Problem session, week 14

9.8 Suppose a discrete-time signal $x$ has DTFT given by

$$X(\omega) = i \sin(K\omega)$$

for some positive integer $K$. Note that $X(\omega)$ is periodic with period $2\pi$, as it must be to be a DTFT.

(a) Determine from the symmetry properties of $X$ whether the time-domain signal $x$ is real.

(b) Find $x$. **Hint:** Use Euler’s relation and the linearity of the DTFT.

9.9 Consider a periodic continuous-time signal $x$ with period $p$ and Fourier series $X: \text{Ints} \to \text{Comps}$. Let $y$ be another signal given by

$$y(t) = x(t - \tau)$$

for some real constant $\tau$. Find the Fourier series coefficients of $y$ in terms of those of $X$.

9.10 Consider the continuous-time signal given by

$$x(t) = \frac{\sin(\pi t/T)}{(\pi t/T)}.$$ 

Show that its CTFT is given by

$$X(\omega) = \begin{cases} T, & \text{if } |\omega| \leq \pi/T \\ 0, & \text{if } |\omega| > \pi/T \end{cases}$$

The following fact from calculus may be useful:

$$\int_a^b e^{\alpha \omega}d\omega = e^{\beta \omega} - e^{\alpha \omega}$$

for real $\alpha$ and $b$ and complex $\beta$.

9.11 If $x$ is a continuous-time signal with CTFT $X$, then we can define a new time-domain function $y$ such that

$$\forall t \in \text{Reals}, \quad y(t) = X(t).$$
That is, the new time domain function has the same shape as the frequency domain function $X$. Then the CTFT $Y$ of $y$ is given by

$$Y(\omega) = 2\pi x(-\omega).$$

That is, the frequency domain of the new function has the shape of the time domain of the old, but reversed and scaled by $2\pi$. This property is called \textbf{duality} because it shows that time and frequency are interchangeable. Show that the property is true.

9.12 Use the results of exercises and to show that a continuous time signal $x$ given by

$$x(t) = \begin{cases} T, & \text{if } |t| \leq \pi/T \\ 0, & \text{if } |t| > \pi/T \end{cases}$$

has CTFT $X$ given by

$$X(\omega) = 2\pi \frac{\sin(\pi \omega/T)}{(\pi \omega/T)}.$$