with period \( p > 0 \ (p \in \text{Ints}) \), and \( \omega_0 = \frac{2\pi}{p} \) (rad/sample) is the fundamental frequency, \( f \) has a unique representation (if \( A_k \geq 0, 0 \leq \phi_k < 2\pi \))

\[
\forall n \in \text{Ints}, \quad f(n) = A_0 + \sum_{k=1}^{[p/2]} A_k \cos(k\omega_0 n + \phi_k)
\]
Why \( \lfloor p/2 \rfloor \)
If \( p = 4 \), \( \omega_0 = 2\pi/4 \) why only the frequencies \( 0, \omega_0 = 2\pi/4, 2\omega_0 = 4\pi/4 \) and not \( 3\omega_0 = 6\pi/4, 4\omega_0 = 8\pi/4, \ldots \)?

Because \( \forall n \in \mathbb{N} \)
\[
\cos(3\omega_0 n + \phi_3) = \cos(6\pi/4 n + \phi_3) = \cos(-6\pi/4 n - \phi_3) = \cos(2\pi/4 n - \phi_3) = \cos(\omega_0 n - \phi_3)
\]

2. Sinusoids in signal compression
Suppose you want to transmit or store a finite signal \( \forall t \in [0, p], t \rightarrow f(t) \).
You can represent \( f \) by its Fourier Series
\[
f(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 + \phi_k)
\]
You can then transmit or store only a finite set of coefficients
\[
A_0, (A_1, \phi_1), \ldots, (A_N, \phi_N)
\]
and reproduce the approximation
\[
f(t) = A_0 + \sum_{k=1}^{N} A_k \cos(k\omega_0 + \phi_k)
\]

The largest frequency in discrete time signals is \( \pi \) radians/sample, i.e. \( k \rightarrow \cos(\pi k) \).

Note that \( \forall k, \cos(2\pi k) = \cos(0k) = 1 \)

This approximation is the basis of most signal compression schemes. Observe how it is suitable for special-purpose hardware or software.
3. FCC regulations

The FCC allocates the (electromagnetic) spectrum to various uses according to this chart

<table>
<thead>
<tr>
<th>1000 m</th>
<th>100 m</th>
<th>30 m</th>
<th>10 m</th>
<th>3 m</th>
<th>1 m</th>
<th>10 cm</th>
<th>30 cm</th>
<th>1 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>LF</td>
<td>MF</td>
<td>HF</td>
<td>VHF</td>
<td>UHF</td>
<td>SHF</td>
<td>Ultra-sonics</td>
<td>FM Broadcast</td>
<td>Microwaves</td>
</tr>
</tbody>
</table>

**THE RADIO SPECTRUM**

For a more complete picture see sidebars/radio/freq

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The AM radio band are in the range 535-1705 kHz
Carriers are separated by 10kHz, 540kHz, 550kHz, ...

The FM radio band are in the range 88-108 MHz
Carriers are separated by 0.2MHz, 88.1MHz, 88.3MHz, ...

Analog TV channels occupy 6 MHz of bandwidth
ch2-4, 54-72MHz; 5-6, 76-88MHz; ch7-13, 174-216MHz