outline

reading assignment:

• Chapter 9 of Lee and Varaiya (new version)

today’s lecture:

• a correction on CTFT

control systems design: an example

• Chapter 10 of Lee and Varaiya
Continuous-time signal from its CTFT

In lecture 39, the incorrect formula for reconstructing a periodic signal was stated as

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{X(\omega)}{i} \, d\omega \]

Correct formula:

\[ x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{X(\omega)}{i} \, d\omega \]

this is inconsistent with the fact that \( X(\omega) \) contains information about \( x(t) \) for \( \omega \) outside \([0, 2\pi] \)

In lecture 39, the inverse CTFT was stated as
General problem: Given an LTI system $S$ and an input $u$, design a control input $u_\text{c}$ so as to achieve a "desired behavior" of the controlled system.

stability (asymptotic behavior)

• frequency domain properties (e.g., disturbance rejection)

• time-domain limits on signals of interest (e.g., for safety reasons)
For LTI systems, we'll prefer a "closed-loop" approach, whereas "open-loop" fashion

controller is designed as a function of states

hence, no notion of (or concern for) stabilility

we don't need to worry about infinite values of states

for finite-state machines, problem is easier (in some sense)

Relationship to finite-state machines
consider a simple first-order model:

\[ \dot{s} = s + x, \quad y = s + n \]

where \( s \in \mathbb{R} \) is the state, \( x \) the input, \( y \) is the output ("measurement") and \( n \) is a (sensor) noise. The zero-input response is \( s(t) = e^{t}s(0) \), it grows unboundedly as \( t \to \infty \) using a control input, we'd like to avoid this unstable behavior.

\[ \infty \leftarrow t \]

Consider a first-order model.
The goal of feedback control is to design the (LTI) system \( \mathcal{C} \) to stabilize via feedback. Feedback can be done ahead of time and implemented in hardware. Computation can be done ahead of time and implemented in hardware. The state-space representation of \( \mathcal{C} \), reduces the problem to computing a few parameters (e.g., state-space representation of \( \mathcal{C} \)).

![Feedback Control Diagram]

Feedback control:

Stabilization via feedback

\[
\begin{align*}
\text{(feedback)} & \quad \mathcal{C} \\
\text{(state)} & \quad S \\
\text{error} & \quad + \text{(state)} + \text{(state)} \\
\end{align*}
\]
Design objectives

• Make closed-loop system noise (e.g., sensor noise) on the closed-loop system stable.

• The control law attenuates the effect of disturbances (e.g., sensor noise). The system responds well to reference inputs (denoted $r$) that zero-input response shows remain bounded.

• The system "responds well" to reference inputs (denoted $r$) that zero-input response shows remain bounded.

• We have not defined stability in general terms; intuitively, means make closed-loop system stable.

• Involves the notion of "steady-state error" seen next.
One of the main reasons: robustness.

Fact: Feedback control is less sensitive to errors in the system than open loop control.

Why use feedback?
proportional state feedback

In 1st order model, let’s use a simple control scheme:

\[ x(t) = -ky(t) + r, \]

where \( k \) is a constant (negative sign above by convention)

\[ u + s = f, \quad u - s + s(\eta - 1) = s, \]

where \( \eta \) is a constant (negative sign above by convention)

\[ \nu + (\nu)f\eta - = (\nu)x \]

In 1st order model, let’s use a simple control scheme:

proportional state feedback
\[ \frac{I - \gamma + \omega}{I} = \mathcal{F} \left( \frac{1}{I - \gamma + \omega} \right) = \mathcal{F} \left( \frac{\omega}{I - \gamma + \omega} \right) = \mathcal{F} \left( \frac{\omega}{I - \gamma + \omega} \right) \int_{-\infty}^{\infty}
\]

Proof: check that

\[ (\mathcal{F}) \frac{1}{\gamma - \omega} = \mathcal{F} \frac{\omega}{I - \gamma + \omega} \]

The impulse response is

\[ \frac{I - \gamma + \omega}{I} = (\mathcal{F}) \frac{1}{\gamma - \omega} = \mathcal{F} \frac{\omega}{I - \gamma + \omega} \]

Yields

\[ (\mathcal{F}) x(\omega) = (\mathcal{F}) s = (\mathcal{F}) \frac{\omega}{I - \gamma + \omega} = (\mathcal{F}) x(\omega) \]

Let's assume \( u > 0 \) and \( \gamma < I \)
Impulse response

$y(t)$

$\text{plot of impulse response}$
Bode Diagrams

From: $U(1)$

To: $Y(1)$

$k = -2$

$k = -10$

Frequency (rad/sec)

Phase (deg); Magnitude (dB)

$10^{-1}$

$10^{-0}$

$10^{1}$

$10^{2}$

$10^{3}$

$10^{4}$

$10^{5}$

$10^{6}$

Plot of Frequency Response
choosing $k$

wecanchoose $k$

soasto

• ensure a fast decay of zero-input response

• ensure a fast impulse response

• hence, closed-loop system will quickly come back to rest after

hence, closed-loop system will quickly forget initial conditions

we can choose $k$ so as to

choosing $k$
at the frequency response for \( m = 0 \) of the system, but the larger the steady-state error (confirm this by looking clearly, the larger the control gain \( k \), the faster the closed-loop steady-state error is defined as the ratio \( \frac{y(t)}{u(t)} \) as \( t \to \infty \)

\[
\infty \leftarrow t \text{ as } (\frac{\tau}{\tau_0})n/(\frac{\tau}{\tau_0})f_i \text{ is defined as the ratio } \frac{1 - \frac{\gamma}{\xi}}{1} = (\frac{\tau}{\tau_0})f_i
\]

Step response describes how the system “tracks” a reference input when

a large \( k \) has also an effect on the steady-state error
the input

A larger $k$ will amplify the effect of (high-frequency) sensor noise on sensor noise to state:

For example, let's look at the closed-loop frequency response from a large value of $k$ has other problems ...

... a large value of $k$ has other problems ...

A trade-off in choosing $k$
Bode Diagrams

- $k = -2$
- $k = -10$

Noise Frequency Response
Control design for LTI systems involves many (possibly conflicting) objectives:

- Stability
- Fast impulse response
- Good tracking properties
- Disturbance rejection
- Robustness to plant imperfections
- Etc.

The area of control systems aims at providing guidelines (and/or software) to design these control laws.