1. Magnitude and phase response

If the frequency response $H : R \rightarrow C$ of a LTI system $S$ is expressed in polar form as

$$\forall \omega, \quad H(\omega) = |H(\omega)| e^{i \angle H(\omega)},$$

the function

$$\omega \rightarrow |H(\omega)|$$

is the magnitude response.

$$\omega \rightarrow \angle H(\omega)$$

is the phase response.

Examples

1. $D_T$ has freq resp $\forall \omega \quad H(\omega) = e^{-i \omega T}$, so

$$\forall \omega \in R, |H(\omega)| = 1, \quad \angle H(\omega) = -\omega T$$

This is an all-pass system.
2. The discrete-time system
\[ y(n) = 1/2[x(n) + x(n - 1)] \]
has freq resp
\[ \forall \omega, H(\omega) = 1/2[1 + e^{-i\omega}], \text{ so} \]
\[ |H(\omega)| = 1/2[1 + e^{-i\omega}] = 1/2[(1 + \cos \omega) - i \sin \omega] \]
\[ = 1/2\sqrt{(1 + \cos \omega)^2 + \sin^2(\omega)} \]
\[ \angle H(\omega) = -\tan^{-1} \left( \frac{\sin \omega}{1 + \cos \omega} \right) \]

the following Matlab program

```
H = (1 + exp(-i*omega))/2;
>> subplot(2,1,1)
>> plot(w, abs(H));
>> subplot(2,1,2)
>> plot(w, angle(H))
>> print -dpdf freqResp
```
gives the plot shown on next slide

3. The RC filter has freq resp
\[ \forall \omega, \quad H(\omega) = \frac{C}{1 + iRC\omega}, \text{ so} \]
\[ |H(\omega)| = \frac{C}{\sqrt{1 + (RC\omega)^2}} \]
\[ \angle H(\omega) = -\tan^{-1} RC\omega \]

This is a low-pass system.

the next slide shows plots for \( R = C = 1 \).
2 Properties

Consider a continuous-time real system, i.e. signals are real-valued.

If the input signal is
\[ t \rightarrow \sum_{k} A_k \cos(\omega_k t + \phi_k) = Re\{\sum_{k} A_k e^{i(\omega_k t + \phi_k)}\} \]
the output signal is
\[ t \rightarrow \sum_{k} |H(\omega_k)|A_k \cos(\omega_k t + \phi_k + \angle H(\omega_k)) \quad (1) \]

Corollary 1

If the input signal is periodic with period \( p \) the output signal is periodic with period \( p \).
Corollary 2

Since
\[ \cos(\omega t) = \cos(-\omega t) \]

it follows that
\[ H(\omega) = H^*(-\omega) \]

i.e.
\[ |H(\omega)| = |H(-\omega)|, \quad \angle H(\omega) = -\angle H(-\omega) \]

So \(|H(\omega)|\) is an even function of \(\omega\)
So \(\angle H(\omega)\) is an odd function of \(\omega\)

Corollary 1

If the input signal is periodic with period \(p\) the output signal is periodic with period \(p\)

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Consider a discrete-time real system, i.e. signals are real-valued.

If the input signal is
\[ n \to \sum_k A_k \cos(\omega_k n + \phi_k) \]

the output signal is
\[ n \to \sum_k |H(\omega_k)| A_k \cos(\omega_k n + \phi_k + \angle H(\omega_k)) \quad (2) \]

Corollary 2

Again, \(\forall \omega\),
\[ |H(\omega)| = |H(-\omega)|, \quad \angle H(\omega) = -\angle H(-\omega) \]

Moreover, since for
\[ e^{i\omega n} = e^{i(\omega+2\pi)n} \]

therefore,
\[ \forall \omega, \quad H(\omega) = H(\omega + 2\pi) \]

i.e. the frequency response of a discrete-time system is periodic in \(\omega\) with period \(2\pi\)
Week 9 Ch 8,
1. Magnitude and phase response
2. Properties
3. Cascade and feedback composition

Suppose open loop frequency response is $H(\omega)$, then 3 db bandwidth is given by

$$Y = H(\omega)E$$
$$R = G(\omega)Y$$
$$E = X + R = X + G(\omega)Y$$
$$Y = H(\omega)X + H(\omega)G(\omega)$$

F.R of feedback composition is

$$\frac{Y}{X} = \frac{H(\omega)}{1 - H(\omega)G(\omega)}$$

Increasing bandwidth by proportional negative feedback

Suppose open loop frequency response is $H(\omega) = \frac{1}{1+\omega}$

So 3 db bandwidth is given by

$$|H(\omega)| = |\frac{1}{\sqrt{3}}| \approx 0.7$$

or $\omega = 1$ rad/sec

Corollary The cascade composition of LTI systems commutes.
The closed loop frequency response is

\[ \frac{Y}{X} = G(\omega) = \frac{KH(\omega)}{1 + KH(\omega)} = \frac{K}{(K + 1) + i\omega} \]

whose 3 db bandwidth \( \approx \omega = K + 1 \text{ rad/sec} \)