Power Distribution Systems: Optimization, Control and Communication

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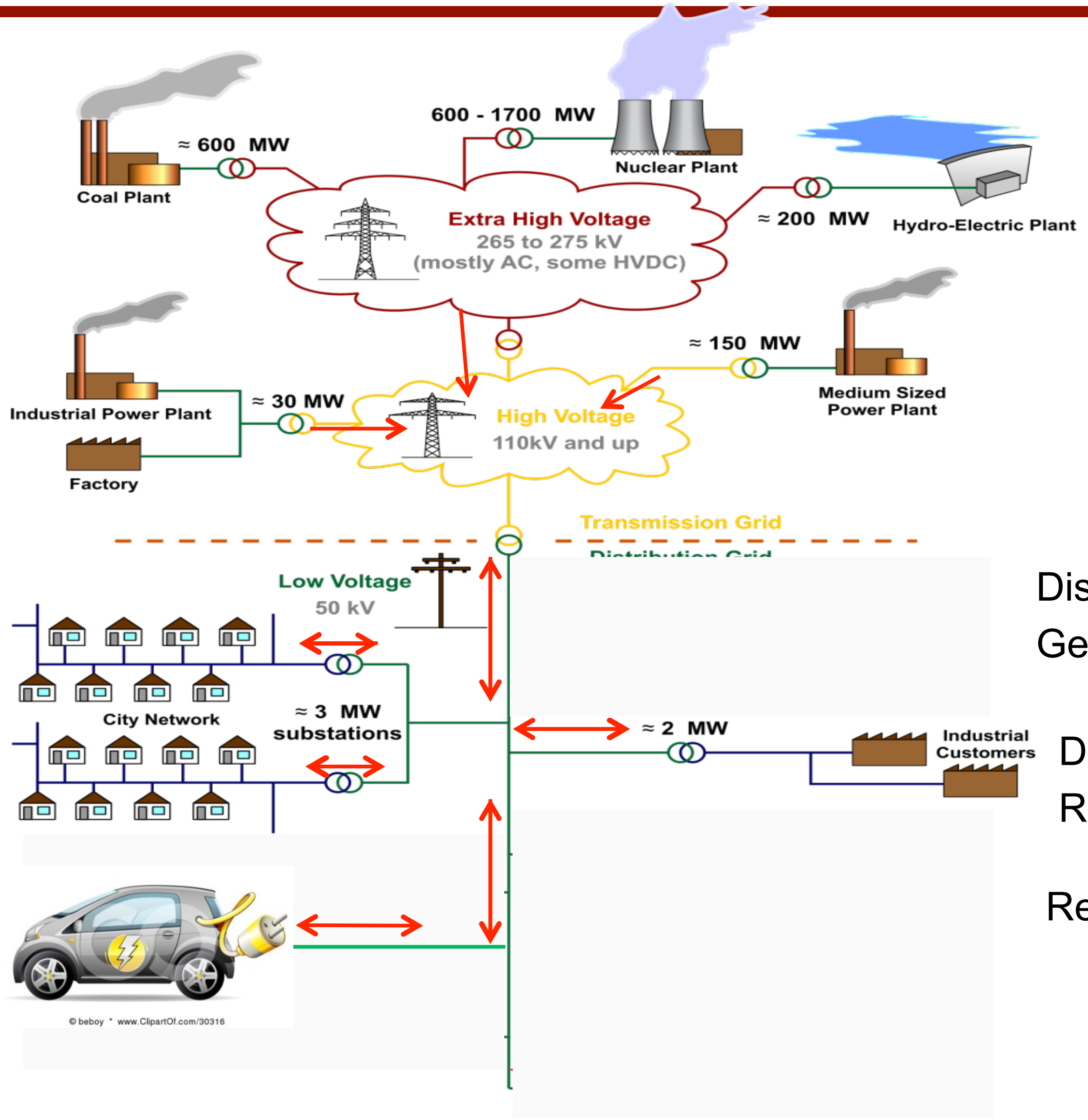
Optimization problems in power networks

- Optimization used for resource allocation
- Increasing complexity:
  - Optimal Power Flow (OPF)
  - Unit Commitment
  - Security Constraint Unit Commitment
- All are done at the level of transmission networks
- Smart grid: Optimization in distribution networks

Power flow in the smart grid



Distributed Generation

Demand Response

Demand Response

EVs

Renewables

Power Flow

- Focus on the basic problem: Optimal Power Flow (OPF)
- Review of AC power flow
- Network with n buses

$$y_{ik} = g_{ik} - jb_{ik} = 1/z_{ik}$$

$$Bus\ matrix: Y \in \mathbb{R}^{n \times n}$$

$$Complex\ Voltage: v \in \mathbb{R}^{n}$$

- Power Flow

$$p + jq = diag(\underbrace{vv^{H}}_{\text{Quadratic}} Y^{H})$$

Optimal power flow

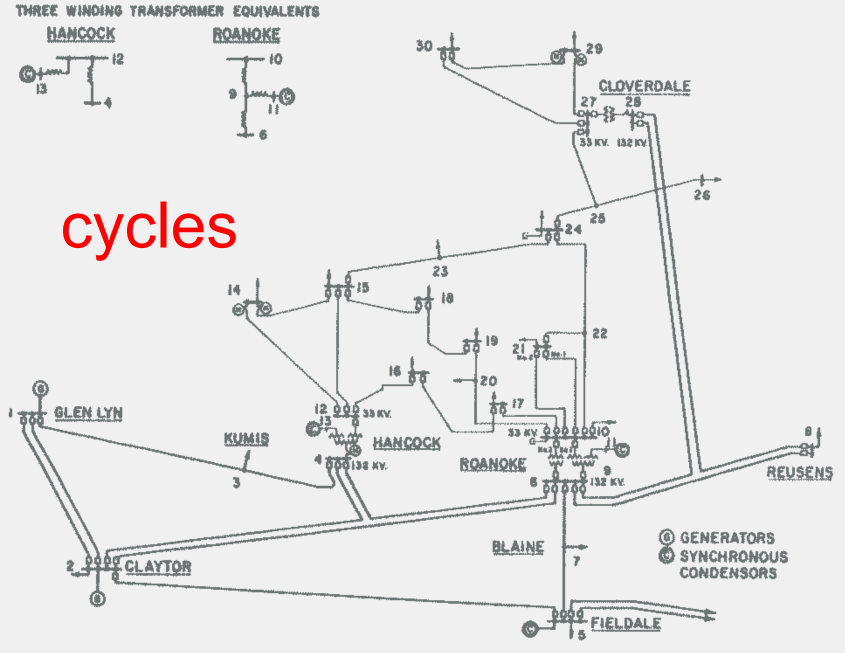
- OPF: optimize over complex voltages

$$\begin{aligned} \text{minimize} \quad & \sum f_i(P_i) \\ \text{subject to} \quad & \text{Voltage Constraints} \\ & \text{Bus Power Constraints} \\ & \text{Network Constraints} \end{aligned}$$

- Non-convex
- DC power flow often used for transmission network
  - Lossless network, ignore reactive power
- Not satisfactory for distribution network
  - Higher losses
  - Reactive power coupled with active power flow

Transmission vs. distribution

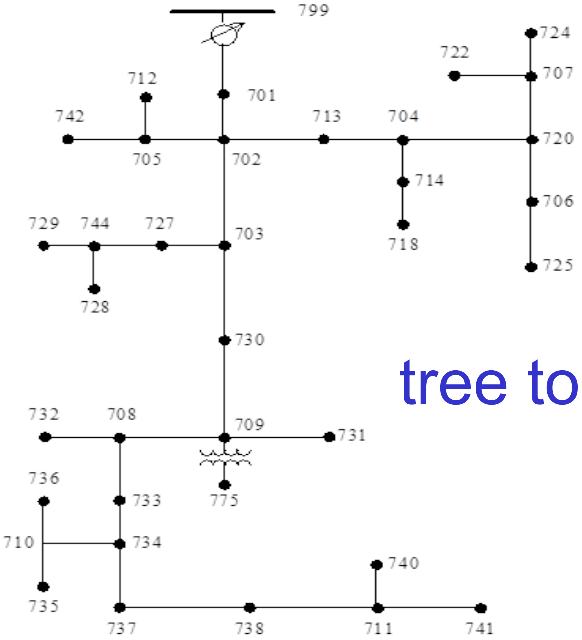
- Transmission Network



cycles

- OPF is non-convex, hard

- Distribution Network



tree topology

- OPF still non-convex
- We show:
  - convex relaxation tight
  - decentralized solution

OPF on trees: take 1

Theorem 1 (Z & Tse., 2011):

Convex rank relaxation for OPF is tight if:

1) the network is a tree

2) no two connected buses have tight bus power lower bounds.

(See also: Sojoudi & Lavaei 11, Steven Low's group)

Geometry of feasible injection region of power networks, IEEE Transaction on Power Systems

Proof approach

- Focus on the underlying injection region and investigate its convexity.
- Used a matrix-fitting lemma from algebraic graph theory.

Drawbacks:

- Role of tree topology unclear.
- Restriction on bus power lower bounds unsatisfactory.

OPF on trees: take 2

Theorem 2 (Lavaei, Tse. & Z 13):

Convex relaxation for OPF is tight if

1) the network is a tree

2) angle differences along lines are "reasonable"

More importantly:

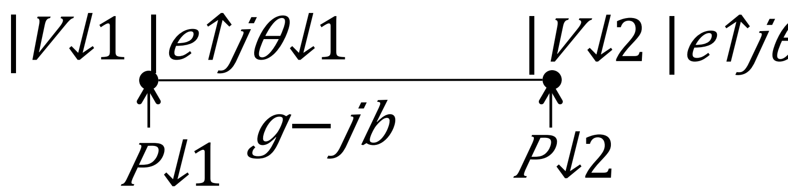
Proof is entirely geometric and from first principles.

Geometry of power flows in tree networks, PES General Meeting, 2012

Outline of talk

- Results on optimal power flow on trees.
- A geometric understanding.
- Application to the voltage regulation problem in distribution networks with renewables.
- An optimal decentralized algorithm for solving this problem.
- What happens when there is no communication?

Example: Two Bus Network

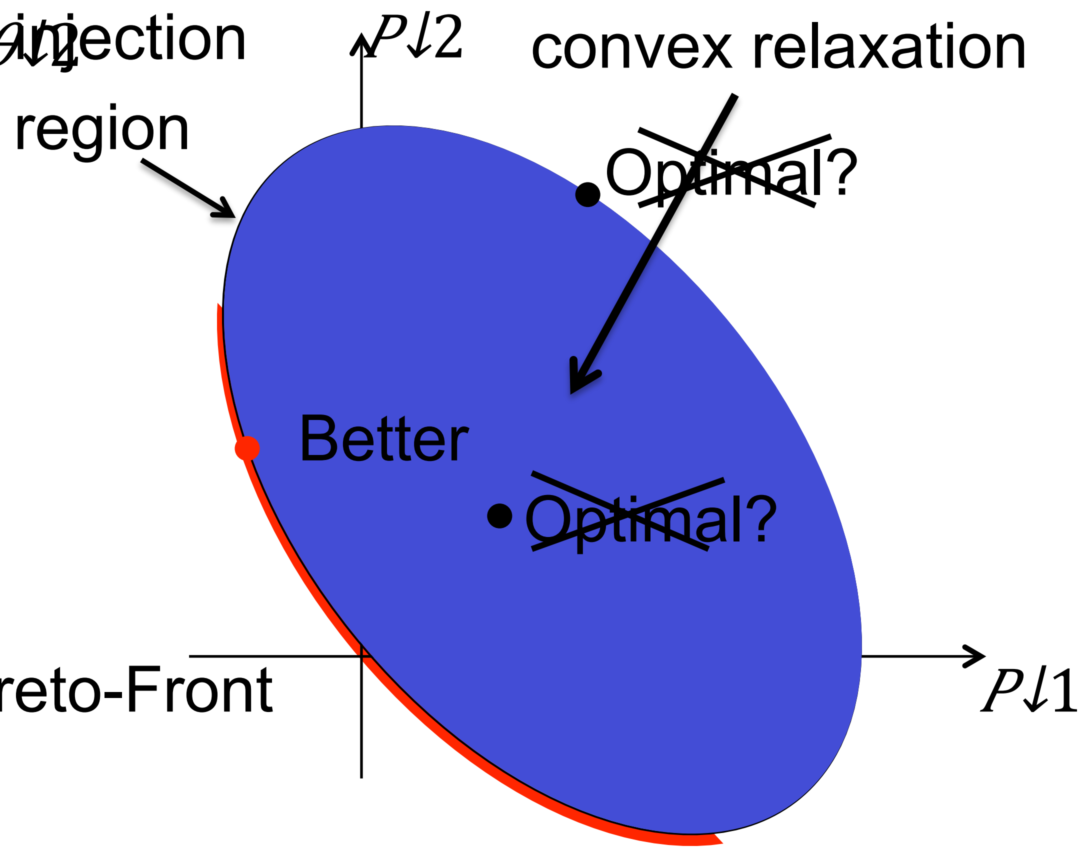


$|V_1|=|V_2|=1 p.u.$

Injection region:
all feasible (P_1,P_2)

$\min f(P_1, P_2)$

f *increasing*

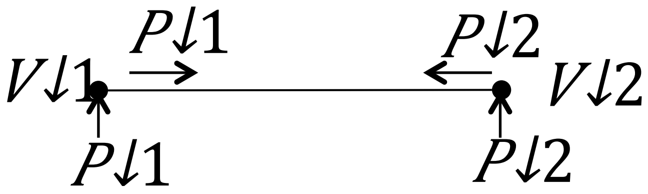


Pareto-Front = Pareto-Front of its Convex Hull

=> convex relaxation is tight.

Add constraints

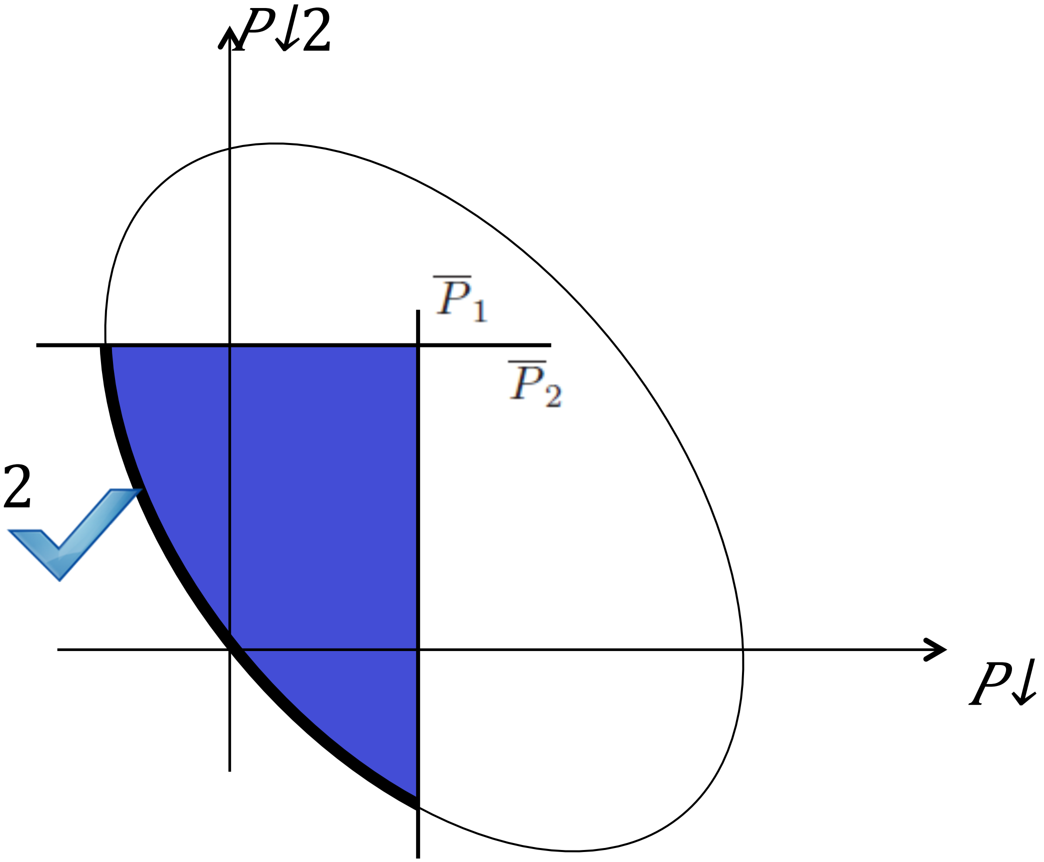
- Two bus network



- Active power upper bounds

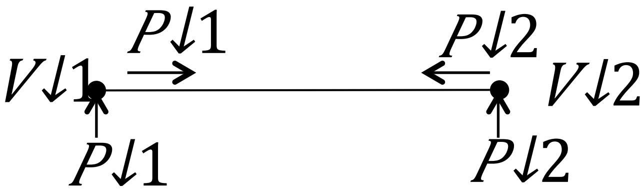
  $P_1 \leq \bar{P}_1 \;, P_2 \leq \bar{P}_2$

- Reactive power upper bounds
- Power lower bounds



Add constraints

- Two bus network



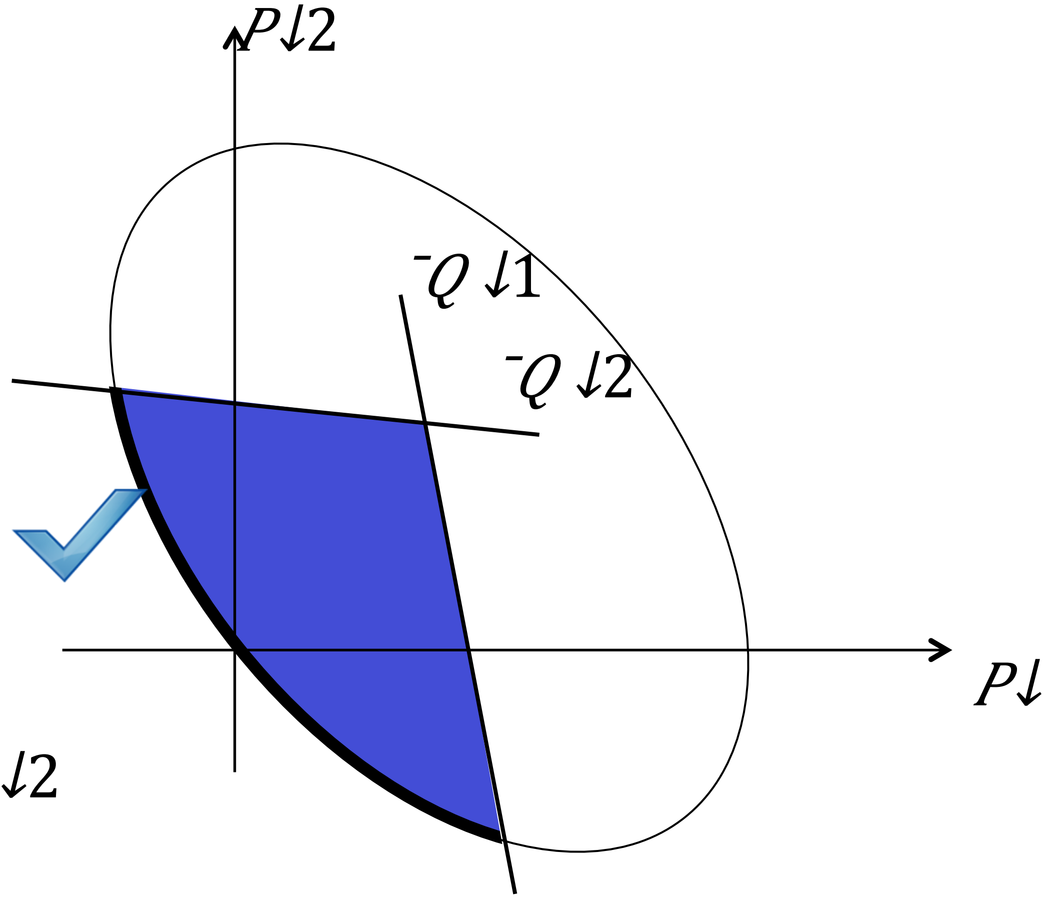
- Active power upper bound

✓

- Reactive power upper bounds

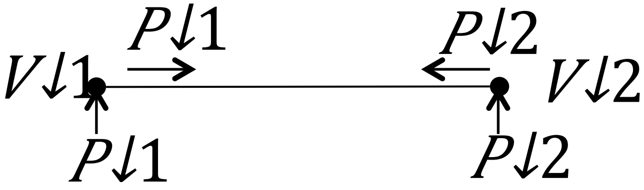
$$Q_1 \leq \bar{Q}_1 \ , Q_2 \leq \bar{Q}_2$$

- Power lower bounds



Add constraints

- Two bus network



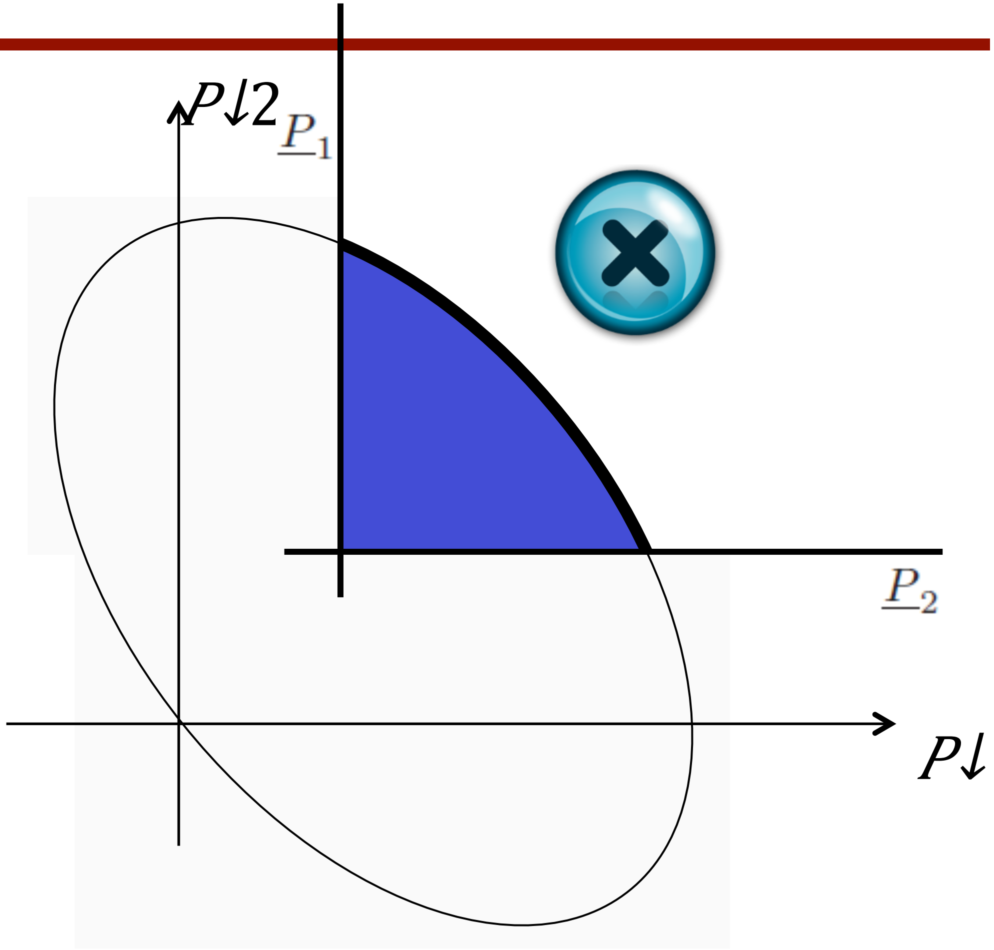
- Loss

✓

- Power upper bounds

✓

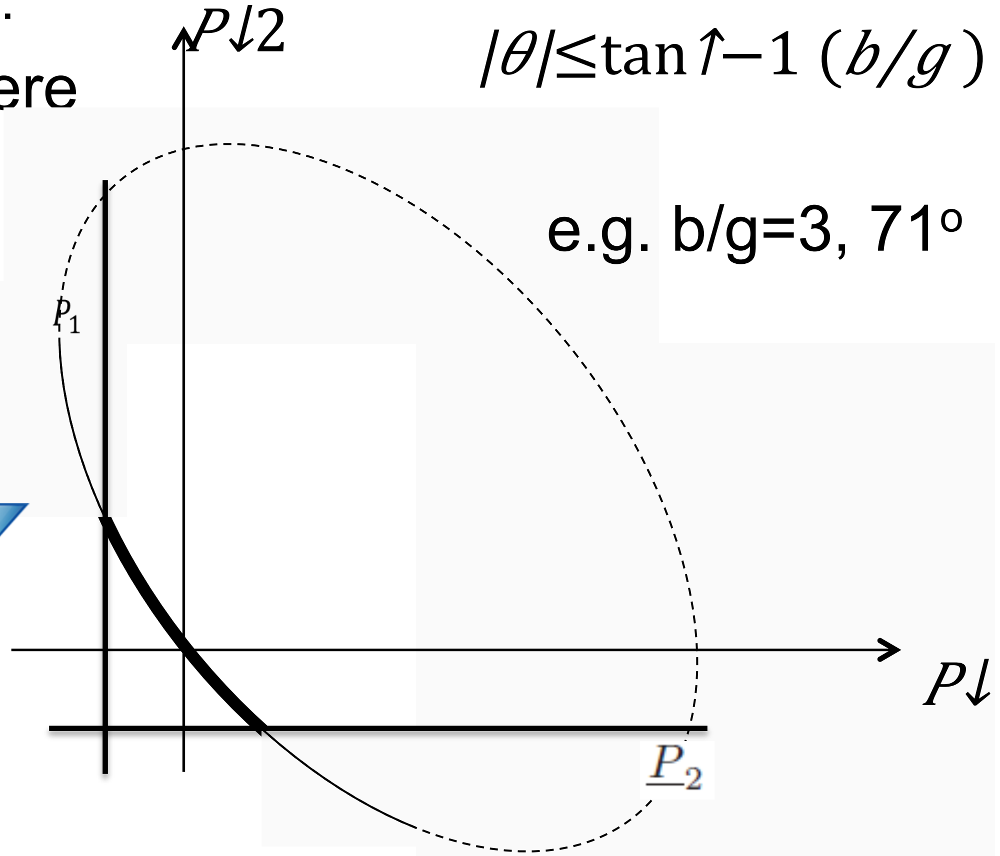
- Power lower bounds



This situation is avoided by adding angle constraints

Angle Constraints

- Angle difference is often constrained in practice
  - Thermal limits, stability, …
- Only a partial ellipse where all points are Pareto optimal.
- Power lower bounds



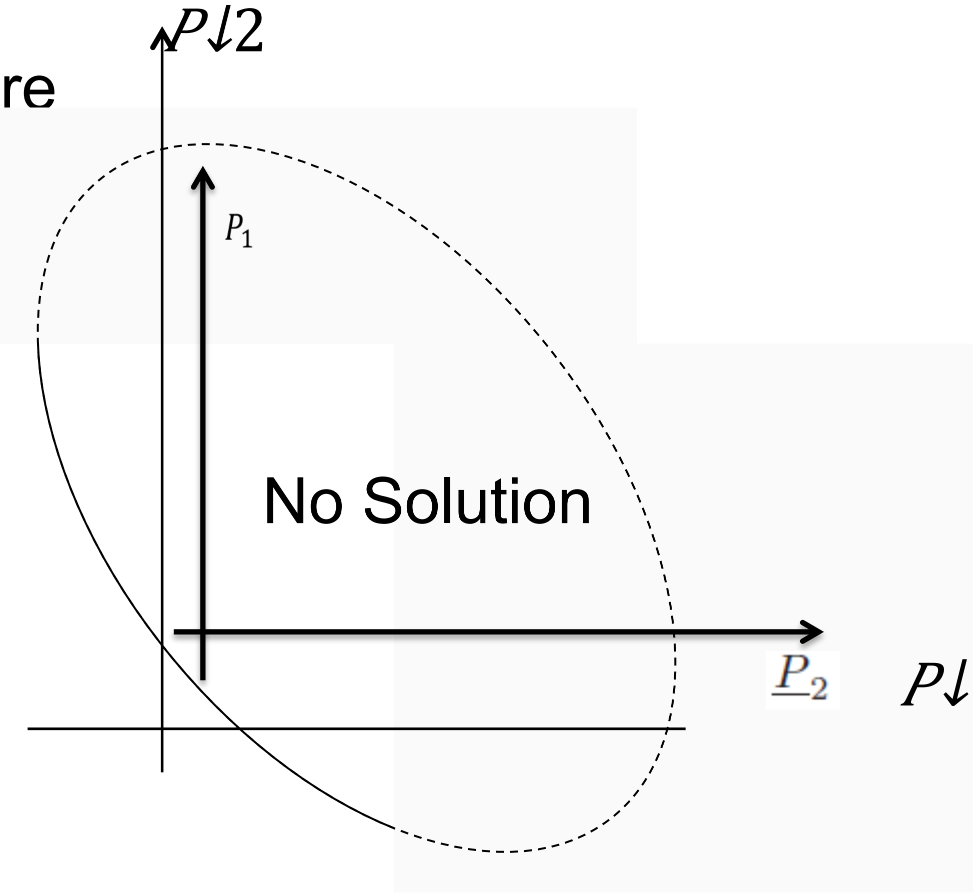
$|\theta| \leq \tan^{-1}(b/g)$

e.g. b/g=3, 71°

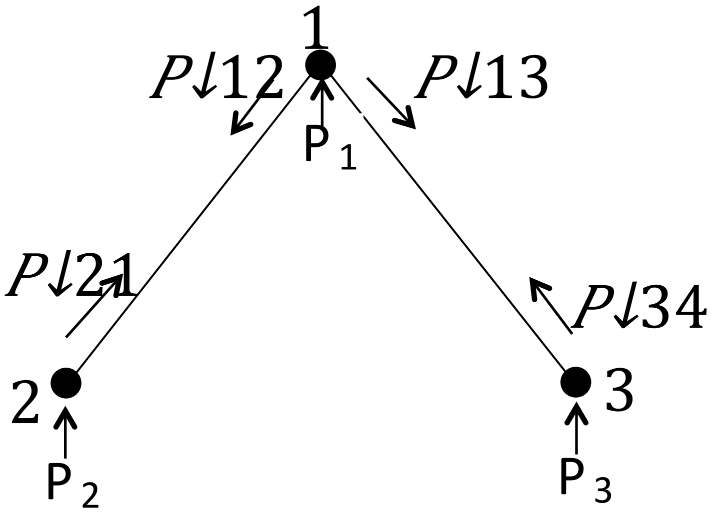
Angle constraints

- Angle difference is often constrained in practice
  - Thermal limits, stability, …
- Only a partial ellipse where all points are Pareto optimal
- Power lower bounds

✓ Or Problem is Infeasible



Injection Region of Tree Networks

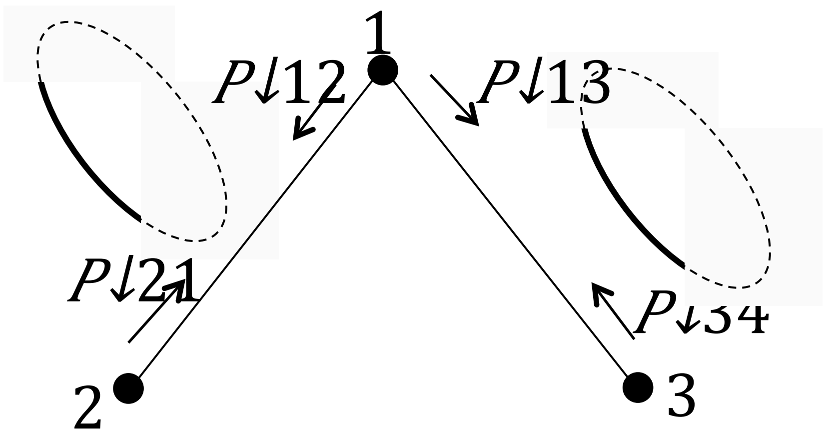


$$P_1 = P_{12} + P_{13}$$
$$\boldsymbol{P_2} = P_{21}$$
$$\boldsymbol{P_3} = P_{34}$$

- Injections are sums of line flows
- Injection region =

  monotone linear transformation of the flow region
- Pareto front of injection region is preserved under convexification if same property holds for flow region.
- Does it?

Flow Region

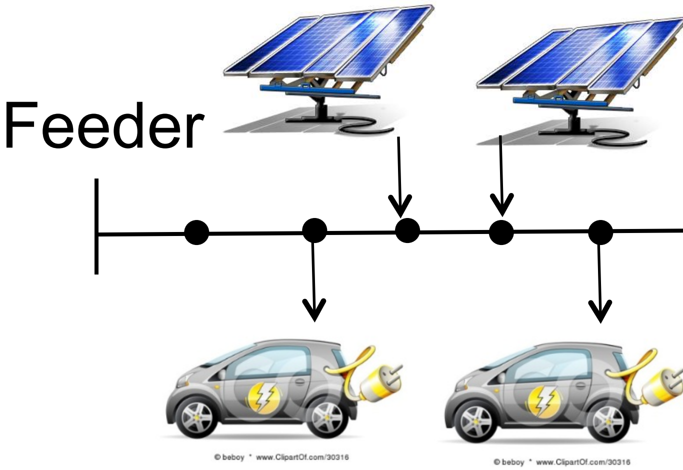


- One partial ellipse per line
- Trees: line flows are decoupled

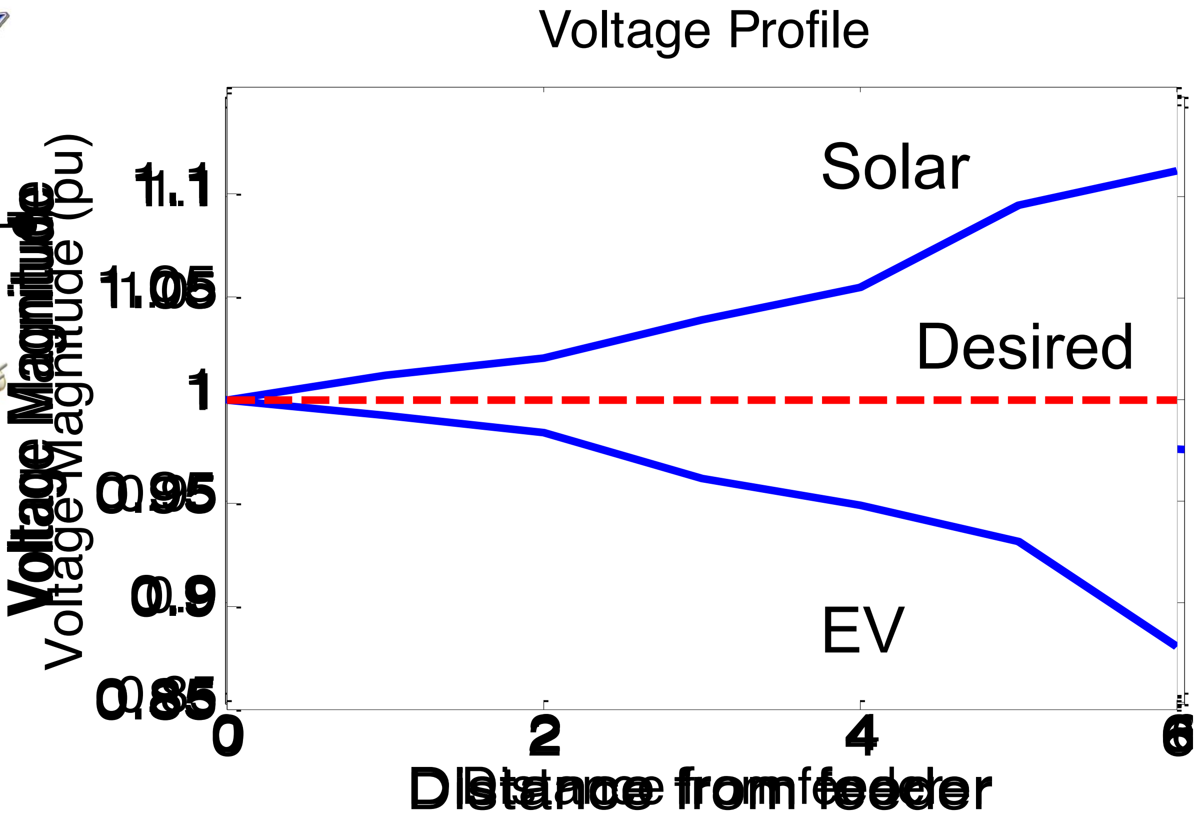
Flow region=Product of *n-1* ellipses

Pareto-Front of Flow Region = Pareto-Front of its Convex Hull

Application: Voltage regulation



(Lam, Z, Dominguez-Garcia & Tse., 12)



Current capacitor banks

- Limited switching
- Small operating range

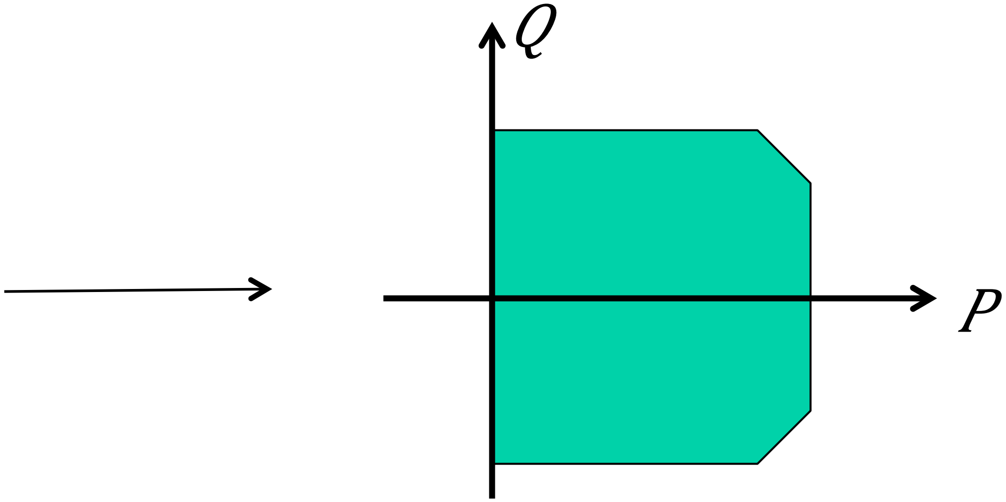
Use power electronics to regulate voltage via reactive power.

Optimal distributed voltage regulation in power distribution networks, ArXiv

Power Electronics

Solar Inverter

P-Q Region

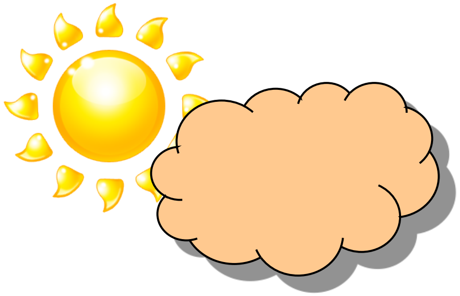


- The reactive powers can be used to regulate voltages

http://en.wikipedia.org/wiki/File:Solar_inverter_1.jpg

Random Solar Injections

- Solar Injections are random

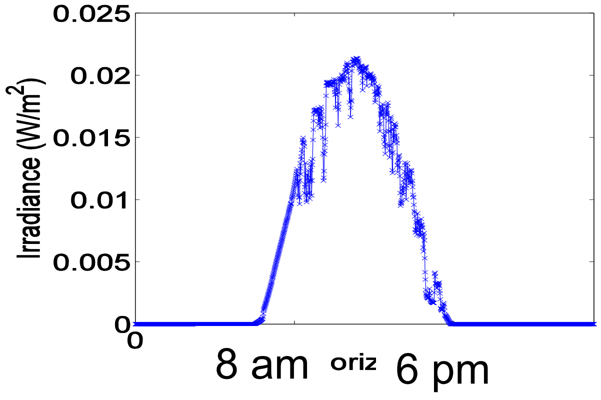


Random

Net Load=Load - Solar

$\in[Load - Solar, Load]$

Random Bus Active Power Constraints

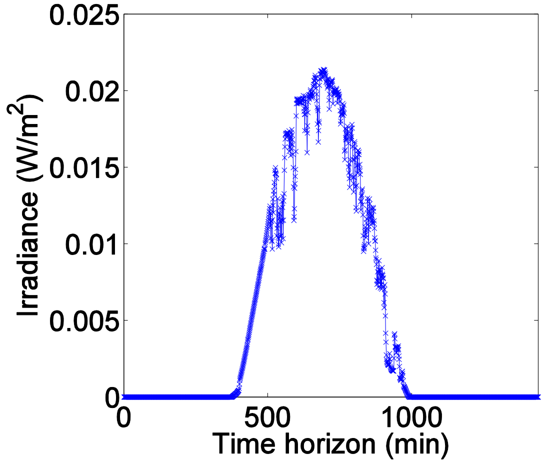


Voltage Regulation Problem

- Can formulate as an loss minimization problem

$$\text{minimize} \sum P_i \quad \longleftarrow \text{System Loss}$$

$$\text{subject to } |V_i| = 1 \quad \longleftarrow \text{Voltage Regulation}$$



$$\underline{P}_i \leq P_i \leq \overline{P}_i \quad \longleftarrow \text{Active Power}$$

$$\underline{Q}_i \leq Q_i \leq \overline{Q}_i \quad \longleftarrow \text{Reactive Power}$$

Control

Network Constraints

How to do relaxation algebraically?

Algebraic Representation

- Everything is linear in VV^H

$$\begin{aligned} &\text{maximize } \sum P_i \\ &\text{subject to } |V|_i^2 = 1,\ \forall i \\ &\qquad\qquad \underline{P}_i \leq P_i \leq \overline{P}_i \\ &\qquad\qquad \underline{Q}_i \leq Q_i \leq \overline{Q}_i \\ &\qquad\qquad \mathbf{p} + j\mathbf{q} = \operatorname{diag}(\mathbf{v}\mathbf{v}^H \mathbf{Y}^H) \end{aligned}$$

- Replace VV^H by W

$$\boldsymbol{W} \succcurlyeq \mathbf{0}$$

$$\cancel{\text{rank}(\boldsymbol{W}) = 1}$$

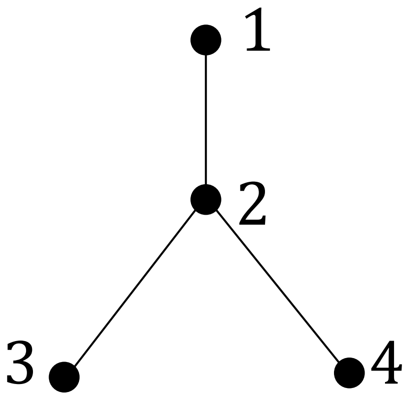
Convex rank relaxation

- SDP

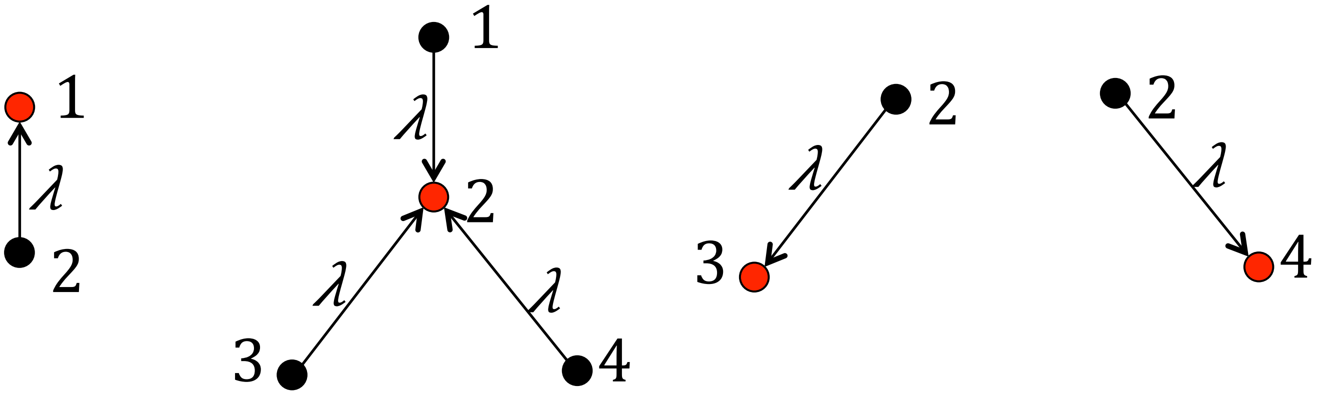
Decentralized Algorithm

- Convex relaxation gives an SDP, does not scale
- No infrastructure to transfer all data to a central node
- We exploit the tree structure to derive a decentralized, asynchronous algorithm
- Communication along physical topology.

Example



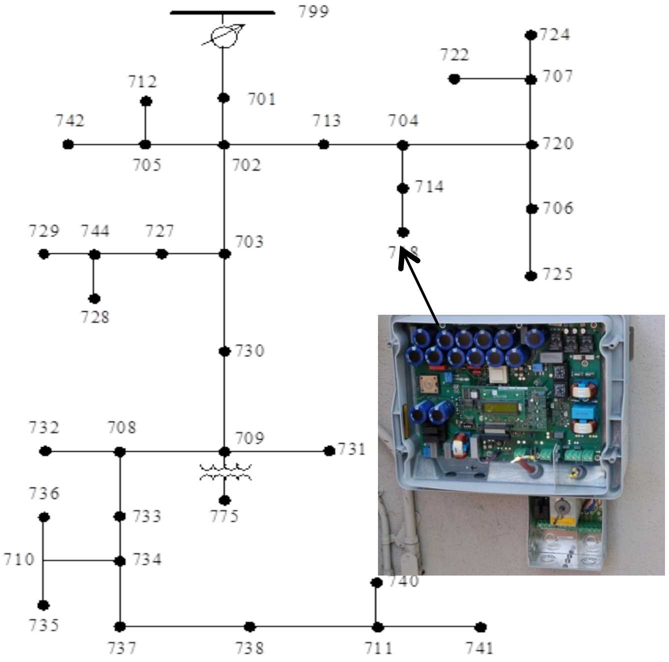
- Each node solves its sub-problem with
  - Its bus power constraints
  - Lagrangian multipliers for its neighbors (line flow constraints)



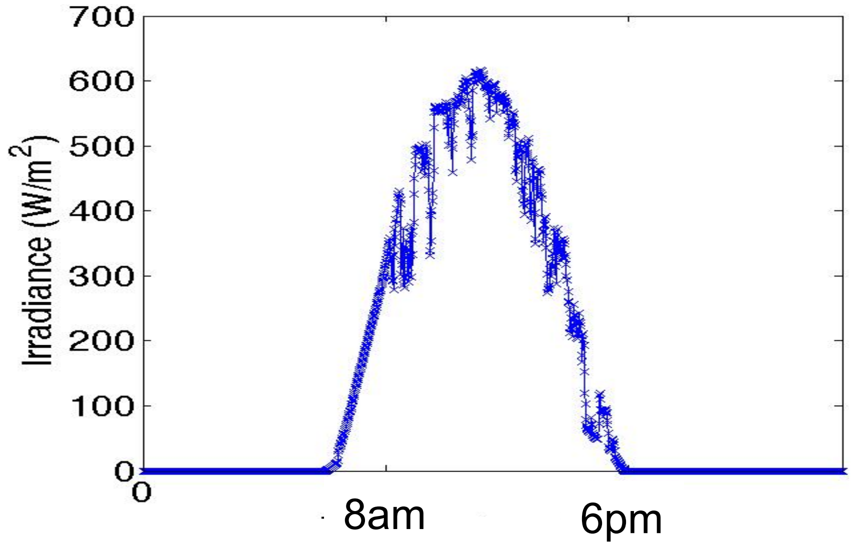
- Update Lagrangian multipliers
- Robust to asynchrony

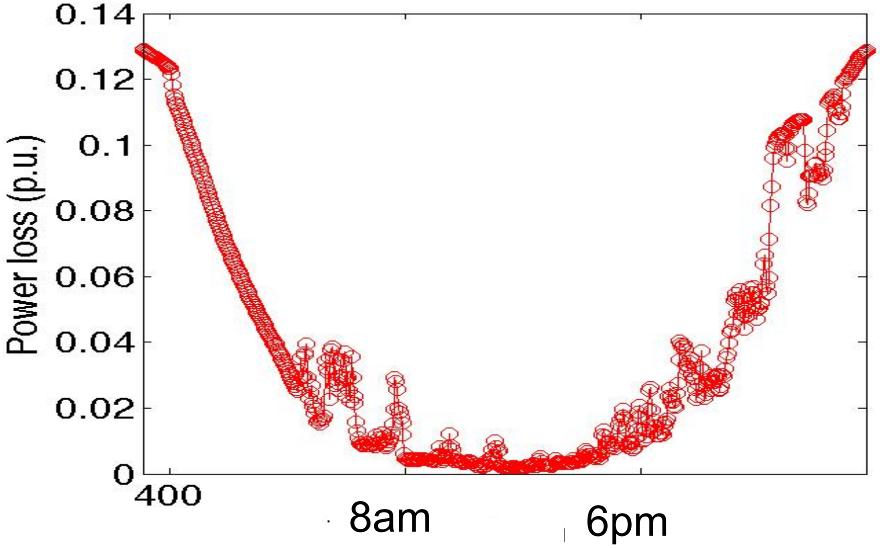
Simulations

- 34 Bus Network



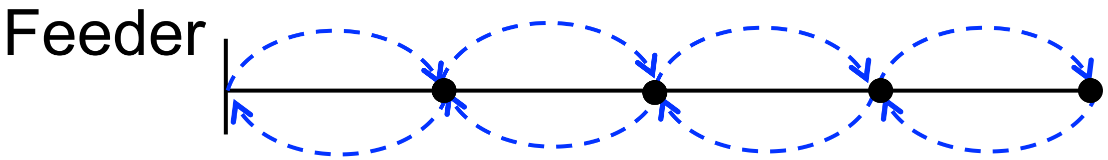
- 2.4 KV
- Bus~ 10 households
- Nominal Loads
- Fixed capacitor banks





Cyber-physical System

- An optimal distributed algorithm for voltage regulation



- PG&E: Communication links fails about 50% of the time (dies completely)

- What happens if communication is not complete?

- Can we maintain voltage stability?

Local Control Scheme

- No real-time communications at all (today's system)
- Solar and EV penetrations are increasing
- How do maintain voltage stability?
- Each bus senses its voltage, adjusts its reactive power (Active power not changed)
- Are local actions enough?

A Local Control Approach to Voltage Regulation in Distribution Networks, NAPS 2013, on ArXiv

Iterative Algorithm

At time t,

- If $|V_i[t]|<1$, Increase Q_i
- If $|V_i[t]|>1$, Decrease Q_i

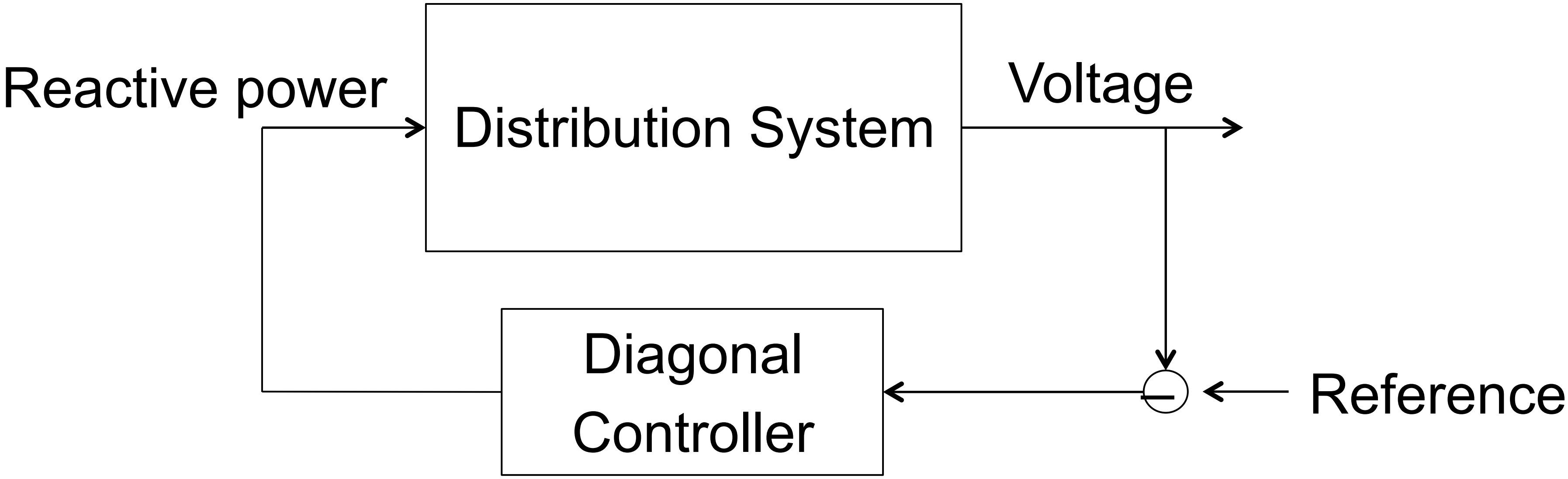
Update algorithm

$$Q_i[t+1]=Q_i[t]+\overset{\text{Gain}}{d_i}(1-|V_i[t]|)$$

Question: Does this algorithm ever terminate?

We show sufficient and necessary conditions

Dynamical System



- Linearize the system

$$\mathrm{v}=Aq$$

- Matrix A depends on active powers

Linear System

- Given active powers,

$$\mathrm{v} = Aq$$

- Does there exist a diagonal controller to stabilize the system?

- Does there exist D, diagonal, such at

$$DA + A^{T} D \succ 0$$

- Given p, easy to check

No Communication

- Active powers lies in a region

$$P_i \leq P_i \leq \bar{P}_i$$

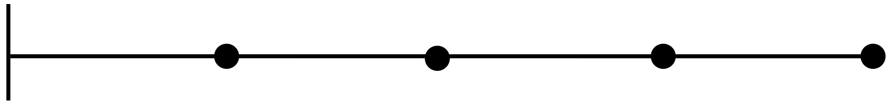
- One diagonal controller needs to work for all active powers

- Theorem: It is sufficient to design a controller with respect to $(P_1, P_2, \ldots, P_n)$

- Proof not trivial, careful analysis of the system matrix

Stability Regions

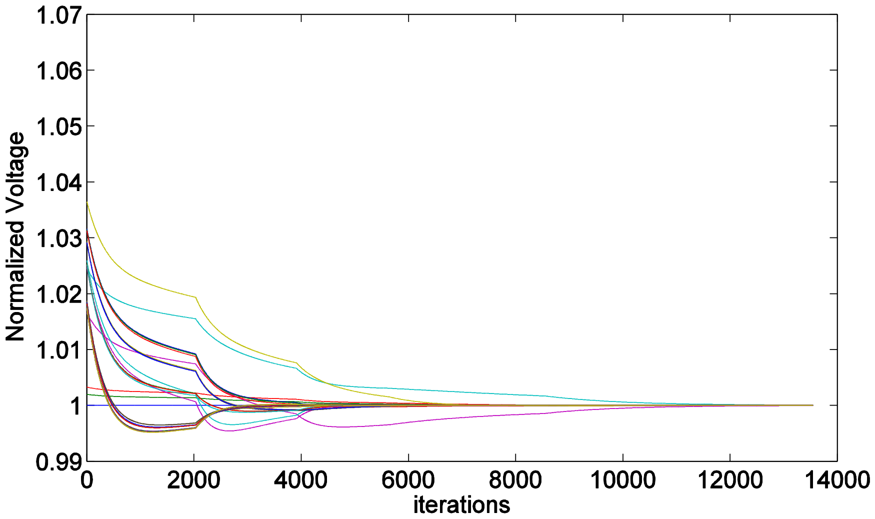
- Stability region: set of all active powers that can be stabilized by some diagonal controller
- Line networks are hardest to control



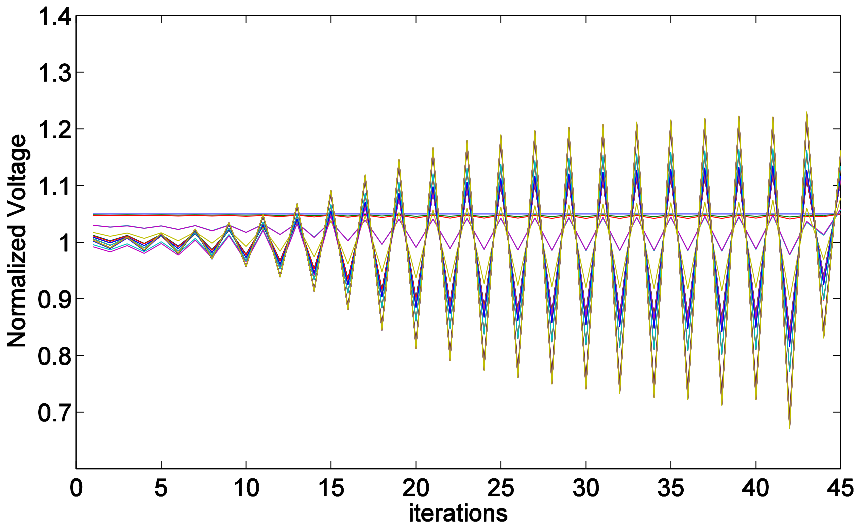
- Size of stability region depends on the depth of the network
- As length go to ∞, stability region goes to a point

Simulation

- IEEE 34-bus test feeder



Normal Load



5x the Load

Summary

- Geometrical view of power flow
- Optimization problems on tree networks can be convexified
- Applied to design an optimal distributed algorithm for voltage regulation.
- Communication important for long networks