Midterm Review

Lecture 30:

EECS 20 N—April 9, 2001
today's lecture: review of probs 1-3 of Midterm II

reading assignment: Chapter 9 of Lee and Varaiya
Problem 1

Problem 1

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Problem 1
Input

A linear system always produces zero output in response to zero

their last argument, if

it is time-invariant iff

nextState and output do not depend on

above system is linear iff

nextState and output are linear

then it is not memoryless (unless vector s has dimension zero)

\[(u,(u)x,(u)s)\text{output} = (u)\text{if}\]

\[\cdots, 1, 2, 3, 0 = u \quad (u,(u)x,(u)s)\text{nextState} = (I + u)s\]

if a system can be represented in state-space form

\[f\]

is a linear function

a memoryless system is always time-invariant, and it is linear iff
Problem 1 (a)

$y(t) = \cos(\omega x(t))$

is not linear, since $\cos$ is a nonlinear function.

For example, take $x(t) = 0$ for all $t$, obtain $y(t) = 1$ (instead of 0).

However, $\mathcal{L}$ is memoryless and hence, time-invariant.
Problem 1(b)

\[ x \ast y + I = h \]

original system can be written as

\[ (I - t)g + (t)g = (t)h \]

\[ (I - t)x + (t)x = (t)h \]

**Note:** the system is LTI, with impulse response

\[ h(t) = \delta(t) + \delta(t-1) \]

original system can be written as

\[ y(t) = x(t) + x(t-1) \]

- system is time-invariant
- system is not memoryless
- system is not linear: take \( I = (t)h \) obtain \( 0 = (t)x \)

\[ y(t) = x(t) + x(t-1) \]

\[ y(t) + I = (t)h \]

\[ x \ast y + I = h \]

\[ (I - t)x + (t)x = (t)h \]

\[ y(t) = x(t) + x(t-1) \]
The discrete-time system $y(n) = x(n) + 0.9x(n-1)$ is LTI; it can be represented in state-space form:

\[
\begin{align*}
(\mu)x + (\mu)s6.0 &= (\mu)\delta \\
(\mu)x &= (I + \mu)s
\end{align*}
\]

The system is not memoryless; it can be represented in state-space form:

\[
\begin{align*}
I &= \mu \neq 0 \\
0 &= \mu \neq 1 \\
\end{align*}
\]

where

\[
(\mu)\eta(\mu - \mu)x \quad \begin{array}{c}
\sum_{\mu=+\infty}^{\infty} \\
\sum_{\mu=-\infty}^{\infty}
\end{array} = (\mu)(x \ast \eta) = (\mu)\delta
\]

represented by the convolution sum.

\[
\begin{align*}
\text{Problem 1(c)}
\end{align*}
\]
Problem 1 (d)

The discrete-time system

\[ ((0)x + \cdots + (1 - u)x + (u)x)z = (u)h \]

is not memoryless:

The system is also defined by convolution with a given \( h \):

\[ (x * y)(z) = h \]

is linear: hence, system is LTI.

\( (0 \leq u \leq 1) \) for every \( u \) is (\( h(y) = (u)h \)), where \( h \) is the unit step (that is, \( h \text{ is the unit step} \))
The discrete-time system defined by
\[ s(n+1) = ns(n) + x(n), \]
\[ y(n) = 2s(n), \]
• system admits state-space representation \( s(n+1) = ns(n) + x(n), \)
\[ y(n) = 2s(n), \]
hence, it is not memoryless.

The system is linear.

The system is not time-invariant (since the state-space representation coefficients depend on \( n \)).

\[ (u)s_2 = (u)h \]
\[ \cdots, 0 = u, (u)x + (u)su = (I + u)s \]

The discrete-time system defined by
\[ (u)x + (u)su = (I + u)s \]

\textbf{Problem 1(e)}
Problem 1

The continuous-time system

\[ y(t) = 0 \]

- System is memoryless, hence, time-invariant
- System is linear (despite the output being constant)

The continuous-time system

\[ y(t) = 0 \]
For system \( S \), the matrix \( L \) describes how the \( n \) first input values relate to the first \( u \) values of \( y \).

Hence, \( L \) is the "upper triangular Toeplitz" matrix:

\[
\begin{bmatrix}
0 & y & \cdots & y \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & 0
\end{bmatrix} = L
\]
$D = \begin{bmatrix}
0 & 1 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & 1 \\
0 & \cdots & 0 & 0
\end{bmatrix}$

plays the same role for the unit delay system.
If $\mathbf{f}$ is the output, then

$$\mathbf{F}_A \mathbf{X} = \mathbf{F}_B \mathbf{X} = \mathbf{F}_C$$

Conversely, take a signal $x$, delay it to get $x_d$, by definition, delay $x$ to get $x_d$.

$$x_d = \mathbf{F}_A x$$

(underline refers to first values of signal)

$$x_d = \mathbf{F}_A x$$

To prove it, take a signal $x$, form $x$, and delay to get $x_d$, by definition.

$$x_d = \mathbf{F}_A x$$

Reflects the time-invariance of $S$.
Consider the system:

\[(1 - u)n_{1-u}(z^-) + (u)n_u(z^-) = (u)y\]

Impulse response: consider system

\[x = \delta\]

Obtain \(h = g\), where \(g(0) = 0\), \(g(1) = -2\), \(g(2) = 4\), ...

More generally, obtain that \(g(n) = (-2)^n u(n)\) (where \(u\) is unit step).

Impulse response: consider system

\[x = \delta_{n-1}\]

Obtain \(y\) response.

Impulse response: consider system

\[(u)x = (1 - u)y + (u)y\]

Impulse response: consider system
\[
\frac{m - \vartheta z + 1}{m - \vartheta + 1} = (m) H
\]

hence

\[m - \vartheta + 1 = m - \vartheta (m) H z + (m) H\]

\((u) x\) after dividing both sides by \((u) x\) obtain

\[(1 - u) x + (u) x = (1 - u) \tilde{z} + (u) \tilde{z}\]

\((m) H\) and eliminate in \(u m \vartheta (m) H = (u) \tilde{z}\) \(u m \vartheta = (u) x\) use frequency response
Let's find an exponential output such that output is zero output.

(\text{say}) \ \nu = \omega \ \text{or} \ \nu = -1, \ \text{that is:} \ \cos \ \omega = -1, \ \text{hence the imaginary part is also zero.}

\text{Hence, response to imaginary part is also zero.}
\[ I = p = c, \quad -q = q, \quad -z = z \]

above is a 1-D, SISO state-space model, with

\[
(u)x + (u)s = (u)\hat{s}
\]

and

\[
(1 - u)x - (1 - u)s\hat{z} = (1 - u)s = \\
\left\{(1 - u)x - \left((1 - u)x - (1 - u)\hat{s}\right)\hat{z}ight\} = \\
(1 - u)x + (1 - u)\hat{s}\hat{z} = (u)s
\]

try

\[
(u)x - (u)\hat{s} = (u)s
\]

state-space model
\[ \frac{m_i - \vartheta \mathcal{Z} + 1}{m_i - \vartheta + 1} = (m)H \]

where

\[ (\frac{u \nu_i \vartheta (\nu)H + z/\nu \nu_i \vartheta z/\nu_i \vartheta (z/\nu)H + (0)H}{e \mathcal{Z}}) \vartheta \mathcal{Z} = (u)(\bar{u}) \]

hence, response is

\[ \left( \frac{u \nu_i \vartheta + z/\nu \nu_i \vartheta z/\nu_i \vartheta + 0 \vartheta}{\mathcal{Z}} \right) \vartheta \mathcal{Z} = (u)x \]

we have

\[ (u \nu \cos + (z/\nu \nu) \sin + z = (u)x \]