Model Predictive Control for Energy Efficient Buildings

Yudong Ma

University of California

Berkeley, USA

www.mpc.berkeley.edu

Outline

- Background
- Nominal MPC Design
- Distributed MPC Design
- Stochastic MPC design
- Conclusions

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  - Motivation
  - HVAC System
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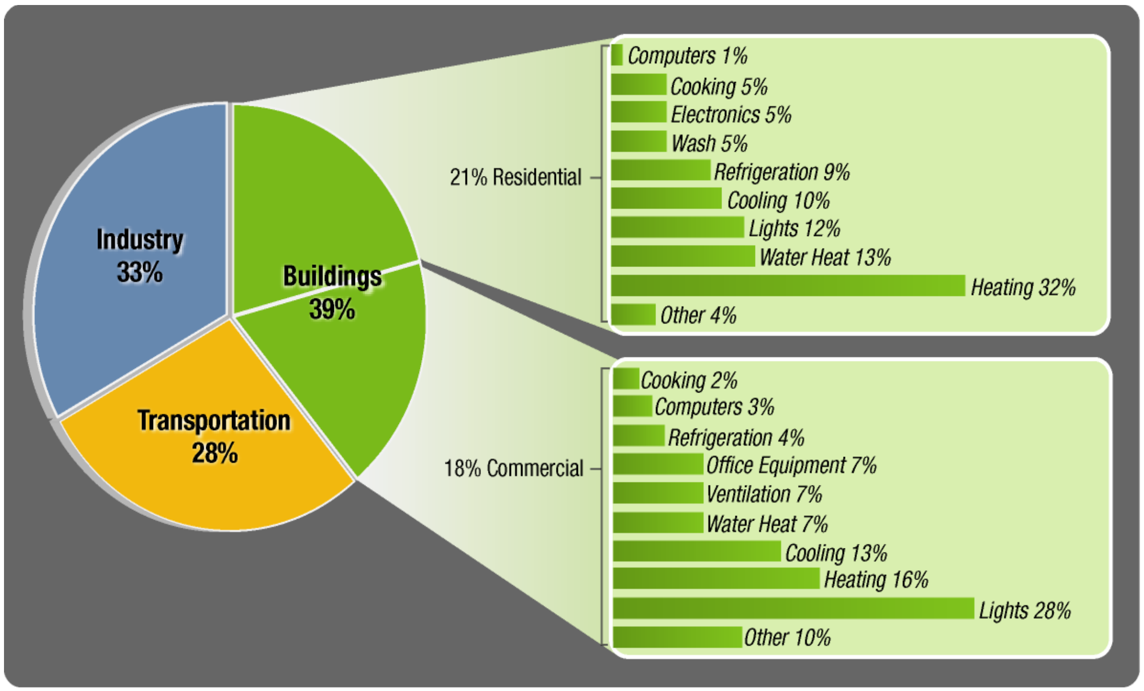
Background

Buildings in USA account for:

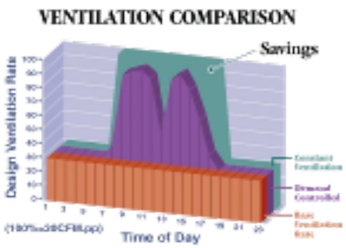


Source: Buildings Energy Data Book

- 48% of total Green House Gas (GHG) emissions over the country.
- Growing GHG emissions at a rate of about 1.8 percent per year over the next 25 years.
- $120 billion in electricity and natural gas used each year.
- Energy consumed in buildings is used wastefully and inefficiently.

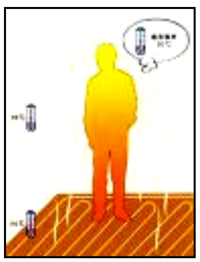
Background

Demand Controlled Ventilation



High Performance Equipment

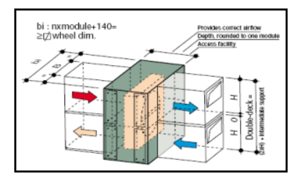
Radiant Heating



Daylighting

Shading

Energy Recovery Ventilation



courtesy of UTRC

Predictive Control of Building Heating Ventilation and Air Conditioning (HVAC) systems

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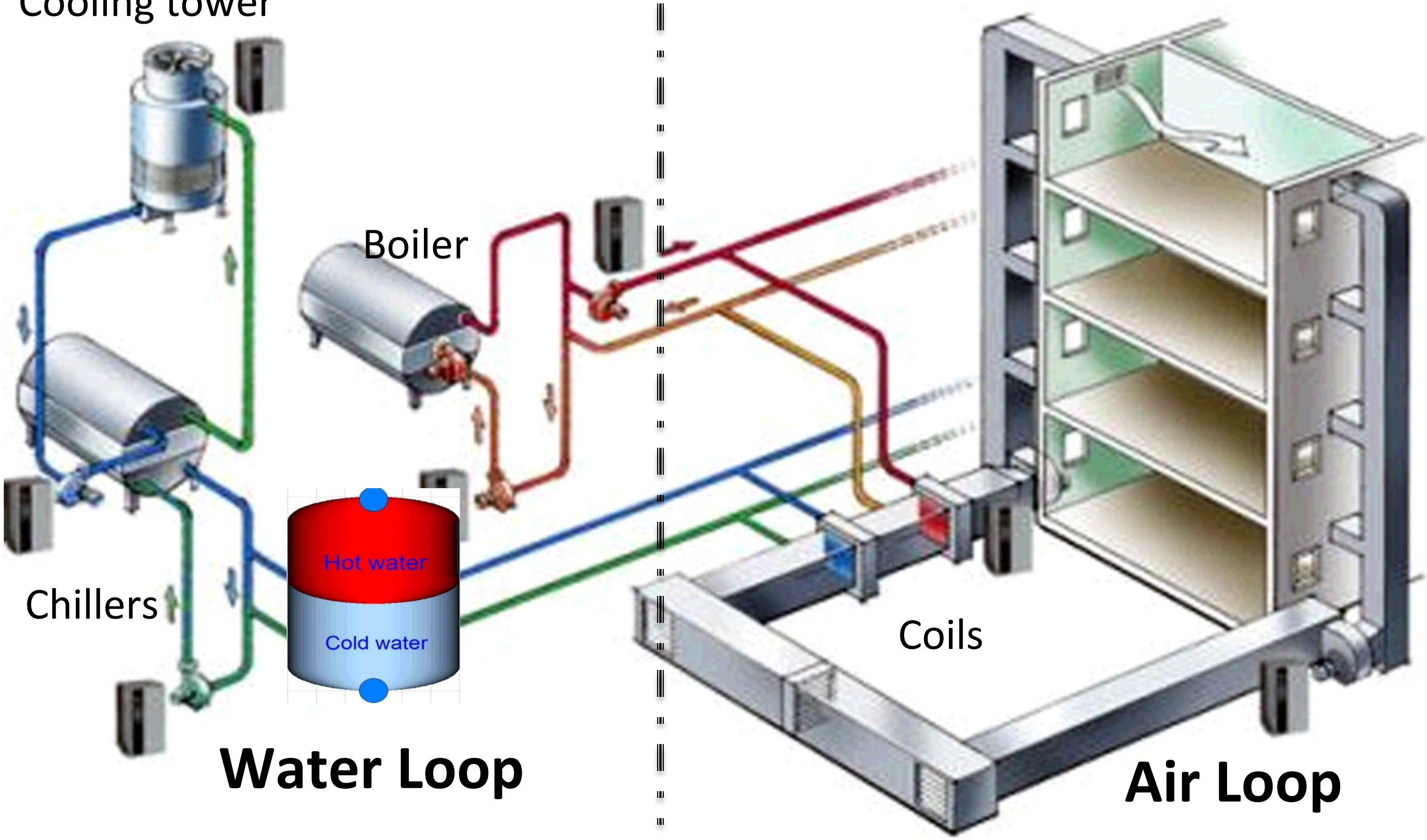
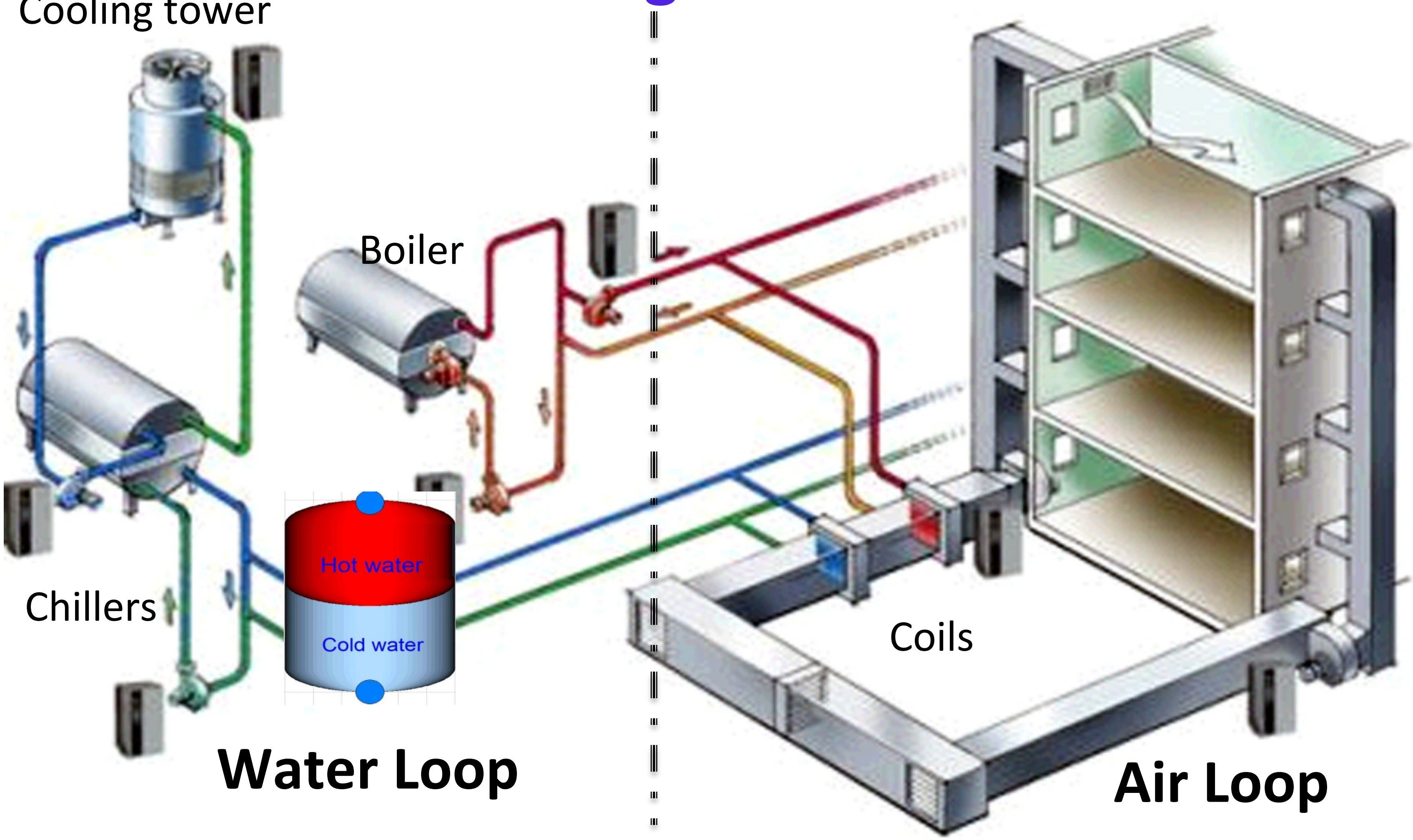


Cooling tower
Boiler
Chillers
Hot water
Cold water
Water Loop
Coils
Air Loop

Background



Focus on Model Predictive Control (MPC) for water loop and air loop with energy storage element

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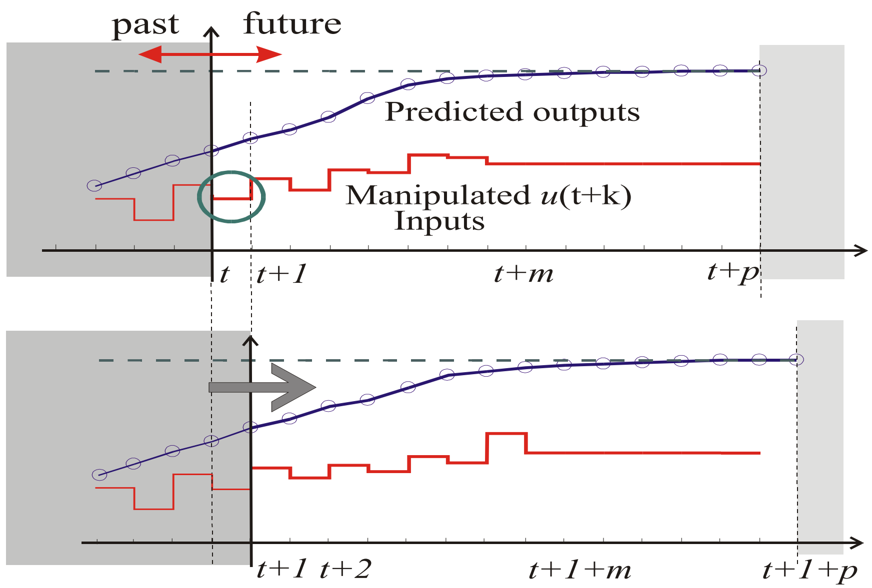
Model Predictive Control (MPC)



<u>At time t:</u>

- Measure (or estimate) the current state x(t).

- Find the optimal input sequence.

- Apply only u(t)=u*(t) , and discard *u*(t+1), u*(t+2)*, …

<u>Repeat the same procedure at time *t +1*</u>

Predictive, Multivariable, Model Based, Constraint satisfaction

Model Predictive Control (MPC)

$$\min_{U} \sum_{k=t}^{t+N-1} \text{Energy}(x_k, u_k, w_k)$$

$$\text{subj. to} \quad \begin{cases} x_{k+1} = f(x_k, u_k, w_k), \ \ k = t, \ldots, t+N-1 \\ u_k \in \mathcal{U}, \ \ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X}, \ \ k = t, \ldots, t+N \\ x_t = x(t) \end{cases}$$

At time t:

- Measure (or estimate) the current state x(t), obtain predictions w_k .
- Find the optimal input sequence.
- Apply only u(t)=u*(t) , and discard *u*(t+1), u*(t+2)*, …

Repeat the same procedure at time *t +1*

Predictive, Multivariable, Model Based, Constraint satisfaction

Steps Towards Success

control action — environment — constraints

$$x_{k+1} = f(x_k, u_k, w_k), \quad x_k \in \mathcal{X}_k$$

At step t decide on $u(t)$ based on prediction on $w_t, w_{t+1}, \dots, w_{t+N-1}$

- "Good" Dynamics Model
- Quantifying Prediction Model

$$w_{t+1} \in \mathcal{W}_{t+1}, \dots, w_{t+N-1} \in \mathcal{W}_{t+N-1}$$

- Predictive Control Design

Main Contributions

- Developed reduced order data driven models
- Studied uncertain building load distribution (learning from historical data statistics)
- Developed a predictive control framework for building HVAC systems
  - Adaptable to buildings with various configurations
  - Implementable on existing distributed low-cost
  - hardware
  - Capable of handling uncertain predictions with finite-support distributions.

Main Contributions

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  - Air-loop System
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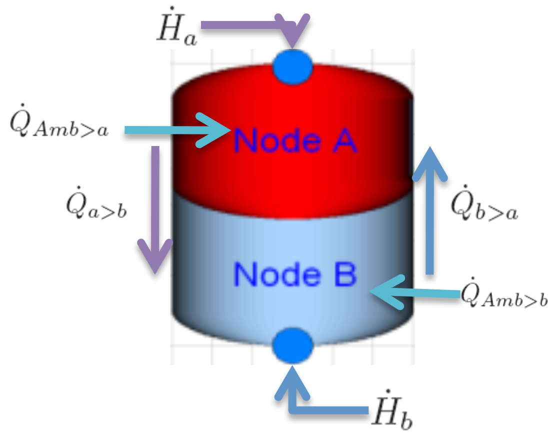
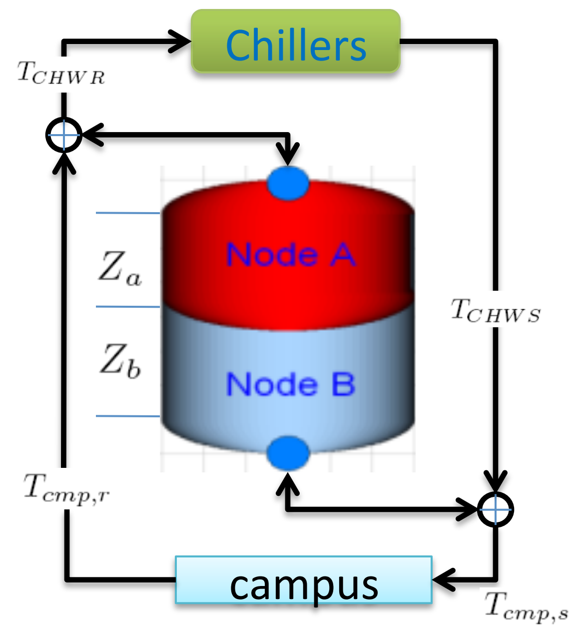
Model Abstraction – Water Loop

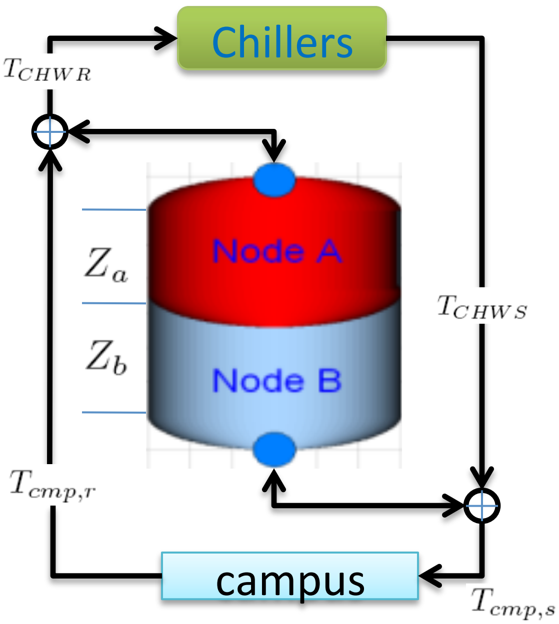
- **Switch linear tank model**

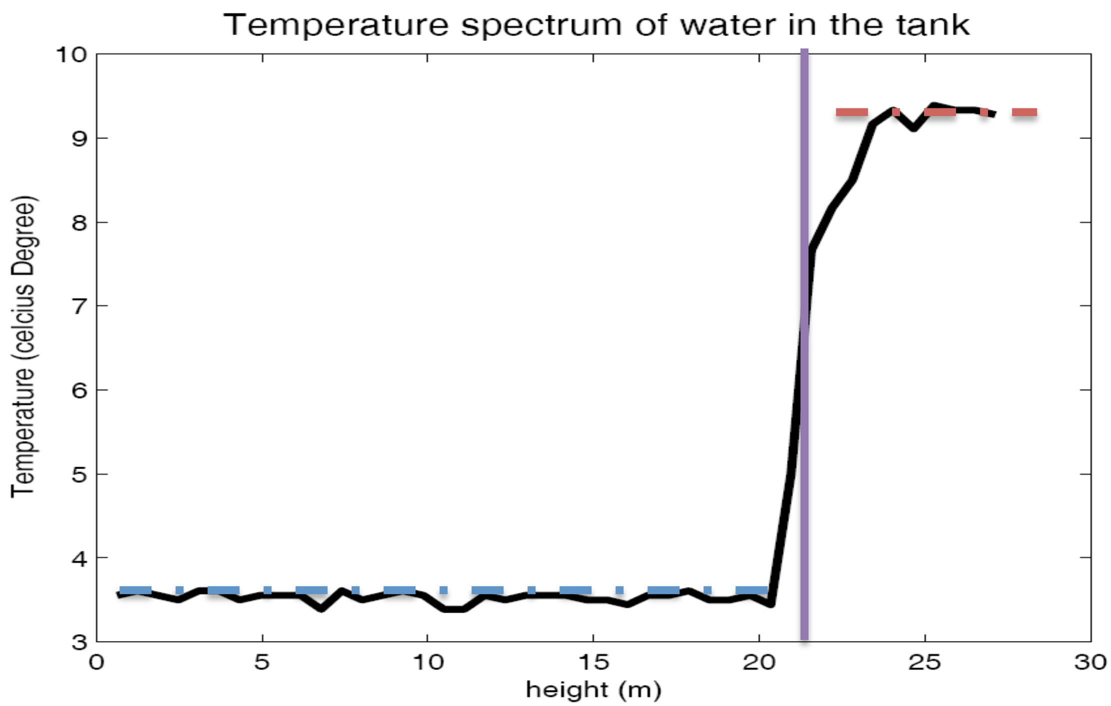




- **Load Predictions**
  - Building Load (required chilled water or hot water to meet building demands)
- **Static Nonlinearities**
  - Equipment Performance Maps (Chillers, Cooling tower, Pumps)
- **Equality and inequality Constraints**
  - Comfort range
  - operational constraints for chillers and cooling towers

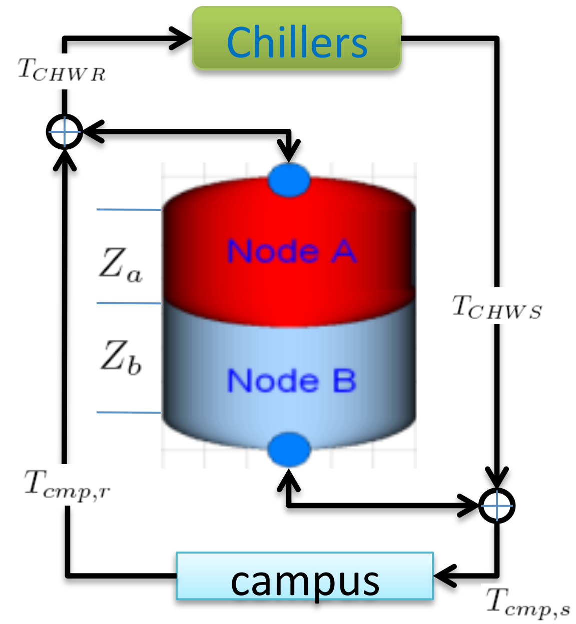
Tank Dynamics





Tank is 2-nodes switch system with a thermo cline that separates the warm water and cool water.

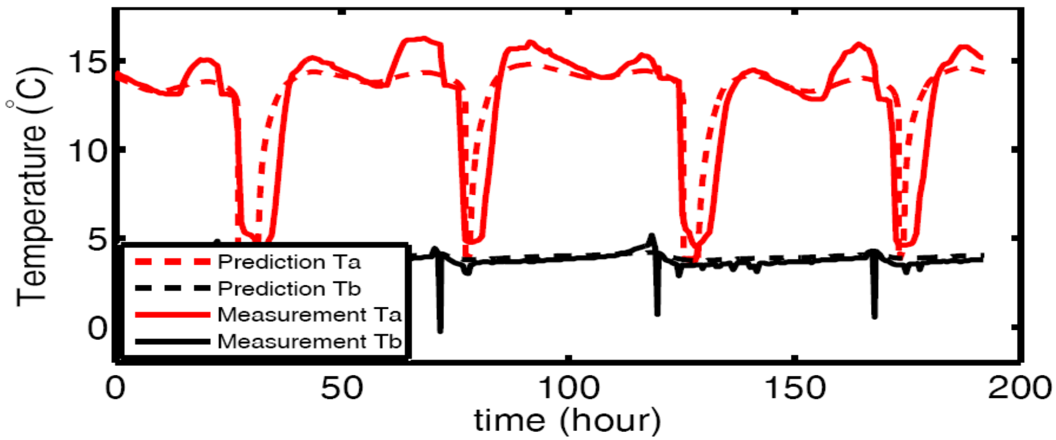
Tank Dynamics

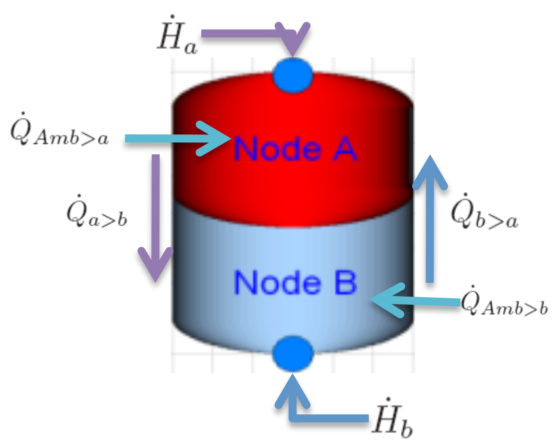


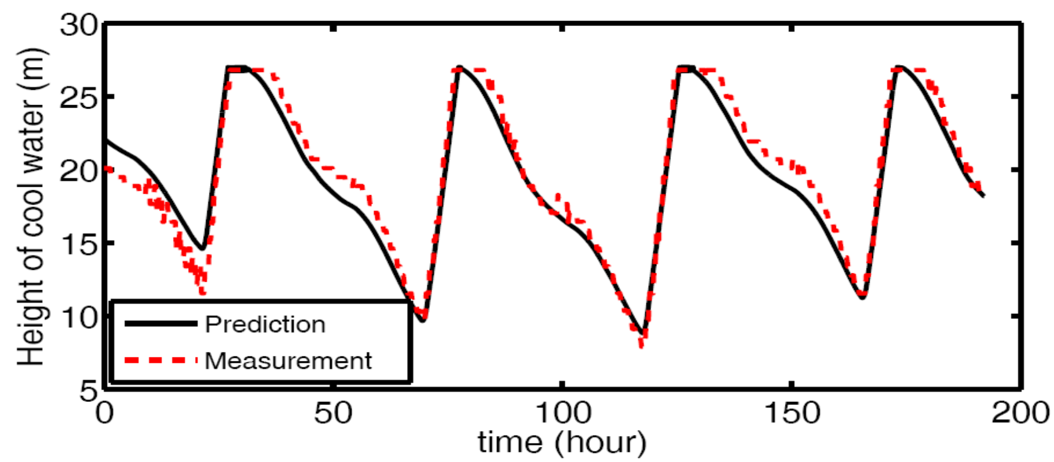
Mass Balance

$$\dot{z}_b = (\dot{m}_{CHWS} - \dot{m}_{cmp,s})/\rho/A_c;$$
$$\dot{z}_a + \dot{z}_b = 0;$$
$$\dot{U}_a = \dot{H}_a + \dot{Q}_{b>a} + \dot{Q}_{Amb>a};$$
$$\dot{U}_b = \dot{H}_b + \dot{Q}_{a>b} + \dot{Q}_{Amb>b};$$

Energy Balance

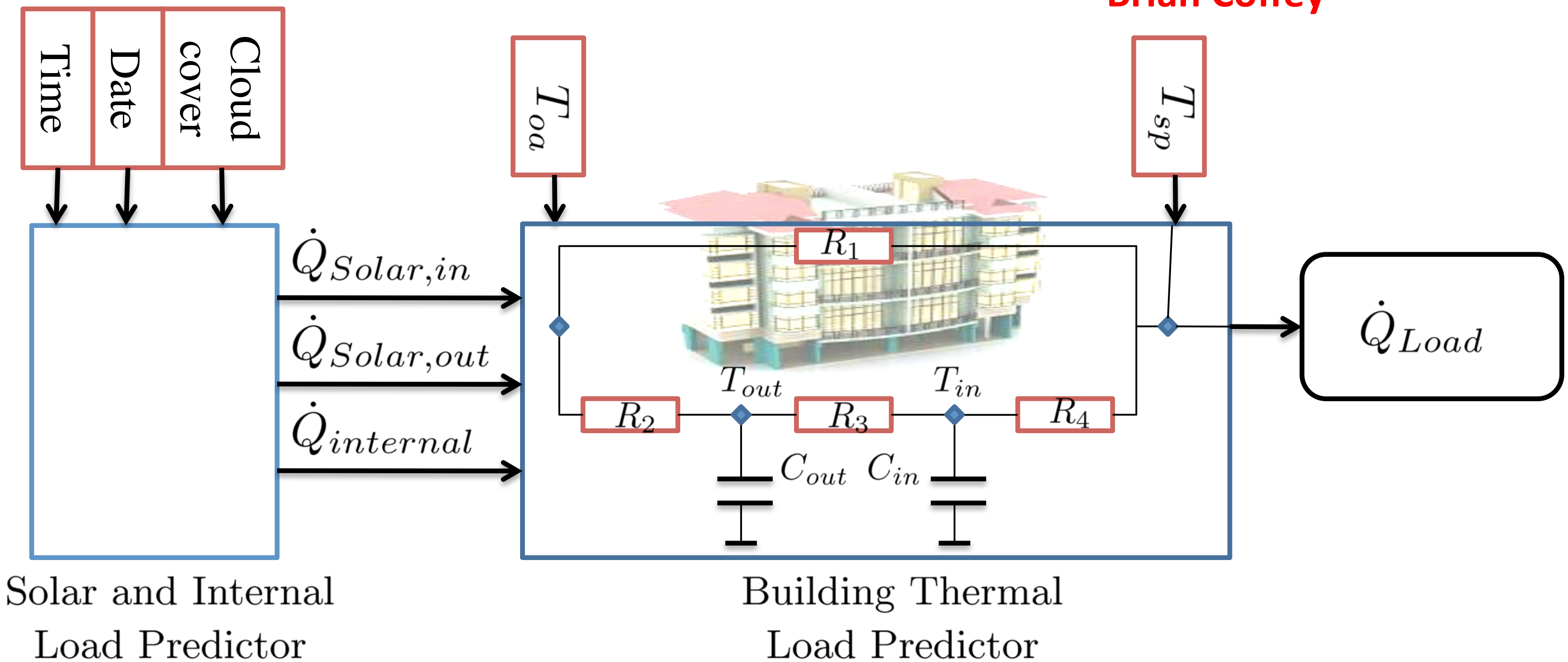




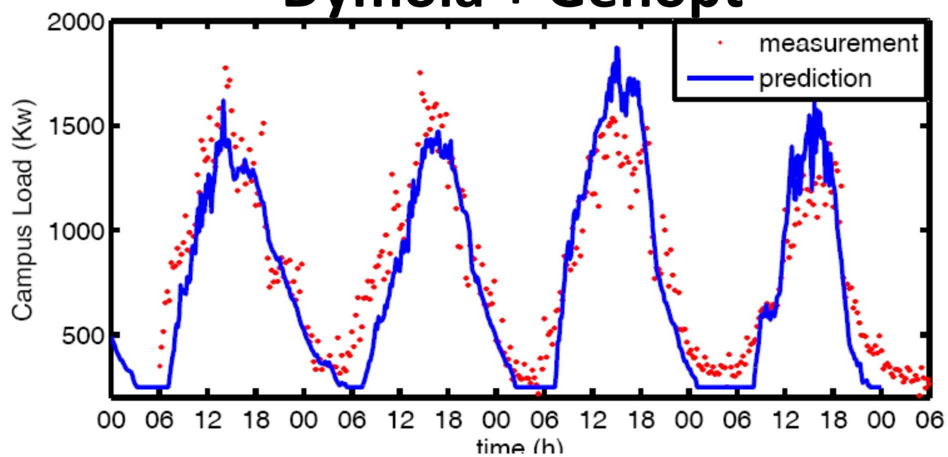


Load Prediction

Brian Coffey

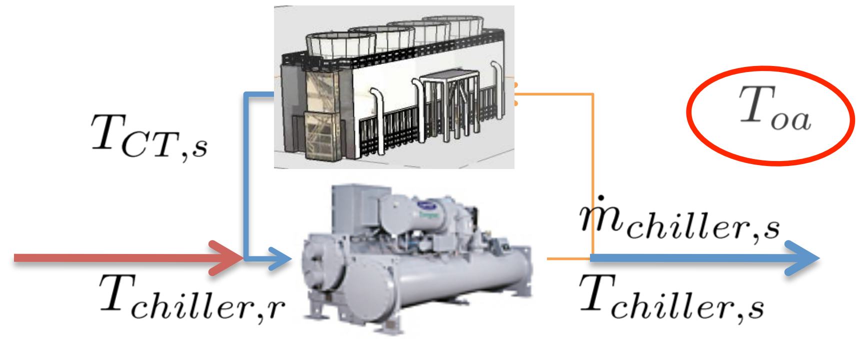


Dymola + Genopt

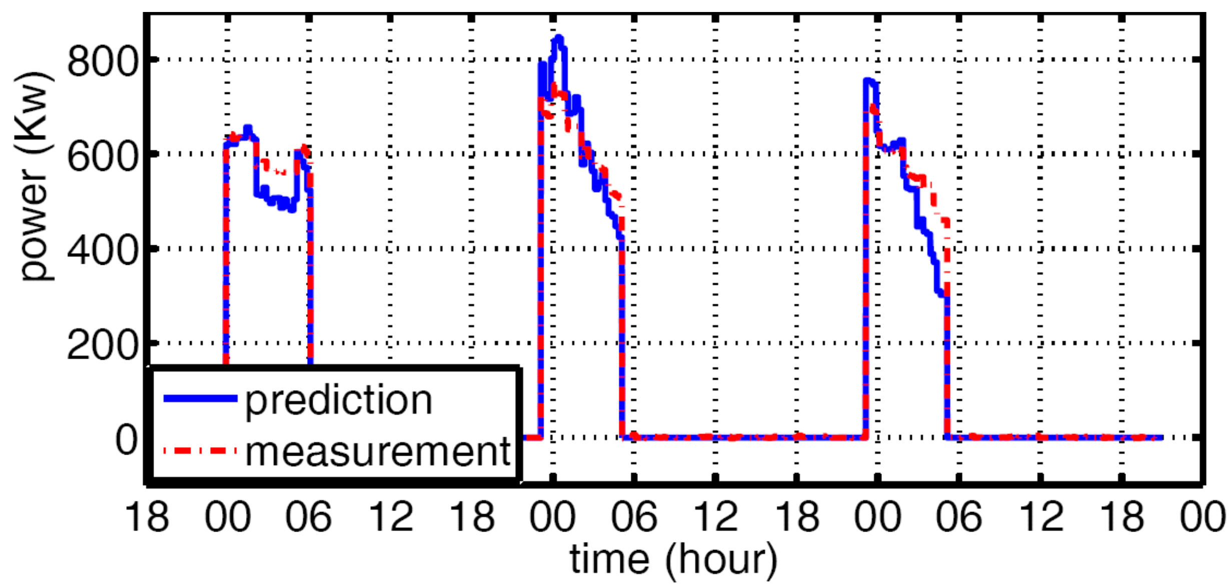


Performance Map

Brandon Hencey

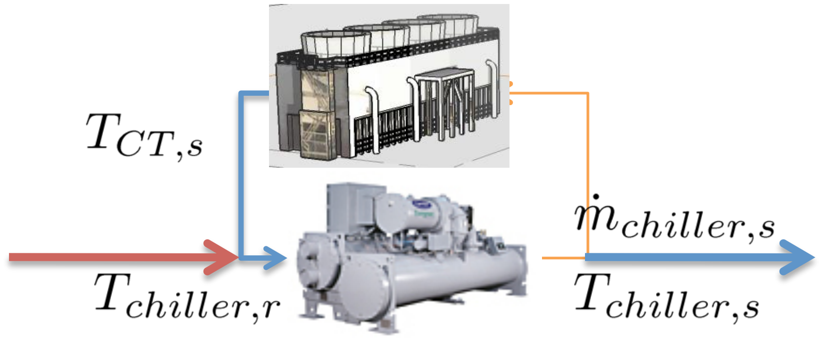


$$\text{Energy}(x_t, u_t, w_t) = \text{Energy}_{\text{chiller}} + \text{Energy}_{\text{CT}} + \text{Energy}_{\text{pump}}$$



Standard in DOE2 modeling libraries

Control Variables and Constraints

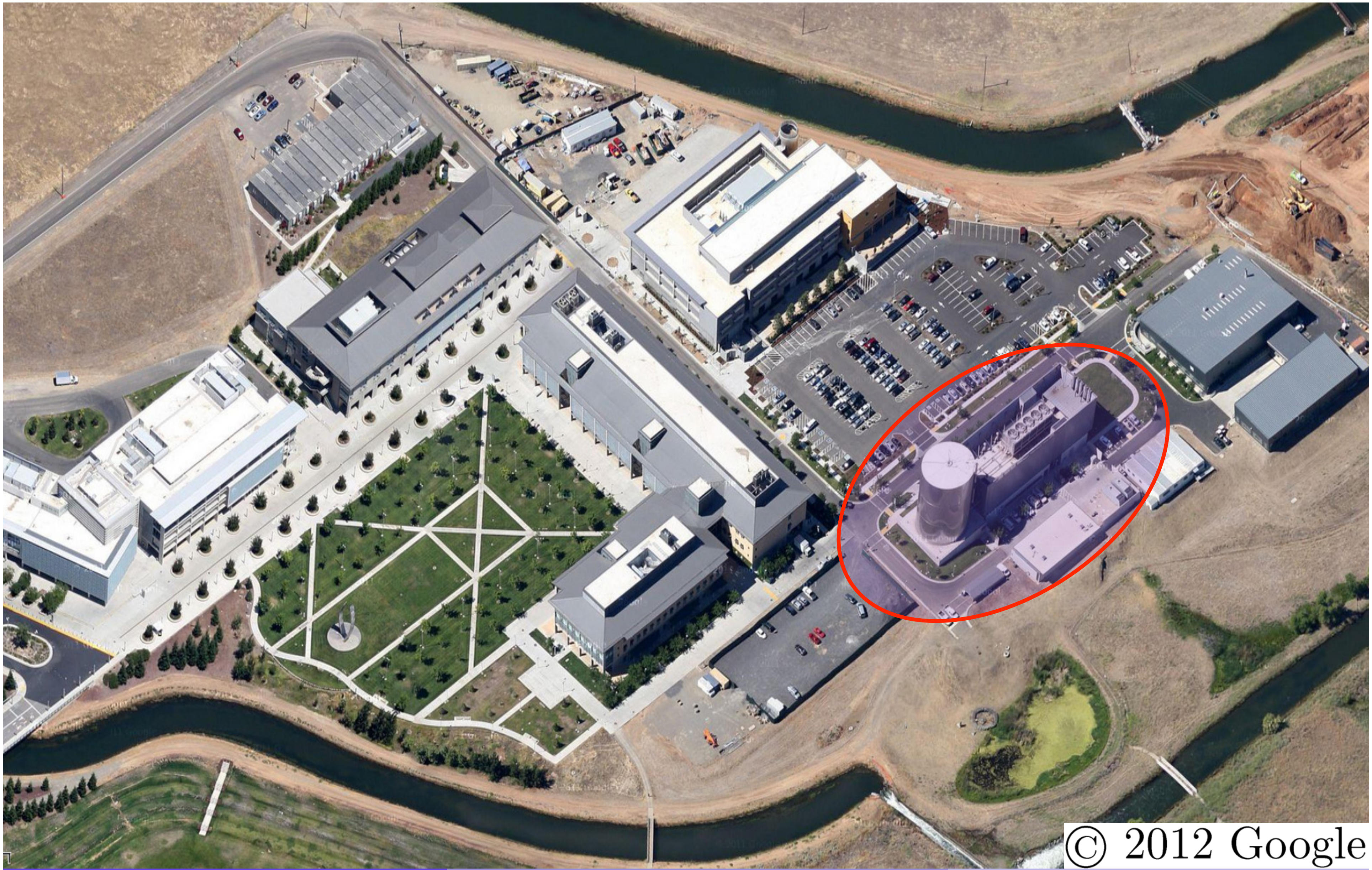


$T_{CT,s}$	Reference of the water temperature exiting the cooling towers
$\dot{m}_{chiller,s}$	Mass flow rate of the chilled water supply
$T_{chiller,s}$	Reference of the water temperature delivered by the chillers
t_{start}, t_{end}	Charging schedule

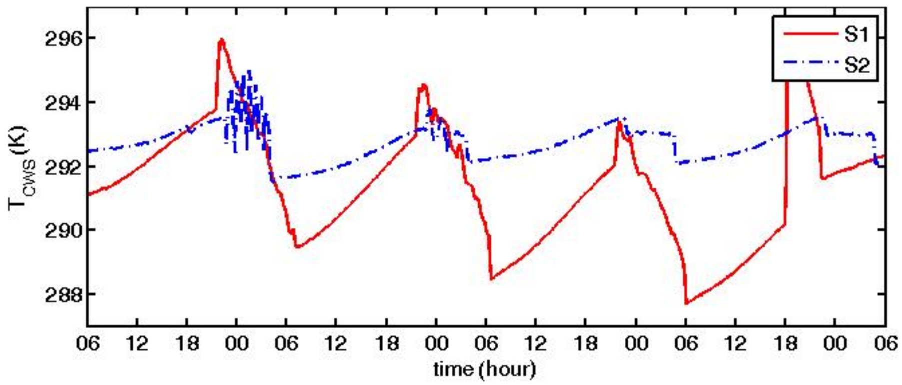
- Operational constraints $u(t) = [T_{CWS,ref}; \dot{m}_{CHWS}; T_{CHWS,ref}] \in \mathcal{U}$
- Tank level $Z_b \in [0.1, 1] z_{tank}$

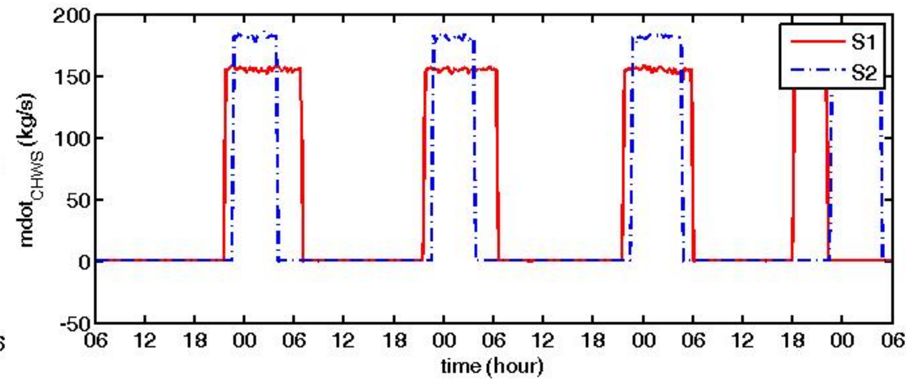
Minimize energy consumption, meet thermal demands, and satisfy operational constraints.

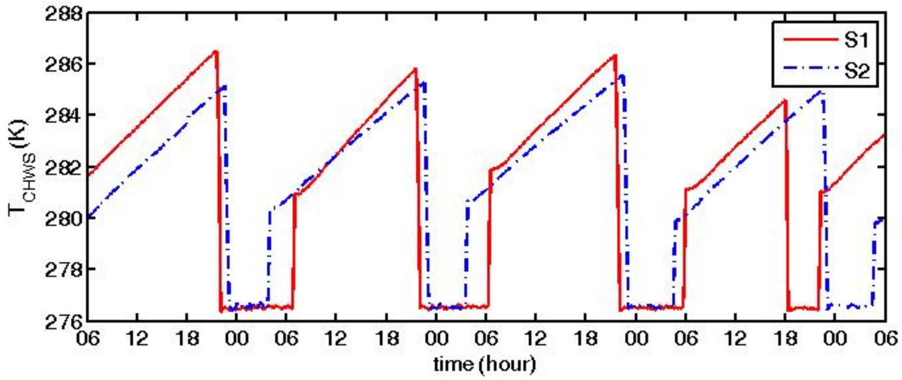
UC Merced Experiment



Results (MPC vs Baseline)







S1 Baseline control

May 24th-28th 2009

S2 Experiment with MPC

Oct 06th-10th 2009

Daily Electricity Bill is reduced by 20%
Central plant efficiency is increased by 7.4%

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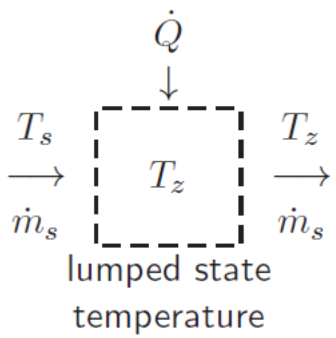
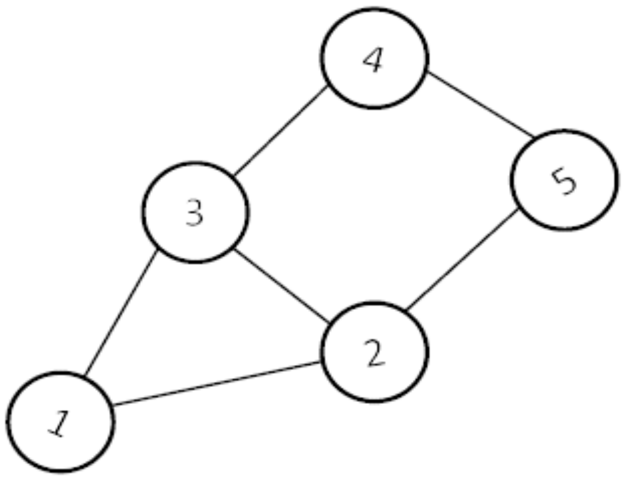
Model Abstraction – Air Loop

- **Network of Bilinear Systems**

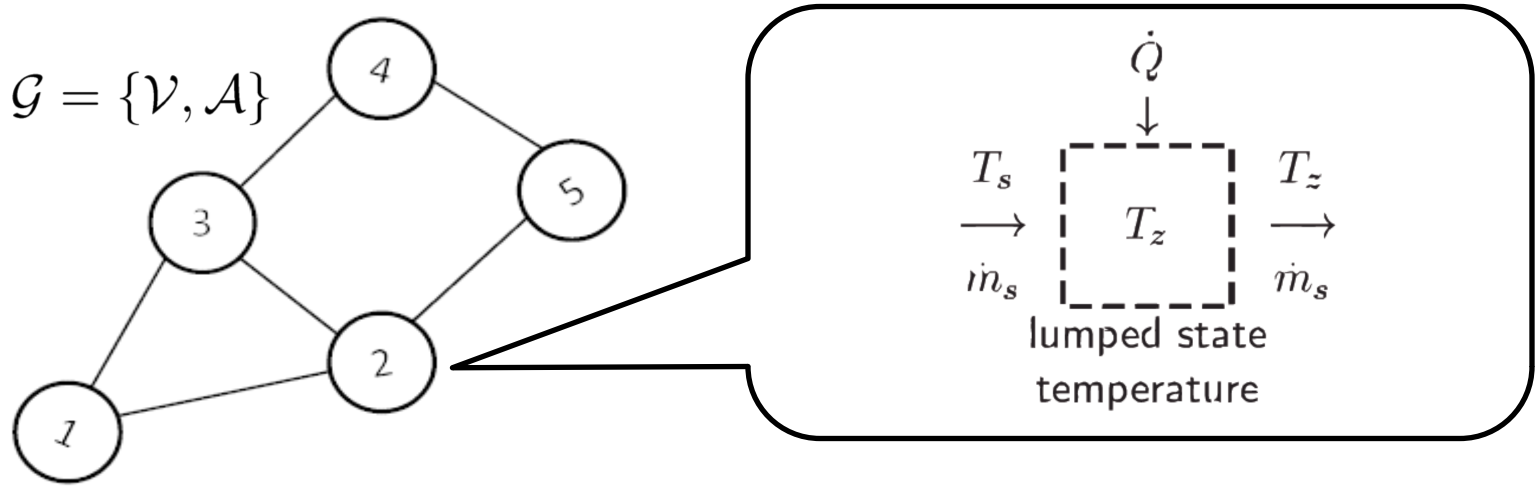


Thermal zone model



- **Load Predictions**
  - Load (Occupancy/Thermal Comfort, Sun radiation, weather)
- **Static Nonlinearities**
  - Equipment Performance Maps (Fan, Coils)
- **Equality and inequality Constraints**
  - Comfort range
  - Operation constraints
  - Dynamic coupling: thermal, supply air & return air

Zone Network Dynamics



- System states

$$x_t^i = [T_z^i(t), T_z^i(t-1), T_z^i(t-2)]$$

- Disturbance

$$w_t^i = [T_{oa}(t), T_{oa}(t-1), T_{oa}(t-2), \dot{Q}^i(t)] \in \mathcal{W}_t$$

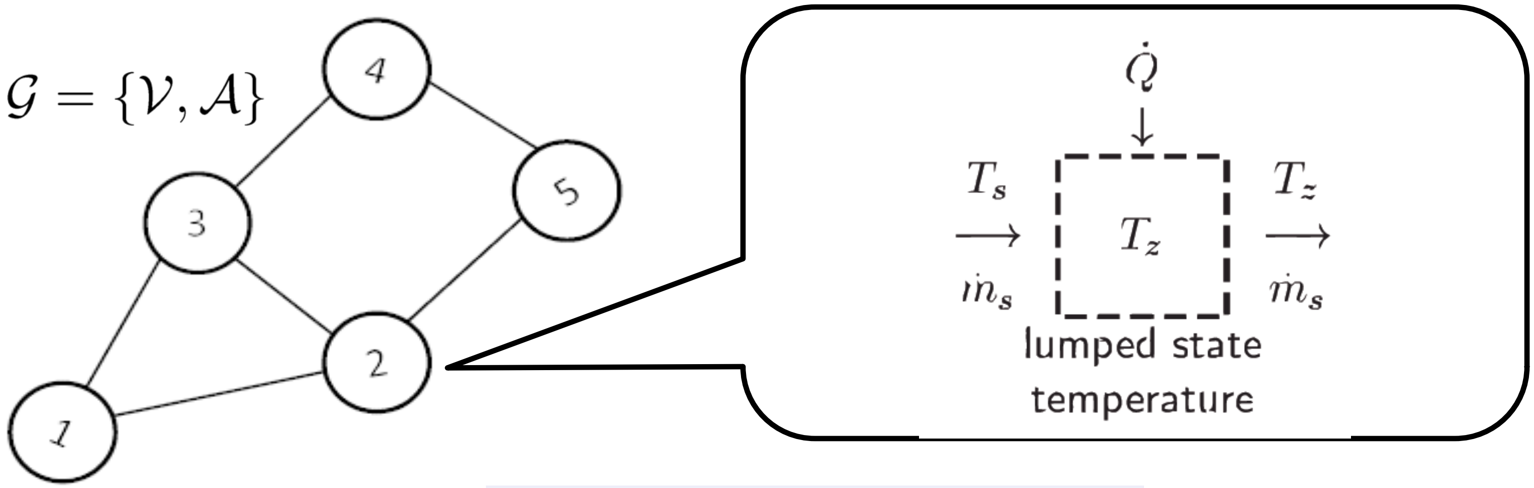
- Control inputs

$$u_t^i = [\dot{m}_s^i(t), T_s^i(t)]$$

$$x_{t+1}^i = A_i x_t^i + B_i \dot{m}_{s\,t}^i (T_{s\,t}^i - C x_t^i) + D_i w_t^i + \sum_{i \in \mathcal{A}(i)} A_{ij} x_t^j$$

Zone dynamics are modeled using ARMAX model of order 2

Zone Network Dynamics



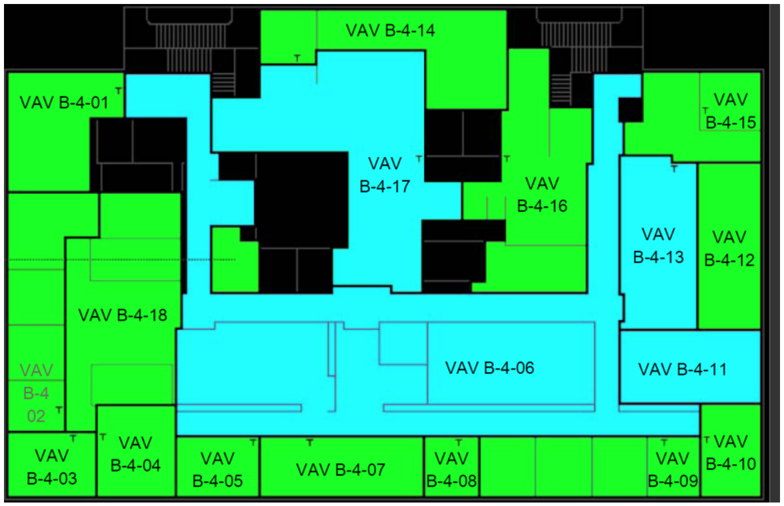
- System states $x_t = [x_t^1, x_t^2, \ldots, x_t^{\mathcal{V}}]$
- Disturbance $w_t = [w_t^1, w_t^2, \ldots, w_t^{\mathcal{V}}]$
- Control inputs $\dot{m}_{st} = [\dot{m}_s^1{}_t, \dot{m}_s^2{}_t, \ldots, \dot{m}_s^{\mathcal{V}}{}_t]$

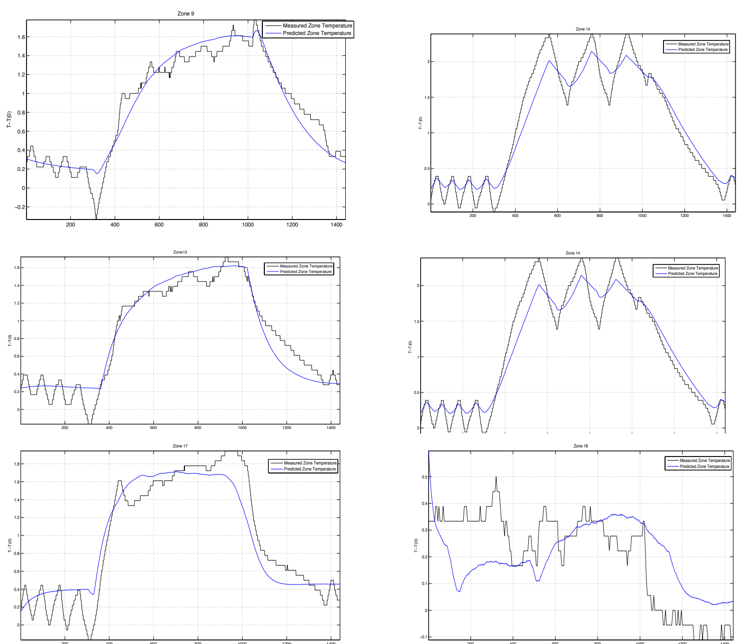
  $T_{st} = [T_s^1{}_t, T_s^2{}_t, \ldots, T_s^{\mathcal{V}}{}_t]$

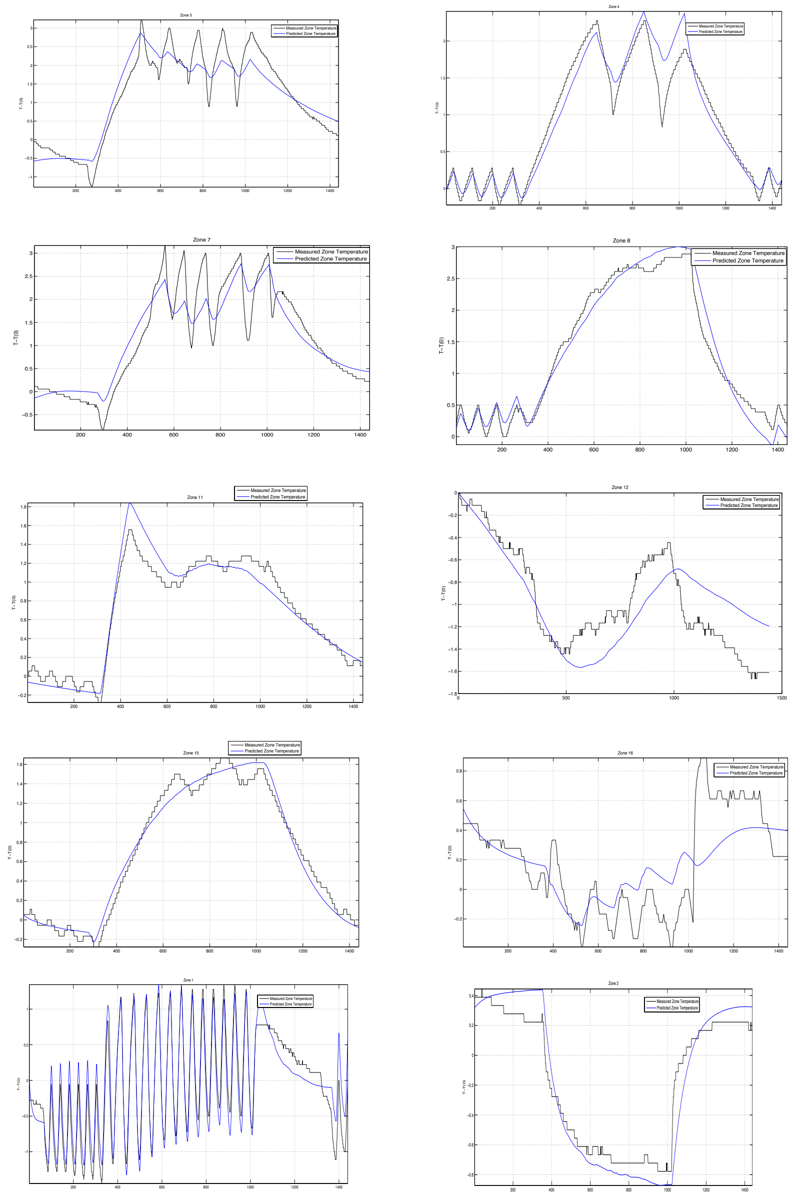
$$x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t$$

Zone dynamics are modeled using ARMAX model of order 2

UC Berkeley Bancroft Library Predictions





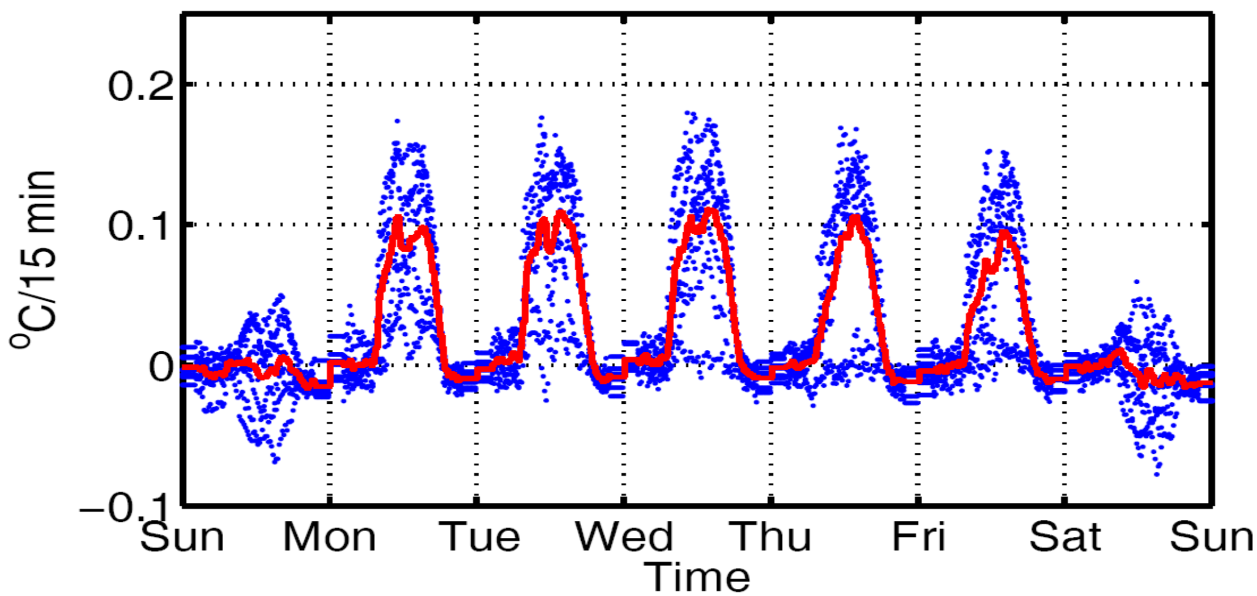


Load Prediction

$$\dot{Q}^i_{t-1} = (T^i_{z\,t} - T^i_{z\,t|t-1})/\Delta t$$

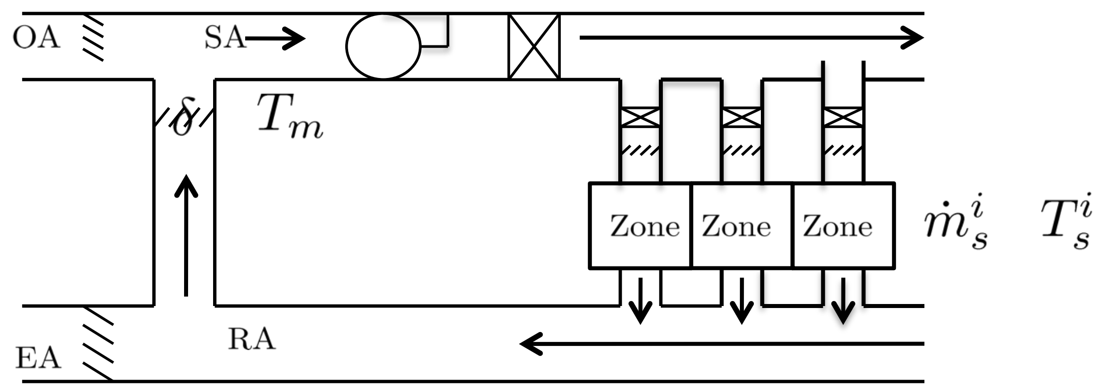
$T^i_{z\,k}$ Measured zone temperature at time t.

$T^i_{z\,t|t-1}$ Predicted zone temperature at time t based on the states and inputs measurement at time t-1.



Load model is extracted from historical profile.

Performance Maps



$$\text{Energy}(x_t, u_t, w_t) = \text{Energy}_{\text{fan}} + \text{Energy}_{\text{coil}}$$

- Fan energy

$$\text{Energy}_{\text{fan}} = f_0 + f_1\Big(\sum_{i\in\mathcal{V}} \dot{m}_s^i\Big) + f_2\Big(\sum_{i\in\mathcal{V}} \dot{m}_s^i\Big)^2$$

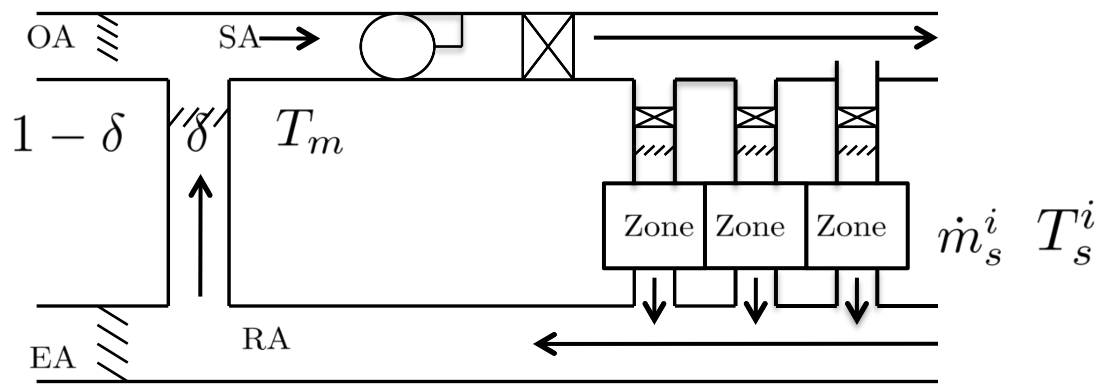
- Coil energy

$$\text{Energy}_{\text{coil}} = c_p \sum_{i\in\mathcal{V}} c^i \dot{m}_s^i |T_s^i - T_m|$$

$$T_m = \delta T_{oa} + (1-\delta)\frac{\sum_{i\in\mathcal{V}} \dot{m}_s^i T_z^i}{\sum_{i\in\mathcal{V}} \dot{m}_s^i}$$

$$\delta_{min} \le \delta \le 1$$

Control Variables and Constraints



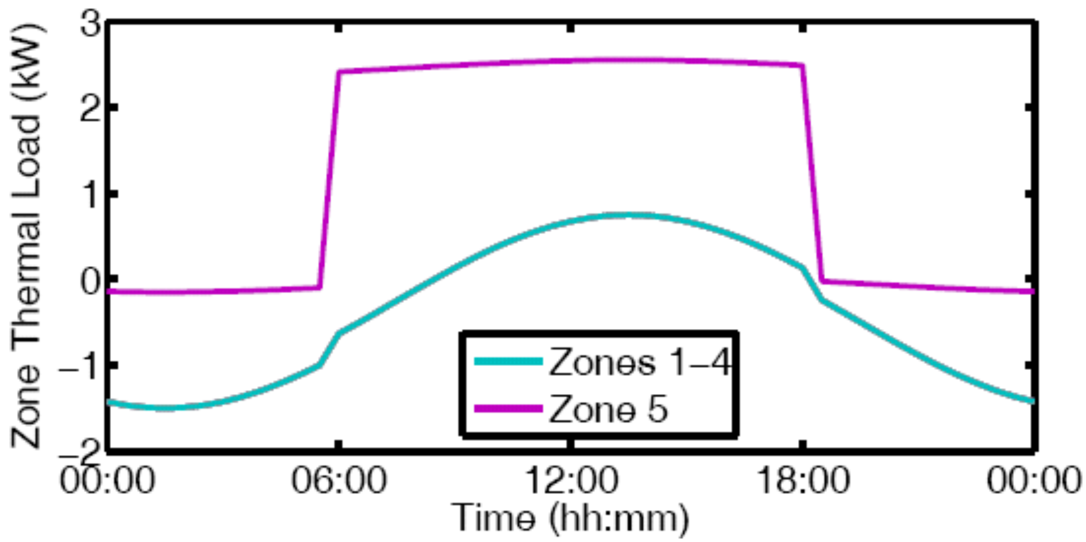
$u =$	T_c	Supply air temperature after the cooling coil in AHU
	T_s^i	Supply air temperature after the VAV box for zone i
	$\dot{m}_s^i$	Air mass flow rate to zone i
	δ	Return air damper position

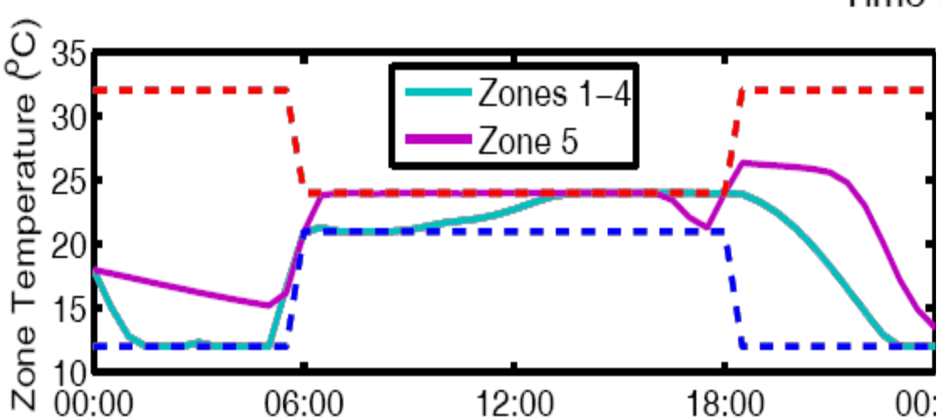
- Operational constraints $u \in \mathcal{U}$
- Thermal comfort $\underline{T}_1^i \leq T_1^i \leq \bar{T}_1^i$

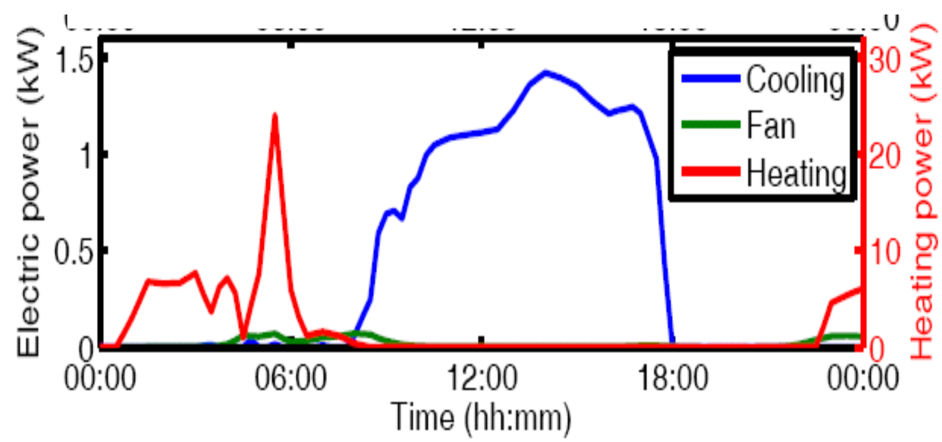
Minimize energy consumption, maintain thermal comfort, and satisfy operational constraints.

Results

A. Kelman, Y. Ma, A. Daly, F. Borrelli, Predictive Control for Energy Efficient Buildings with Thermal Storage: Modeling, Stimulation, and Experiments, *IEEE Control System Magazine*, 32(1), page 44-64, February 2012.







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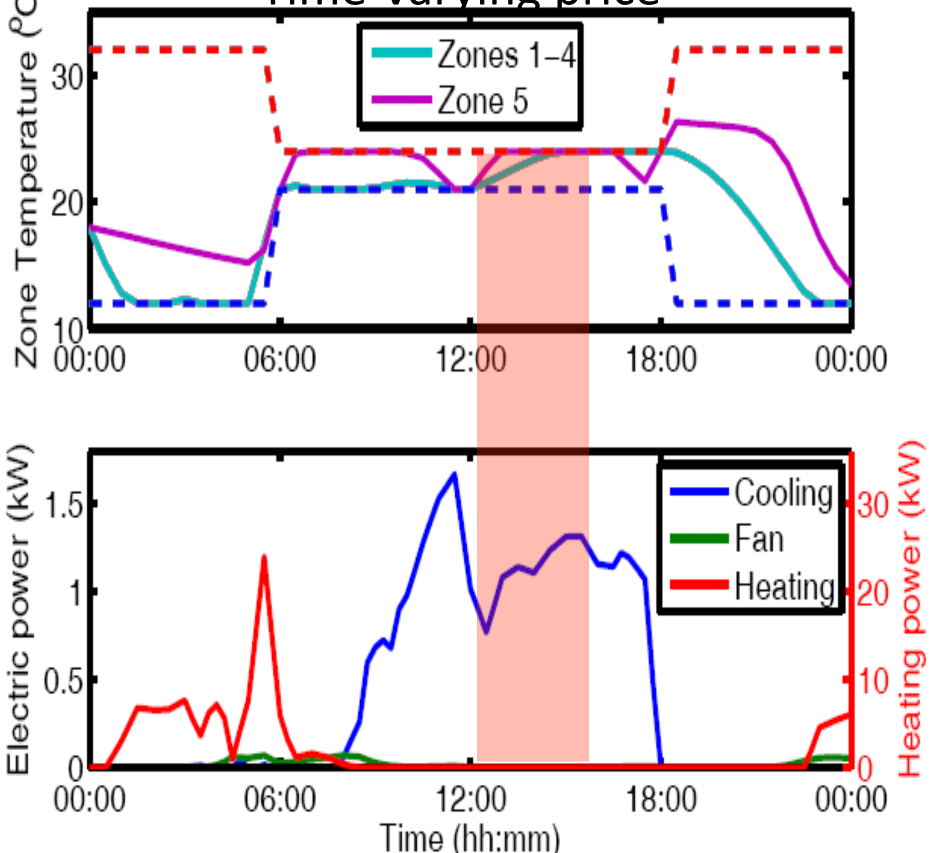
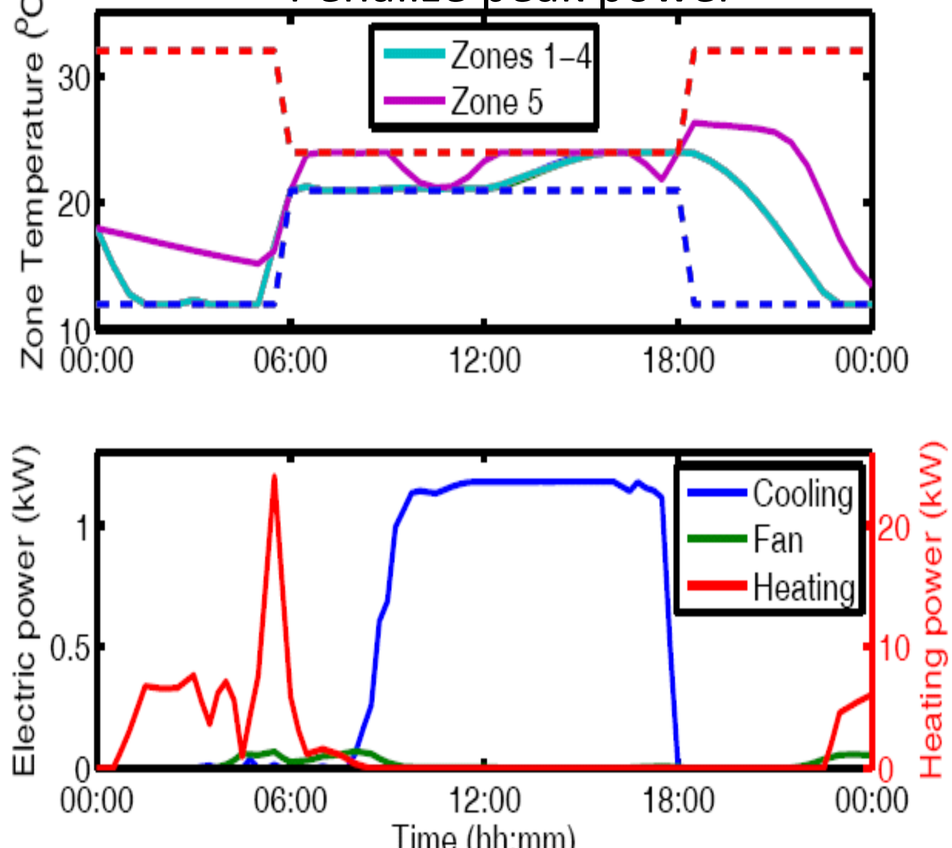
Y. Ma, A. Kelman, A. Daly, F. Borrelli, Predictive Control for Energy Efficient Buildings with Thermal Storage: Modeling, Simulation, and Experiments, *IEEE Control System Magazine*, 32(1), page 44-64, February 2012.

Time-varying price



Penalize peak power



MPC is able to incorporate time-varying energy price and reduce peak power consumption

Nominal MPC issues

- Computational complexity
  - Tailored MPC Solvers
  - Distributed MPC
- Role of Prediction Errors
  - Stochastic MPC
  - Robust MPC
- Stability and persistent feasibility
- Global vs Local Optima

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Centralized MPC

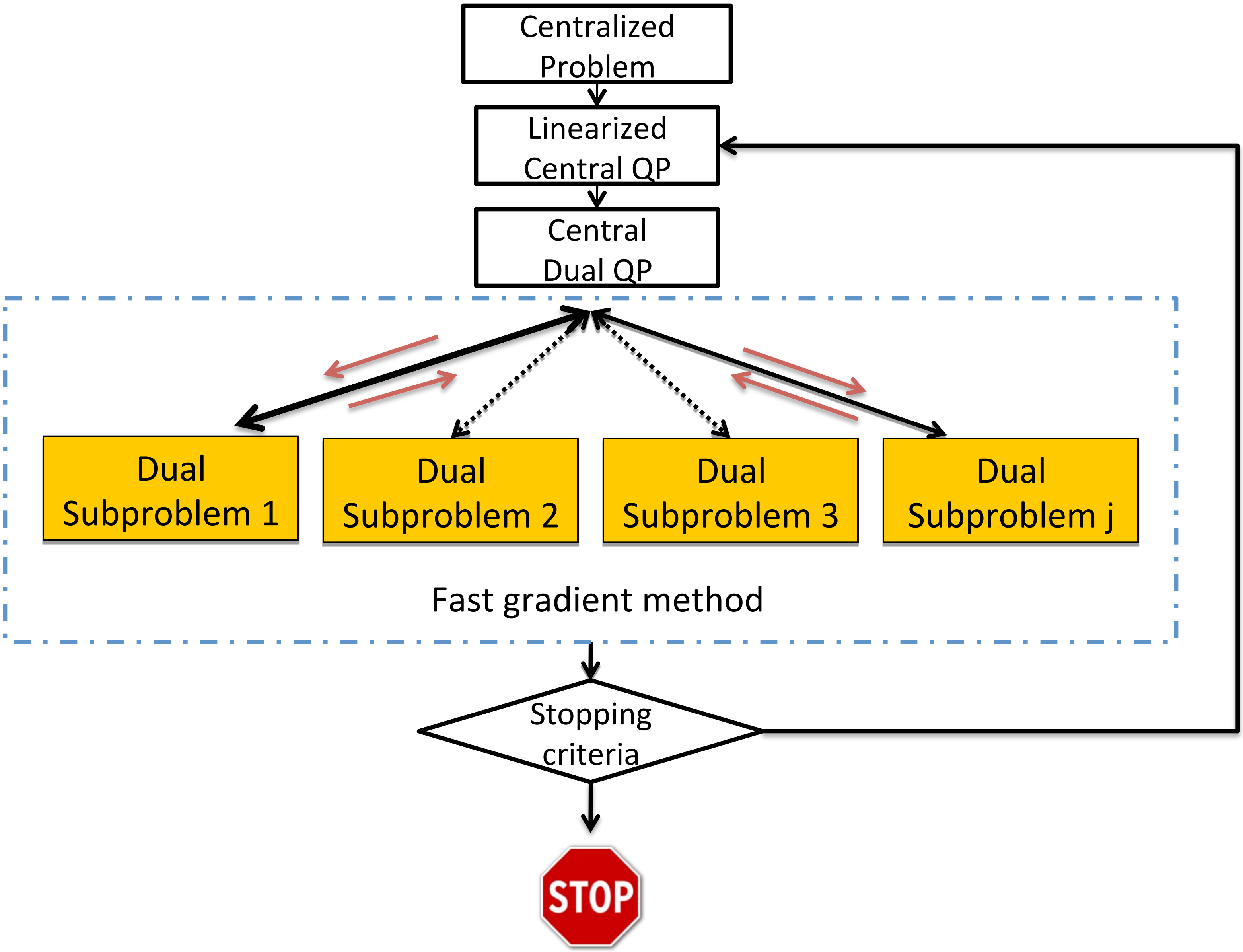
$$\min_{U,\ X} \sum_{k=t}^{t+N-1} \text{Energy}(x_k, u_k)$$

$$\text{subj. to} \begin{cases} x_{k+1} = f(x_k, u_k),\ k = t, \ldots, t+N-1 \\ u_k \in \mathcal{U},\ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X},\ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

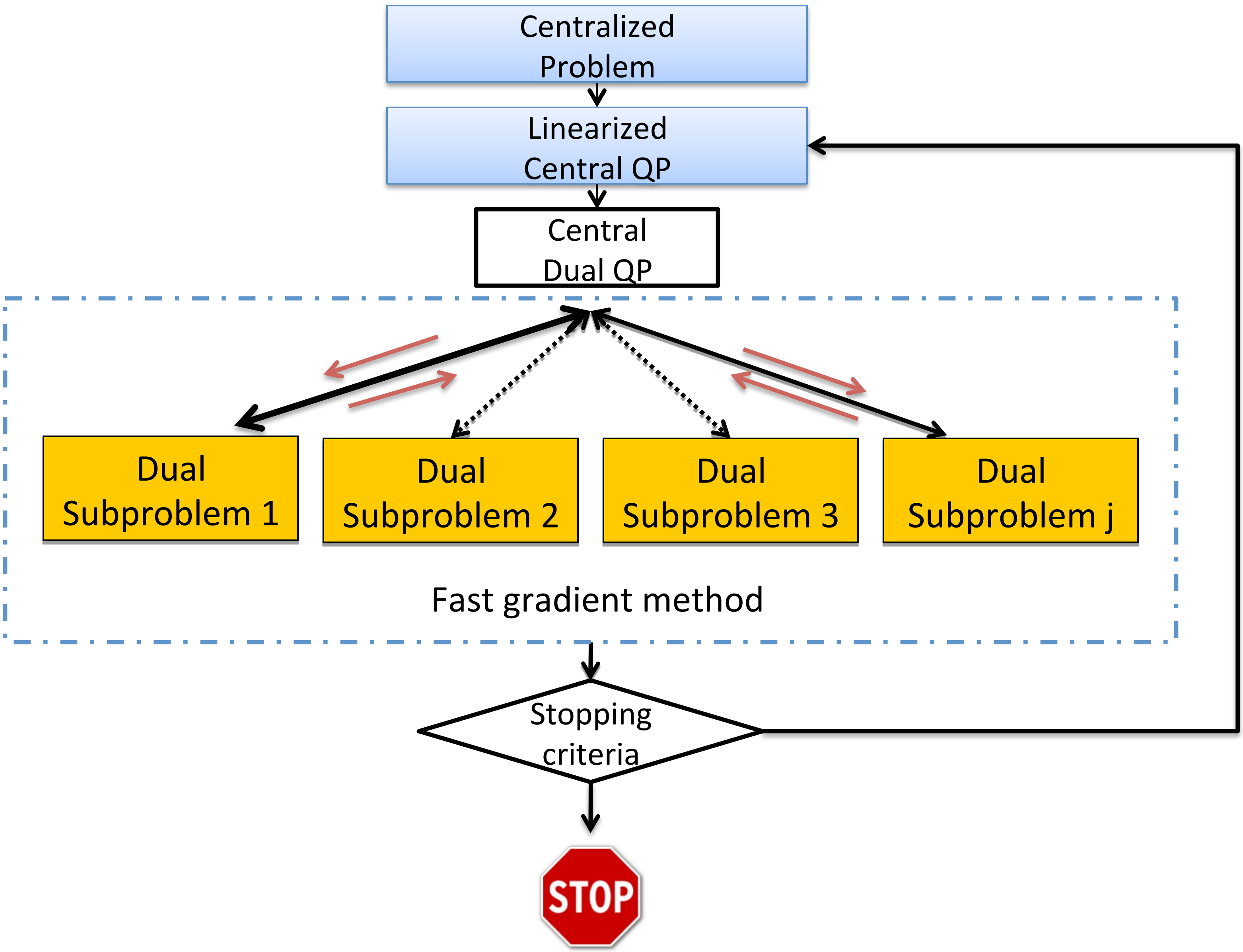
Centralized MPC:

1) computationally prohibitive with increasing complexity of building HVAC systems
2) not readily implementable on current distributed low-cost control hardware for buildings control.

Distributed MPC



Distributed MPC



Distributed MPC

$$\min_{U,\ X} \sum_{k=t}^{t+N-1} \text{Energy}(x_k, u_k)$$

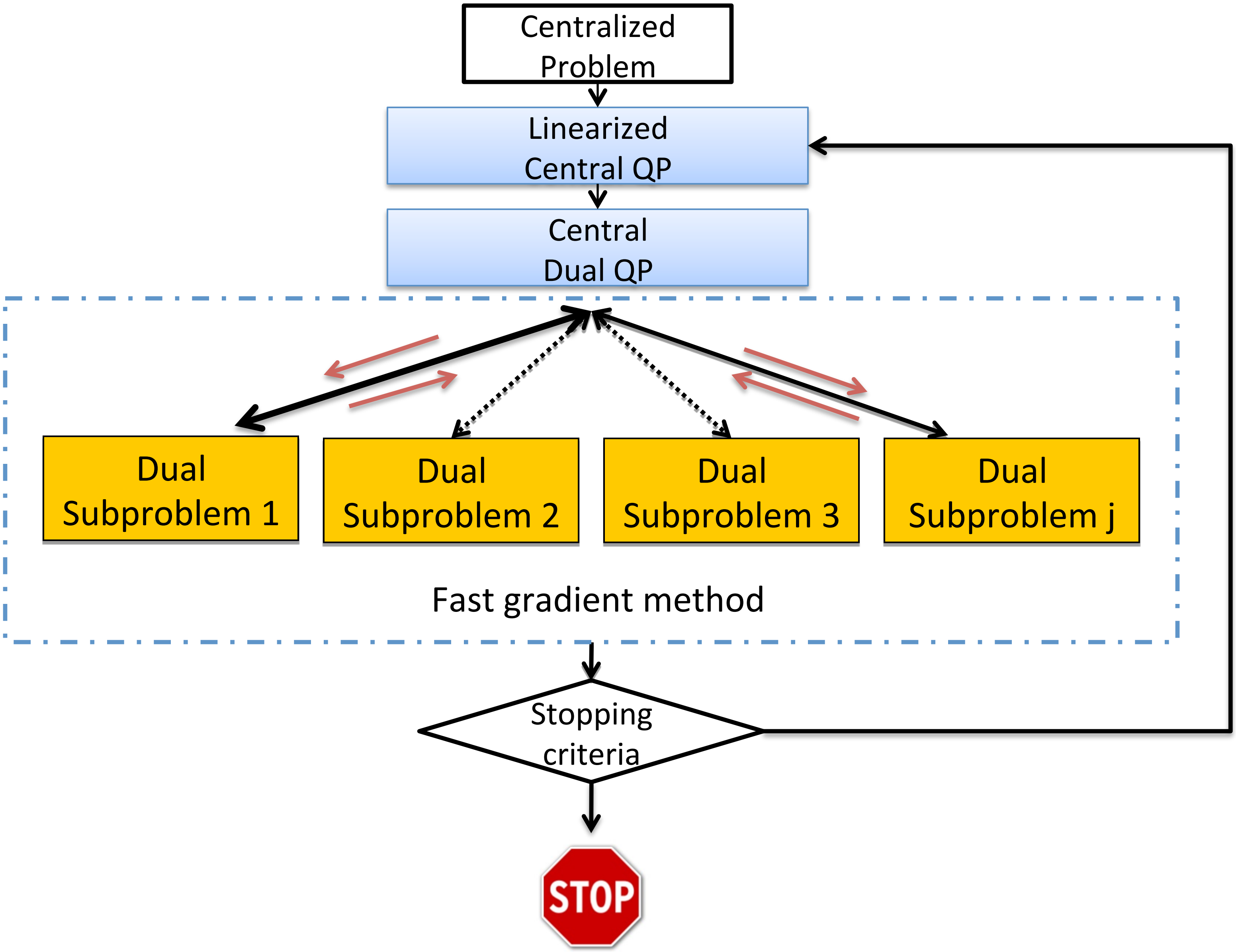
$$\text{subj. to} \begin{cases} x_{k+1} = f(x_k, u_k),\ k = t, \ldots, t+N-1 \\ u_k \in \mathcal{U},\ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X},\ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

$$\min_{U,\ X} \sum_{k=t}^{t+N-1} x_k^T Q_k^x x_k + c_k^{x\,T} x_k + u_k^T Q_k^u u_k + c_k^{u\,T} u_k$$

$$\text{subj. to} \begin{cases} x_{k+1} = A_k x_k + B_k u_k + d_k,\ k = t, \ldots, t+N-1 \\ u_k \in \mathcal{U},\ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X},\ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

$Q_k^u,\ Q_k^x$ are diagonal and semi positive definite

Distributed MPC



Distributed MPC

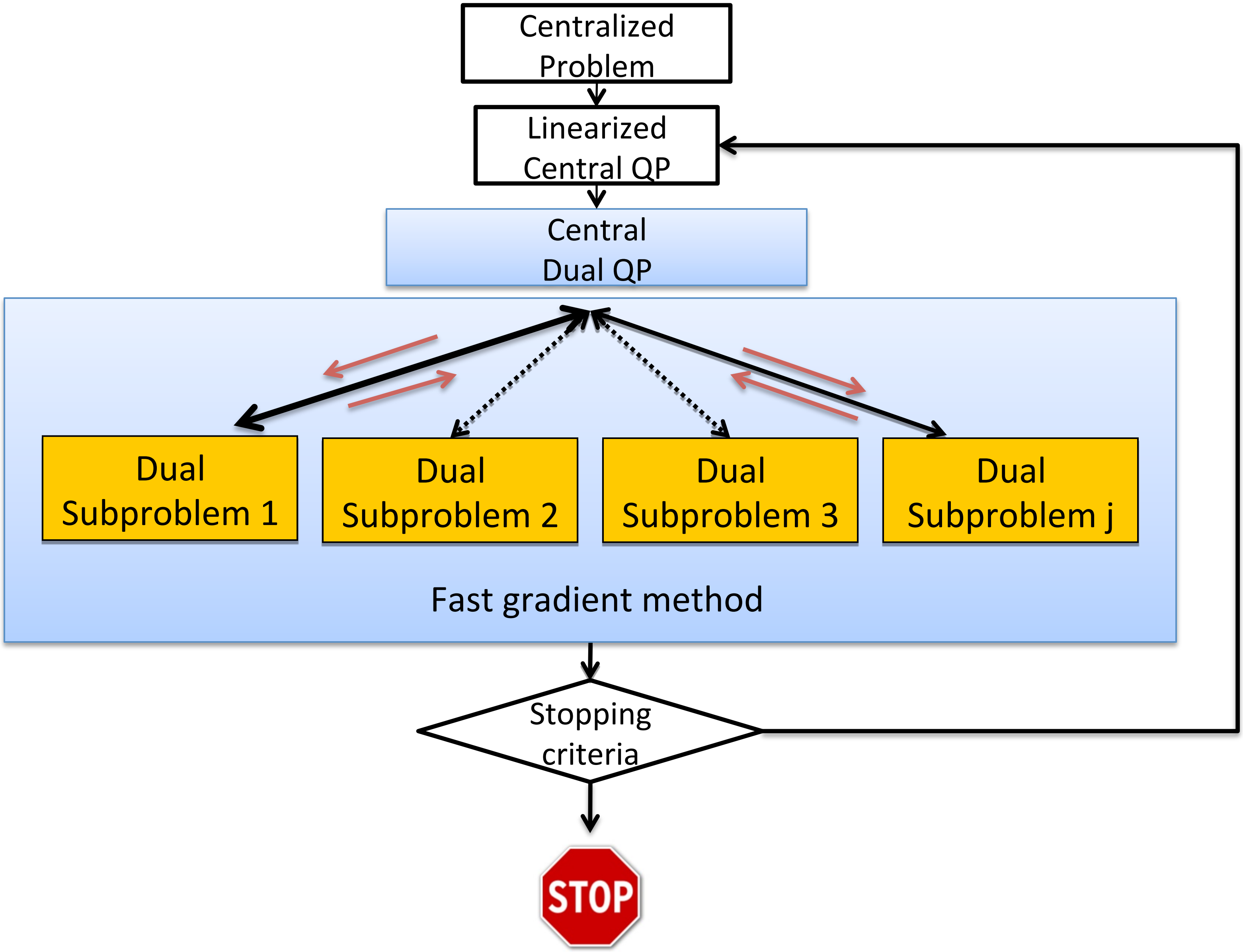
$$\min_{U,\ X} \sum_{k=t}^{t+N-1} x_k^T Q_k^x x_k + {c_k^x}^T x_k + u_k^T Q_k^u u_k + {c_k^u}^T u_k$$

$$\text{subj. to} \begin{cases} x_{k+1} = A_k x_k + B_k u_k + d_k, \ \ k = t, \ldots, t+N-1 \\ u_k \in \mathcal{U}, \ \ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X}, \ \ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

$$\max_{\lambda} \min_{U,\ X} \sum_{k=t}^{t+N-1} x_k^T Q_k^x x_k + {c_k^x}^T x_k + u_k^T Q_k^u u_k + {c_k^u}^T u_k + \lambda_k^T (A_k x_k + B_k u_k + d_k - x_{k+1})$$

$$\text{subj. to} \begin{cases} u_k \in \mathcal{U}, \ \ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X}, \ \ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

Distributed MPC



Distributed MPC

$$\max_{\lambda} \min_{U,\ X} \sum_{k=t}^{t+N-1} x_k^T Q_k^x x_k + c_k^{x\,T} x_k + u_k^T Q_k^u u_k + c_k^{u\,T} u_k$$
$$+\lambda_k^T (A_k x_k + B_k u_k + d_k - x_{k+1})$$

$$\text{subj. to} \quad \begin{cases} u_k \in \mathcal{U},\ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X},\ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

Coordinator – Fast Gradient Methods

$$\lambda_k^i := \text{FGM}\left(\lambda_k^i, h_{\lambda_k^i} = (A_k x_k + B_k u_k + d_k - x_{k+1})_i\right)$$

$\downarrow \lambda$ $\qquad\qquad$ $x \uparrow u$

Subproblem

$$(x_k)_i = -\frac{1}{2(Q_k^x)_{ii}} \left[\lambda_{k+1}^i - (c_k^x)_i - (A_k^T \lambda_k)_i\right] \qquad (x_k)_i \in \mathcal{X}_i$$

$$(u_k)_i = -\frac{1}{2(Q_k^u)_{ii}} \left[-(c_k^u)_i - (B_k^T \lambda_k)_i\right] \qquad (u_k)_i \in \mathcal{U}_i$$

L. S. Lasdon. *Duality and decomposition in mathematical programming*. Systems Science and Cybernetics, IEEE Transactions on, 4(2), July 1968.

Distributed MPC

$$\max_{\lambda} \min_{U,\ X} \sum_{k=t}^{t+N-1} x_k^T Q_k^x x_k + c_k^{x\,T} x_k + u_k^T Q_k^u u_k + c_k^{u\,T} u_k$$
$$+\lambda_k^T (A_k x_k + B_k u_k + d_k - x_{k+1})$$

$$\text{subj. to} \begin{cases} u_k \in \mathcal{U},\ \ k = t, \ldots, t+N-1 \\ x_k \in \mathcal{X},\ \ k = t, \ldots, t+N-1 \\ x_t = x(t) \end{cases}$$

Coordinator – Fast Gradient Methods

$$\lambda_k^i := \text{FGM}\left(\lambda_k^i, h_{\lambda_k^i} = (A_k x_k + B_k u_k + d_k - x_{k+1})_i\right)$$

$$\lambda_k^{i,n} = \bar{\lambda}_k^{i,n-1} + \frac{1}{L} h_{\lambda_k^i}(\bar{\lambda}_k^{n-1})$$

$$\gamma^n = \frac{\gamma^{n-1}}{2}\left(\sqrt{(\gamma^{n-1})^2 + 4} - \gamma^{n-1}\right)$$

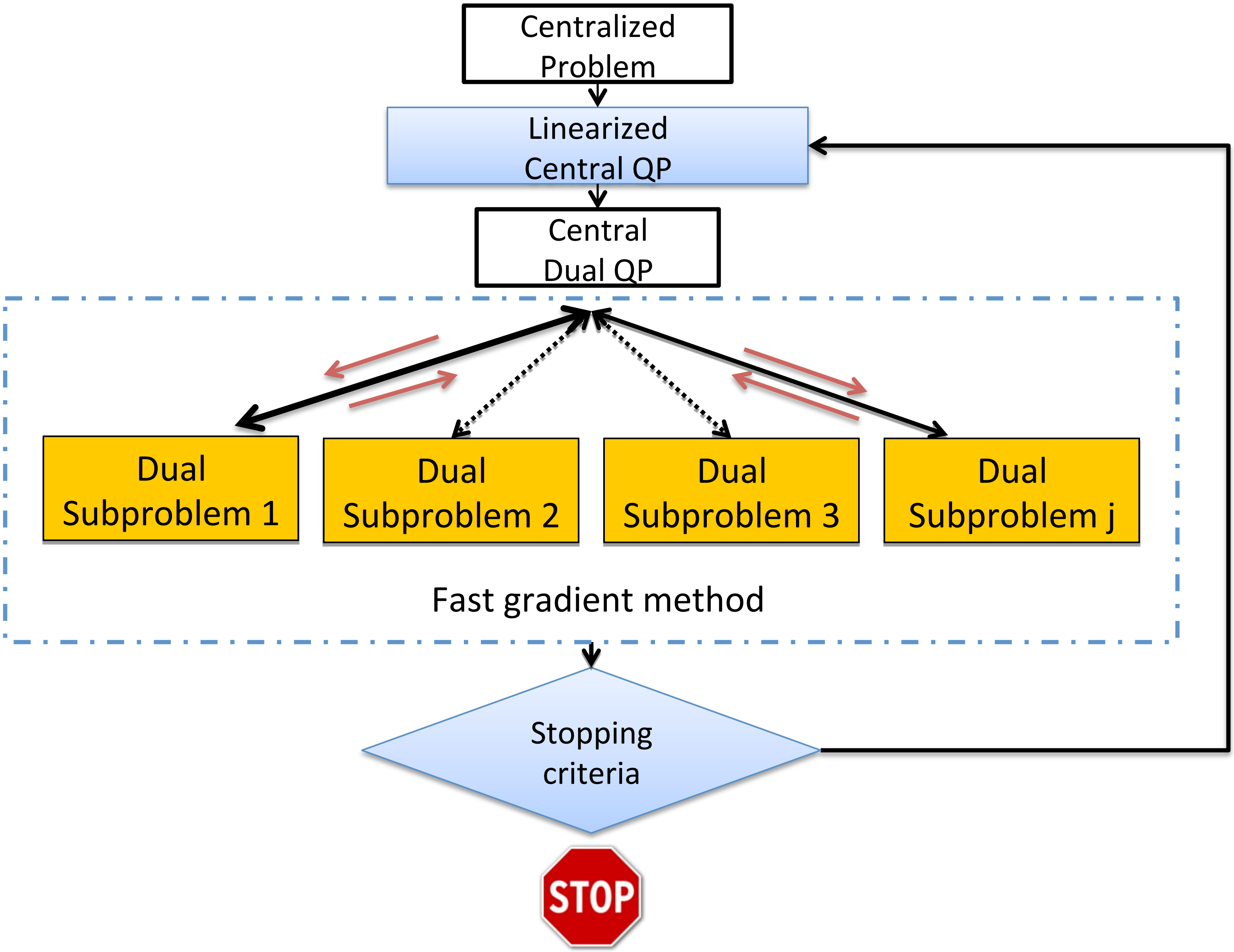
$$\beta = \frac{\gamma^{n-1}(1-\gamma^{n-1})}{(\gamma^{n-1})^2 + \gamma^n}$$

$$\bar{\lambda}_k^{i,n} = \lambda_k^{i,n} + \beta(\lambda_k^{i,n} - \lambda_k^{i,n-1})$$

Yurii Nesterov

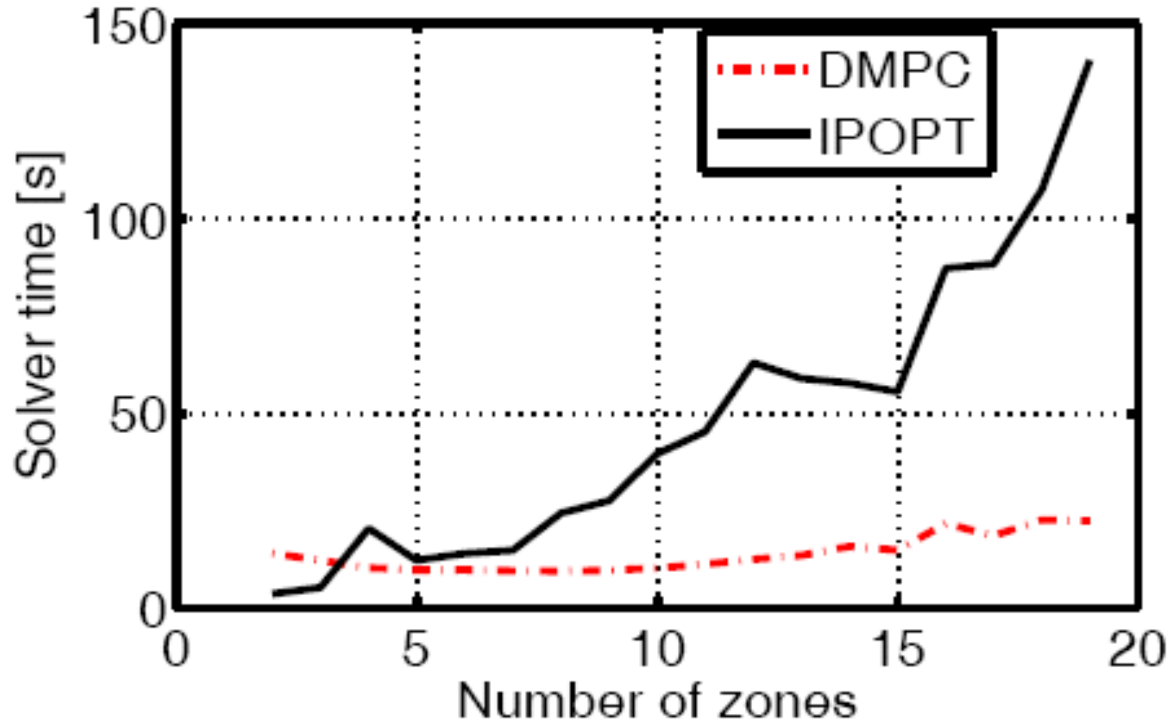
Stefan Richt

Distributed MPC



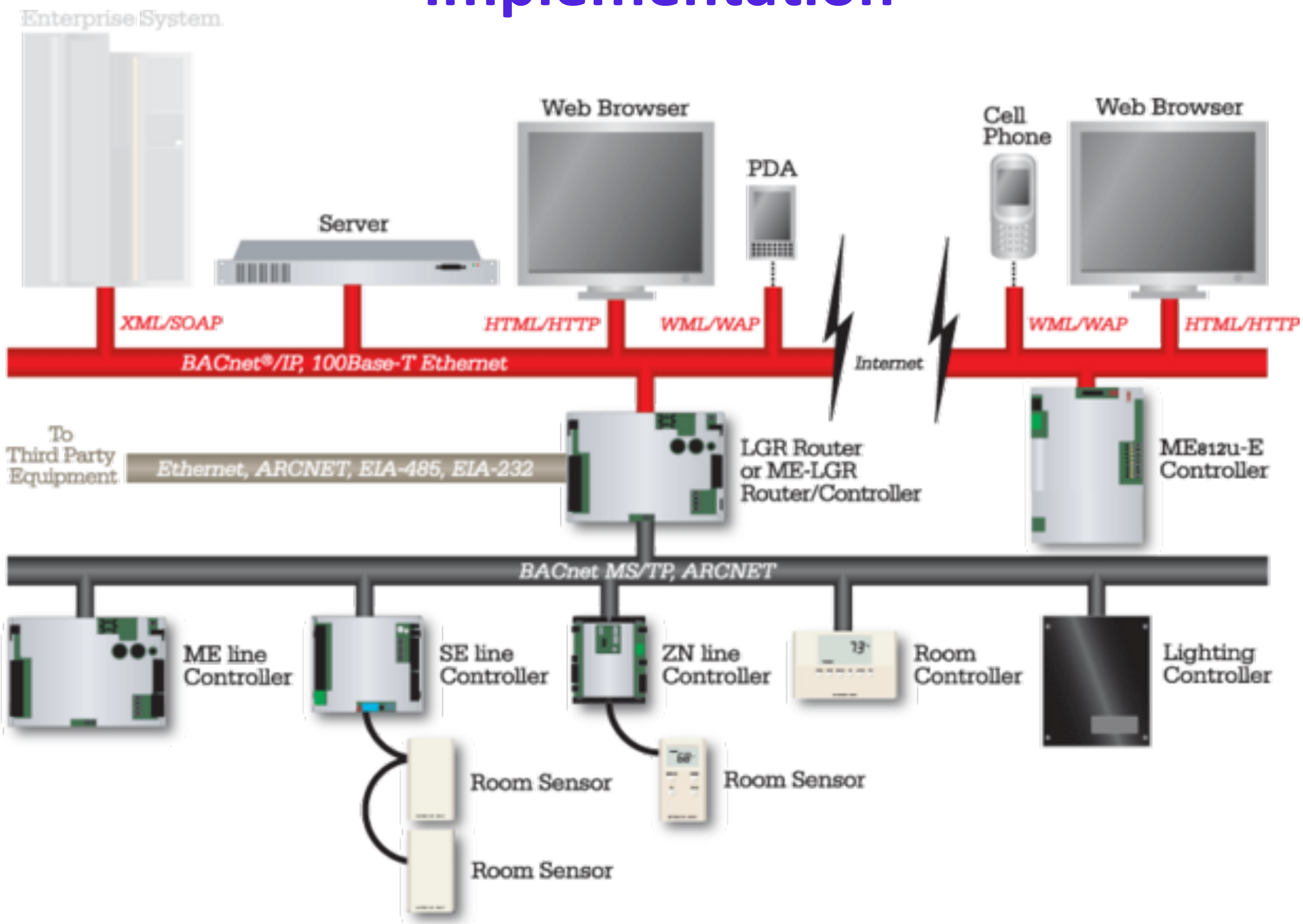
Results

IPOPT interfaced via AMPL

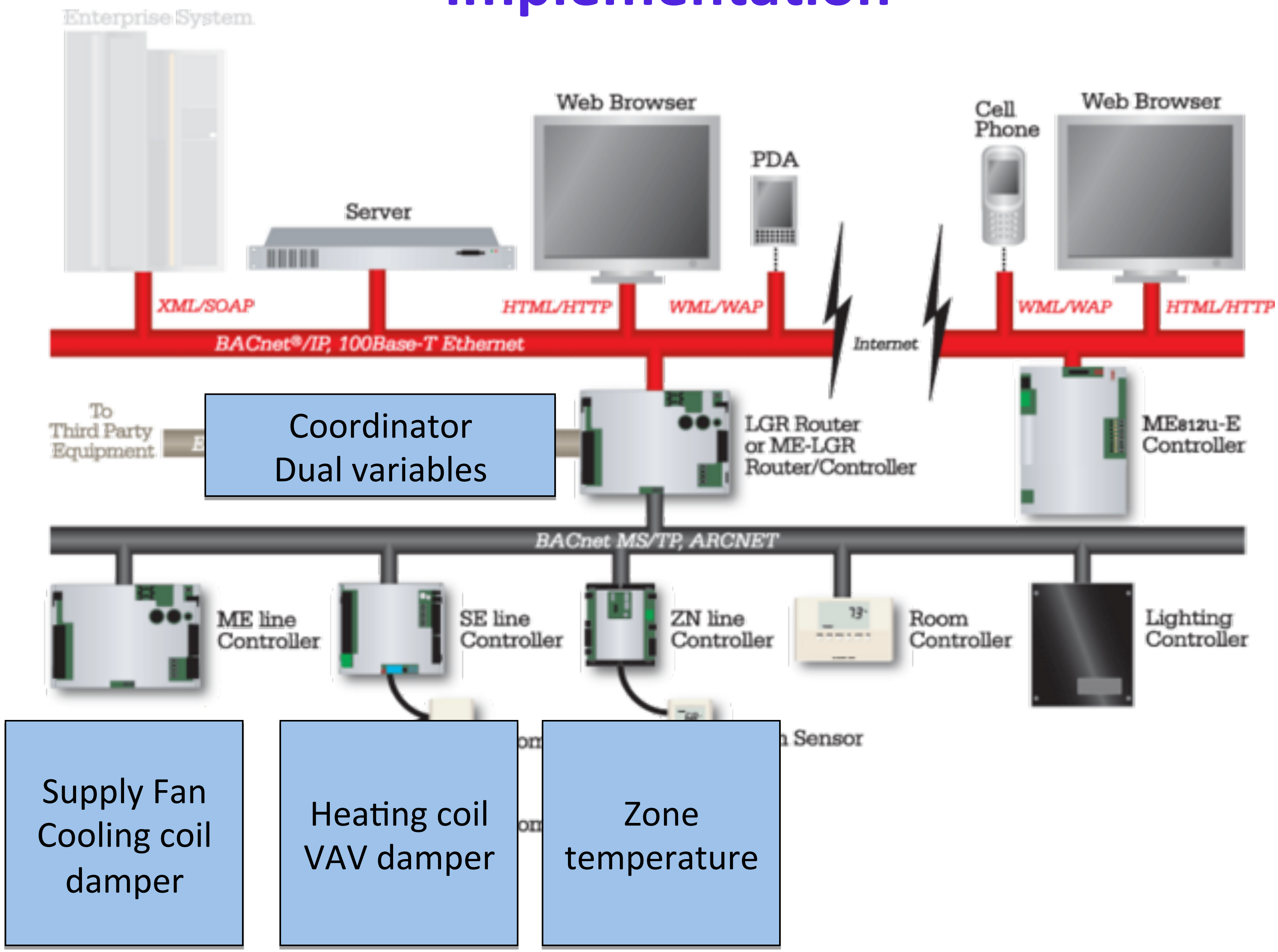


Distributed model predictive control enables real-time implementation on low cost distributed computational platform

Implementation



Implementation



Outline

- Background
- Nominal MPC Design
- Distributed MPC Design
- Stochastic MPC design
- Conclusions

Predictive Control Design for Large Scale Systems

$$\min_{\pi_0,\pi_1,\ldots,\pi_N} \mathbb{E}\{\sum_{t=0}^{t=N} \text{Energy}(x_t, u_t, w_t)\}$$

$$\text{s.t.} \quad x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t$$

$$u_t = \pi_t(x_t)$$

$$\mathbb{P}\left\{x_k^{i\in\mathcal{V}} \in \mathcal{X}\right\} > 1 - \epsilon$$

$$u_t^{i\in\mathcal{V}} \in \mathcal{U} \quad \forall w_t \in \mathcal{W}_t$$

Predictive Control Design for Large Scale Systems

$$\min_{\pi_0,\pi_1,\ldots,\pi_N} \mathbb{E}\{\sum_{t=0}^{t=N} \text{Energy}(x_t,u_t,w_t)\}$$

$$\begin{aligned} \text{s.t.} \quad & x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t \\ & u_t = \pi_t(x_t) \\ & \mathbb{P}\left\{x_k^{i\in\mathcal{V}} \in \mathcal{X}\right\} > 1-\epsilon \\ & u_t^{i\in\mathcal{V}} \in \mathcal{U} \quad \forall w_t \in \mathcal{W}_t \end{aligned}$$

- **Computational tractability**
  - **Non convex**
  - **Large-scale**
- **Value of uncertain forecast**
  - **Building load non-Gaussian in practice**
  - **Nominal predictions**
  - **Gaussian approximation**

Predictive Control Design for Large Scale Systems

$$\min_{\pi_0,\pi_1,\ldots,\pi_N} \mathbb{E}\{\sum_{t=0}^{t=N} \text{Energy}(x_t, u_t, w_t)\}$$

$$\begin{aligned} \text{s.t.} \quad & x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t \\ & u_t = \pi_t(x_t) \\ & \mathbb{P}\left\{x_k^{i\in\mathcal{V}} \in \mathcal{X}\right\} > 1 - \epsilon \\ & u_t^{i\in\mathcal{V}} \in \mathcal{U} \quad \forall w_t \in \mathcal{W}_t \end{aligned}$$

- **Linearize system**

$$x_{t+1} = A_t x_t + B_t u_t + D_t w_t$$

Feedback Linearization

$$x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t$$

$$w_t \sim \text{finitely supported PDF}$$

- **Feedback linearization**

$$\text{Let } \dot{m}^i_{st} = \frac{u^i_{2t}}{T^i_{zt}},\ T^i_{st} = \frac{u^i_{1t}}{u^i_{2t}} T^i_{zt},\ u^i_{1t} > 0,\ u^i_{2t} > 0$$

$$u_t = [u^{i\in\mathcal{V}}_{1t}, u^{i\in\mathcal{V}}_{2t}]$$

$$x_{t+1} = Ax_t + B_f u_t + Dw_t$$

- **Mean and error dynamics**

$$\hat{x} = \mathbf{E}\{x\} \qquad \tilde{x} = x - \mathbf{E}\{x\}$$

$$\hat{x}_{t+1} = A\hat{x}_t + B_f \hat{u}_t + D\hat{w}_t$$

$$\tilde{x}_{t+1} = A\tilde{x}_t + D\tilde{w}_t$$

Predictive Control Design for Large Scale Systems

$$\min_{\pi_0,\pi_1,\ldots,\pi_N} \mathbb{E}\{\sum_{t=0}^{t=N} \text{Energy}(x_t, u_t, w_t)\}$$

$$\text{s.t.} \quad x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t$$

$$u_t = \pi_t(x_t)$$

$$\mathbb{P}\left\{x_k^{i\in\mathcal{V}} \in \mathcal{X}\right\} > 1 - \epsilon$$

$$u_t^{i\in\mathcal{V}} \in \mathcal{U} \quad \forall w_t \in \mathcal{W}_t$$

- **Linearize system**

$$x_{t+1} = Ax_t + Bu_t + Dw_t$$

- **Robustify input constraints**

Robustify Input Constraints

- Constraints on mass flow rate $\dot{m}_s$

$$u_2 = \dot{m}_s T_z = \dot{m}_s C x$$

$$\dot{m}_s^{\min} \leq \dot{m}_s = \frac{u_2}{C(\hat{x}+\tilde{x})} \leq \dot{m}_s^{\max}, \ \forall \tilde{w}$$

$$u_2 - \dot{m}_s^{\max} C\hat{x} \leq \dot{m}_s^{\max} \min_{\tilde{w}}(C\tilde{x})$$

$$u_2 - \dot{m}_s^{\min} C\hat{x} \geq \dot{m}_s^{\min} \max_{\tilde{w}}(C\tilde{x})$$

Robustify Input Constraints

- **Constraints on mass flow rate** $\dot{m}_s$

$$u_2 = \dot{m}_s T_z = \dot{m}_s C x$$

$$\dot{m}_s^{\min} \leq \dot{m}_s = \frac{u_2}{C(\hat{x}+\tilde{x})} \leq \dot{m}_s^{\max}, \ \forall \tilde{w}$$

$$u_2 - \dot{m}_s^{\max} C\hat{x} \leq \dot{m}_s^{\max} \min_{\tilde{w}}(C\tilde{x}) = \epsilon_x^{upper}$$

$$u_2 - \dot{m}_s^{\min} C\hat{x} \geq \dot{m}_s^{\min} \max_{\tilde{w}}(C\tilde{x}) = \epsilon_x^{lower}$$

Right hand side can be computed offline as the bounds of $\tilde{x}$ is know.
The resulting constraints are LINEAR

Robustify Input Constraints

- Constraints on mass flow rate $\dot{m}_s$

$$u_2 = \dot{m}_s T_z = \dot{m}_s C x$$

$$\dot{m}_s^{\min} \leq \dot{m}_s = \frac{u_2}{C(\hat{x}+\tilde{x})} \leq \dot{m}_s^{\max}, \ \forall \tilde{w}$$

$$u_2 - \dot{m}_s^{\max} C\hat{x} \leq \dot{m}_s^{\max} \min_{\tilde{w}}(C\tilde{x}) = \epsilon_x^{upper}$$

$$u_2 - \dot{m}_s^{\min} C\hat{x} \geq \dot{m}_s^{\min} \max_{\tilde{w}}(C\tilde{x}) = \epsilon_x^{lower}$$

- Constraints on supply air temperature T_s

$$u_1 = \dot{m}_s T_s$$

$$T_s^{\min} \leq T_s = \frac{u_1}{u_2} C(\hat{x}+\tilde{x}) \leq T_s^{\max}, \ \forall \tilde{w}$$

$$u_1 \max_w(C\tilde{x}) + u_1 C\hat{x} \leq T_s^{\max} u_2$$

$$u_1 \min_w(C\tilde{x}) + u_1 C\hat{x} \geq T_s^{\min} u_2$$

Robustify Input Constraints

- Constraints on mass flow rate $\dot{m}_s$

$$u_2 = \dot{m}_s T_z = \dot{m}_s C x$$

$$\dot{m}_s^{\min} \leq \dot{m}_s = \frac{u_2}{C(\hat{x}+\tilde{x})} \leq \dot{m}_s^{\max}, \ \forall \tilde{w}$$

$$u_2 - \dot{m}_s^{\max} C\hat{x} \leq \dot{m}_s^{\max} \min_{\tilde{w}}(C\tilde{x}) = \epsilon_x^{upper}$$

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- Constraints on supply air temperature T_s

$$u_1 = \dot{m}_s T_s$$

$$T_s^{\min} \leq T_s = \frac{u_1}{u_2} C(\hat{x}+\tilde{x}) \leq T_s^{\max}, \ \forall \tilde{w}$$

$$u_1 \epsilon_x^{upper} = u_1 \max_w (C\tilde{x}) + u_1 C\hat{x} \leq T_s^{\max} u_2$$

$$u_1 \epsilon_x^{lower} = u_1 \min_w (C\tilde{x}) + u_1 C\hat{x} \geq T_s^{\min} u_2$$

Predictive Control Design for Large Scale Systems

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- **Linearize system**

$$x_{t+1} = A_t x_t + B_t u_t + D_t w_t$$

- **Robustify input constraints**

- **Handle state chance constraints**

$$\Pr\left\{\bigcup_{k=1}^{N} x_k \notin \mathcal{X}\right\} \leq \epsilon, \quad \forall\, w_k \in \mathcal{W}$$

Chance constraints (Gaussian)

- Conservatism in Boole's inequality and risk allocations

$$\Pr\left\{\bigcup_{k=1}^{N} x_k \notin \mathcal{X}_k\right\} \leq \sum_{k=1}^{N} \sum_{i=1}^{M} \Pr\left\{{h_k^i}^T x_k > g_k^i\right\} < \epsilon$$

- Require computation of CDF

$$\Pr\left\{{h^i}^T x_k > g\right\} = \int_g^\infty \mathrm{pdf}({h^i}^T x_k) d({h^i}^T x_k) < \epsilon/(NM)$$

- Linear Gaussian systems: convex problem [Lars Blackmore and Masahiro Ono]

$$h^{iT}(\hat{x}_k) \leq g - \mathrm{cdf}^{-1}_{h^T \tilde{x}_k}(1-\epsilon_i)$$ CONVEX

Mean Offset

$$\sum \epsilon_i = \epsilon$$ Risk Allocation

The resulting problem can be solved efficiently using **tailored sparse solver**

Chance constraints (non-Gaussian)

- **Conservatism in Boole's inequality and risk allocations**

$$\Pr\left\{\bigcup_{k=1}^{N} x_k \notin \mathcal{X}_k\right\} \leq \sum_{k=1}^{N}\sum_{i=1}^{M} \Pr\left\{h_k^{i\,T} x_k > g_k^i\right\} < \epsilon$$

- **Require computation of CDF**

$$\Pr\left\{h^{iT} x_k > g\right\} = \int_g^{\infty} \mathrm{pdf}(h^{iT} x_k)\, d(h^{iT} x_k) < \epsilon/(NM)$$

- **Linear non-Gaussian systems: fix risk allocation**

$$\text{Let } x_k = \hat{x}_k + \tilde{x}_k, \qquad h^T \tilde{x}_k = h^T A_{cl}^k \tilde{x}_0 + h^T \sum_{i=0}^{i=k-1} A_{cl}^{k-i-1} D\tilde{w}_i$$

$$h^T(\hat{x}_k) \leq g - \mathrm{cdf}_{h^T\tilde{x}_k}^{-1}(1 - \epsilon/(Nm))$$

Mean   Offset

The inverse cdf $\mathrm{cdf}_{h^T\tilde{x}_k}^{-1}$ can be evaluated offline by discretizing the PDF of w and discrete convolution

Predictive Control Design for Large Scale Systems

$$\min_{\pi_0,\pi_1,\ldots,\pi_N} \mathbb{E}\{\sum_{t=0}^{t=N} \text{Energy}(x_t, u_t, w_t)\}$$

$$\begin{aligned}\text{s.t.}\quad & x_{t+1} = Ax_t + B\text{diag}(\dot{m}_{st})(T_{st} - Cx_t) + Dw_t \\ & u_t = \pi_t(x_t) \\ & \mathbb{P}\left\{x_k^{i\in\mathcal{V}} \in \mathcal{X}\right\} > 1-\epsilon \\ & u_t^{i\in\mathcal{V}} \in \mathcal{U} \quad \forall w_t \in \mathcal{W}_t\end{aligned}$$

- **Linearize system**

$$x_{t+1} = A_t x_t + B_t u_t + D_t w_t$$

- **Robustify input constraints**
- **Handle state chance constraints**

$$\Pr\left\{\bigcup_{k=1}^{N} x_k \notin \mathcal{X}\right\} \leq \epsilon, \quad \forall\, w_k \in \mathcal{W}$$

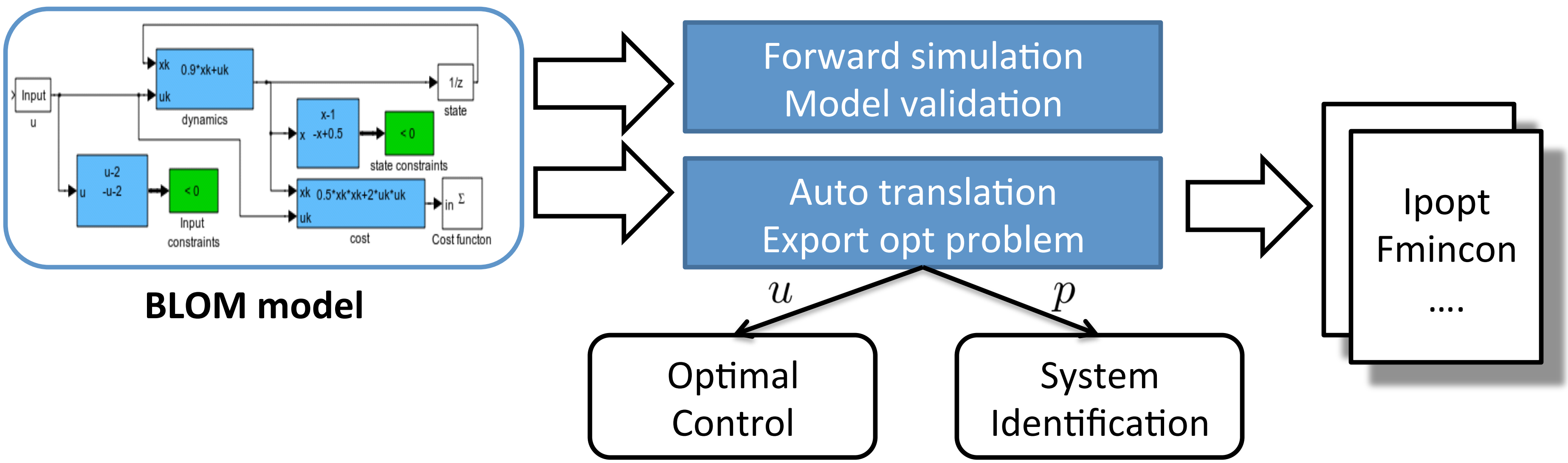
- **Ipopt for resulting optimization problem**

BLOM work flow

A. Kelman, S. Vichik

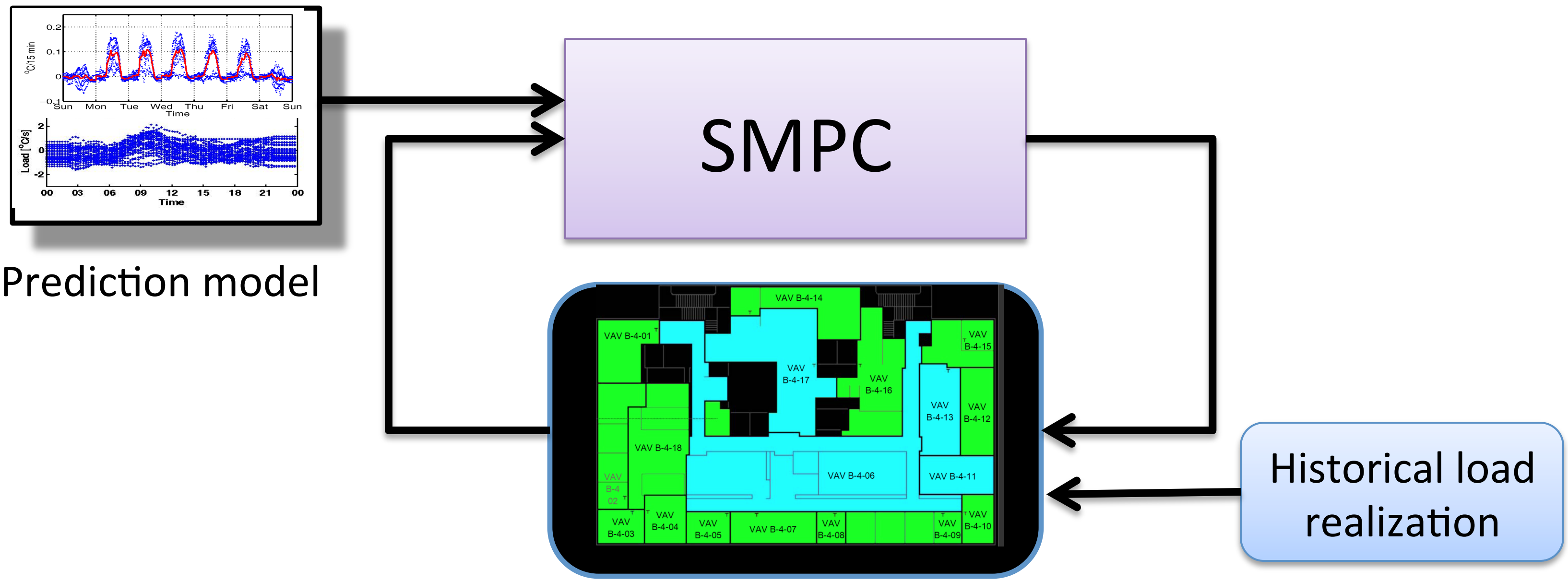
Berkeley Library for Optimization Modeling

http://www.mpc.berkeley.edu/software/blom

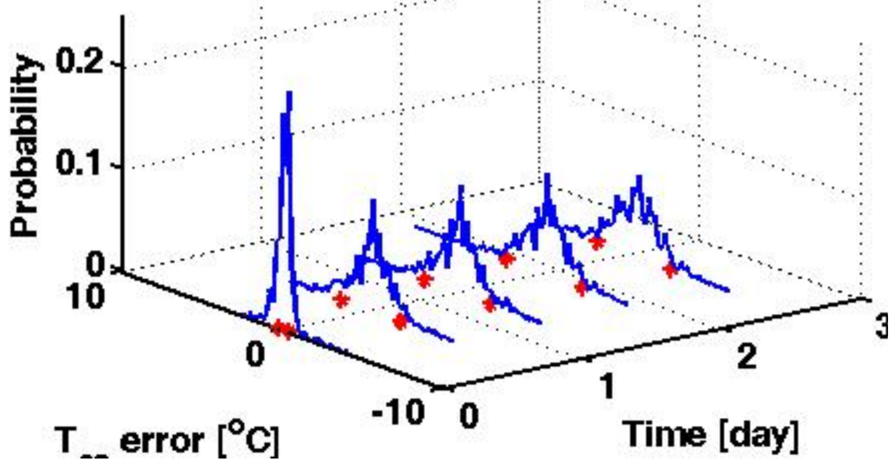


- **Create model using Simulink with BLOM library, Run and compare the model to reference data**
- **Export the problem to a solver**
- **Used to create MPC controller for a large HVAC system: 41 zones, 430 states, 30 time steps, 37000 variables, 40000 constraints needs < 1minutes solver time with Ipopt**

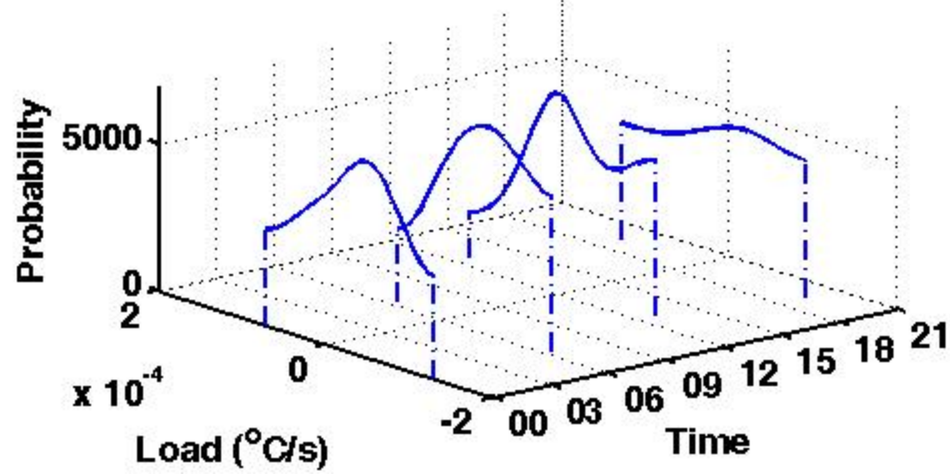
Simulation results



Ambient temperature



Load



Data are collected from DOE library Nov 2011 –Jan 2012

Simulation results

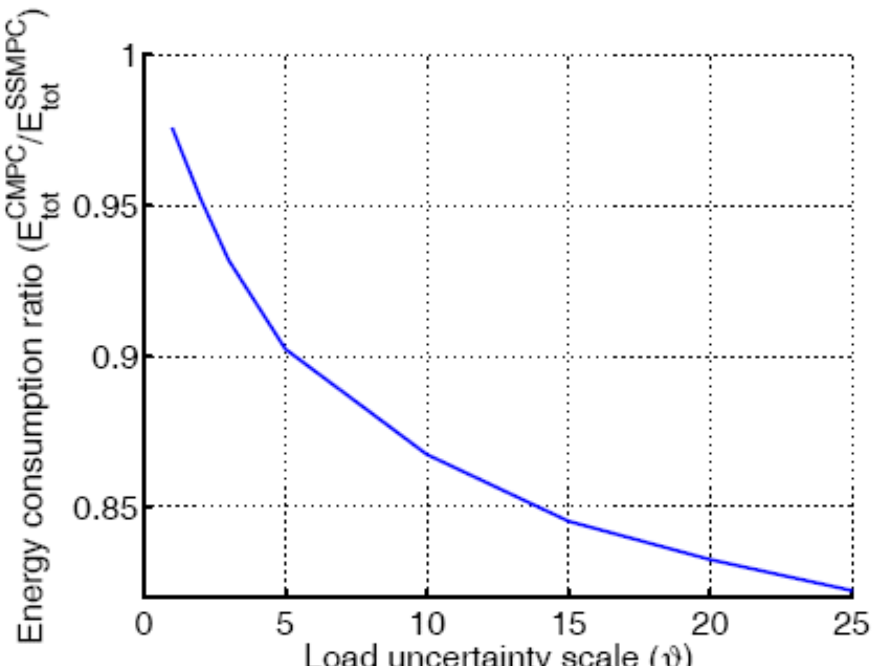
PMPC	MPC with perfect load predictions
CMPC	MPC with mean load predictions at design stage
GSMPC	SMPC with prediction uncertainties approximated as Gaussian
ESMPC	Proposed stochastic MPC

Controller	PMPC	CMPC	GSMPC	ESMPC
Energy savings S (%)	22.54	22.43	20.39	20.33
omfort improvement Δ (%)	96.25	-121.91	17.26	88.07

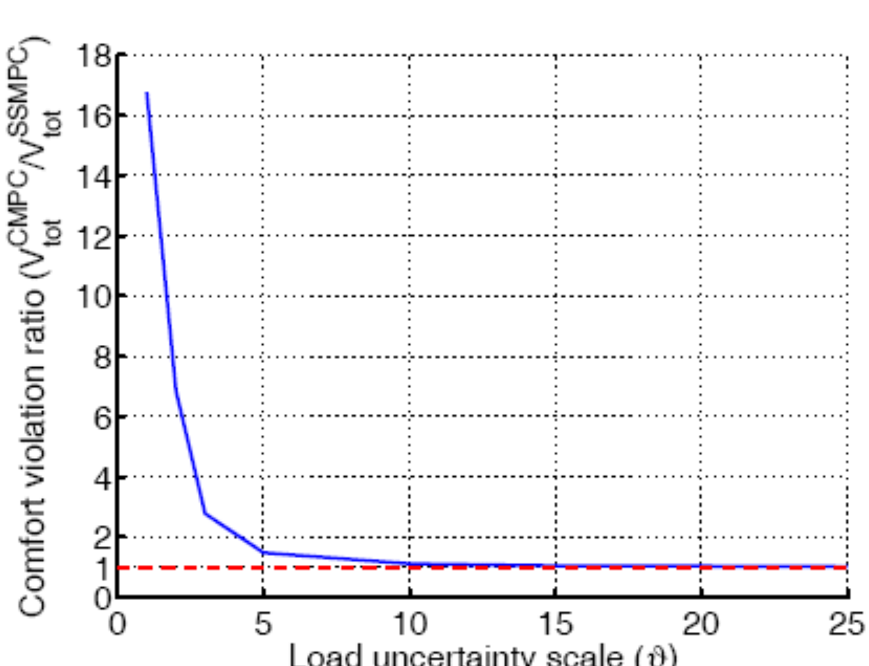
- **PMPC** has the best performance
- **CMPC** violates the most comfort constraints
- **GSMPC** fails to respect the chance constraints
- **ESMPC** has best performance, same complexity as CMPC (online)

Simulation results

$$w(t)_\vartheta = \hat{w}(t) + \vartheta(w(t) - \hat{w}(t))$$



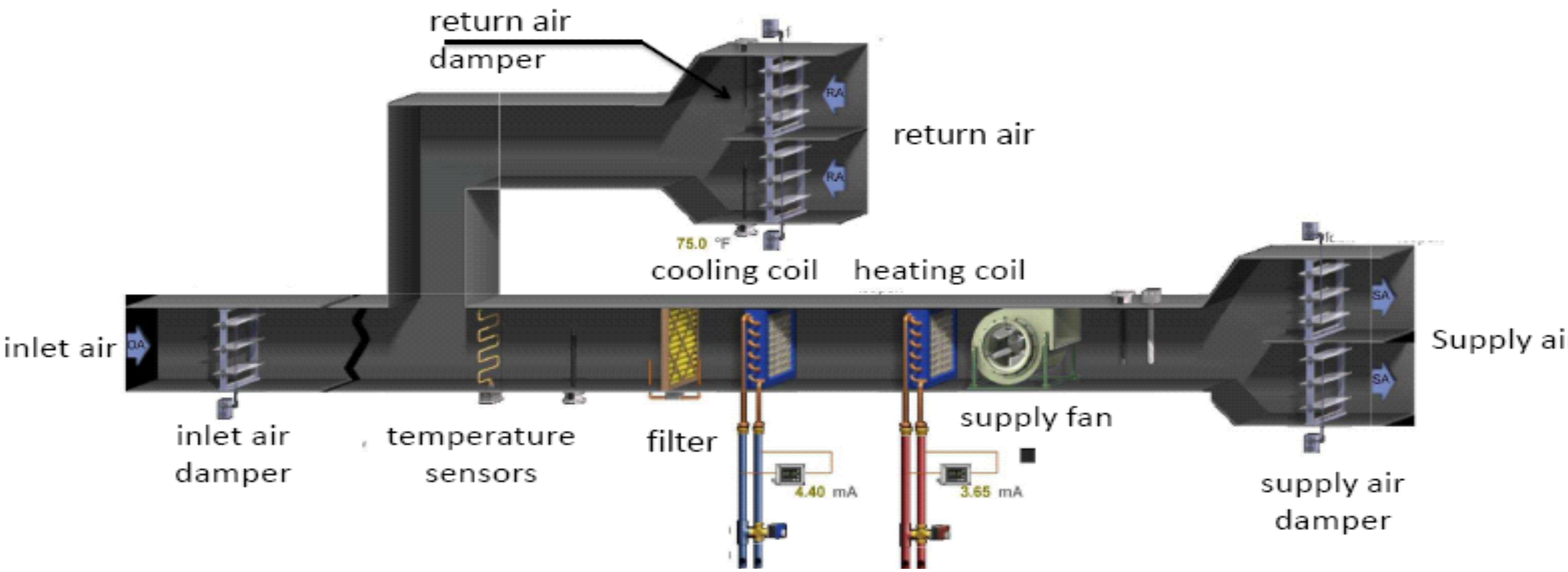
(a) Energy consumption vs load uncertainty

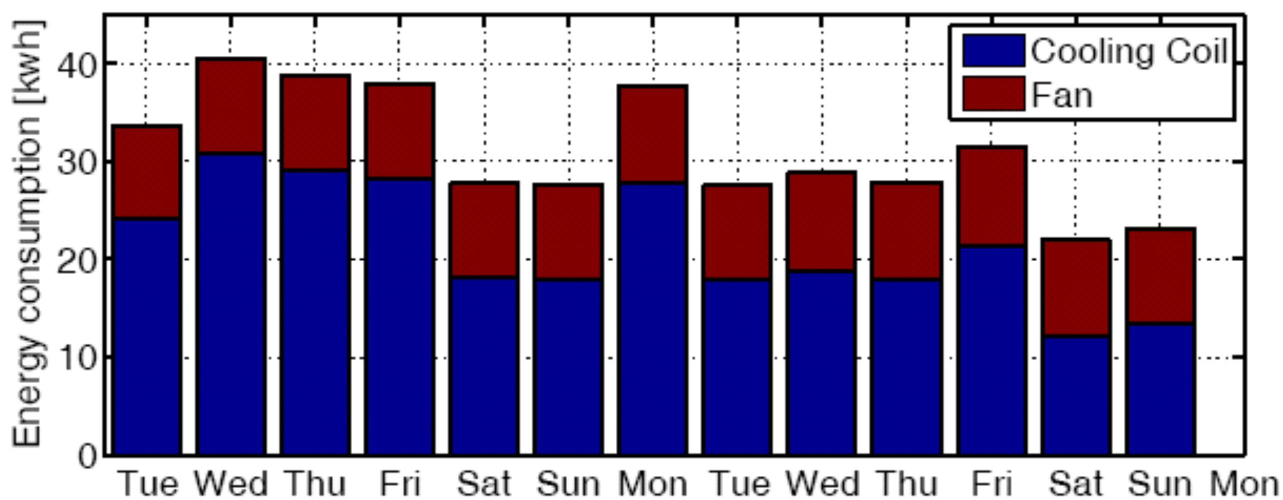


(b) Comfort violations vs load uncertainty

The performance of ESMPC degrades as load uncertainty level increases

MPC lab implementation





Outline

- Background
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- Stochastic MPC design
- Conclusions

Conclusions

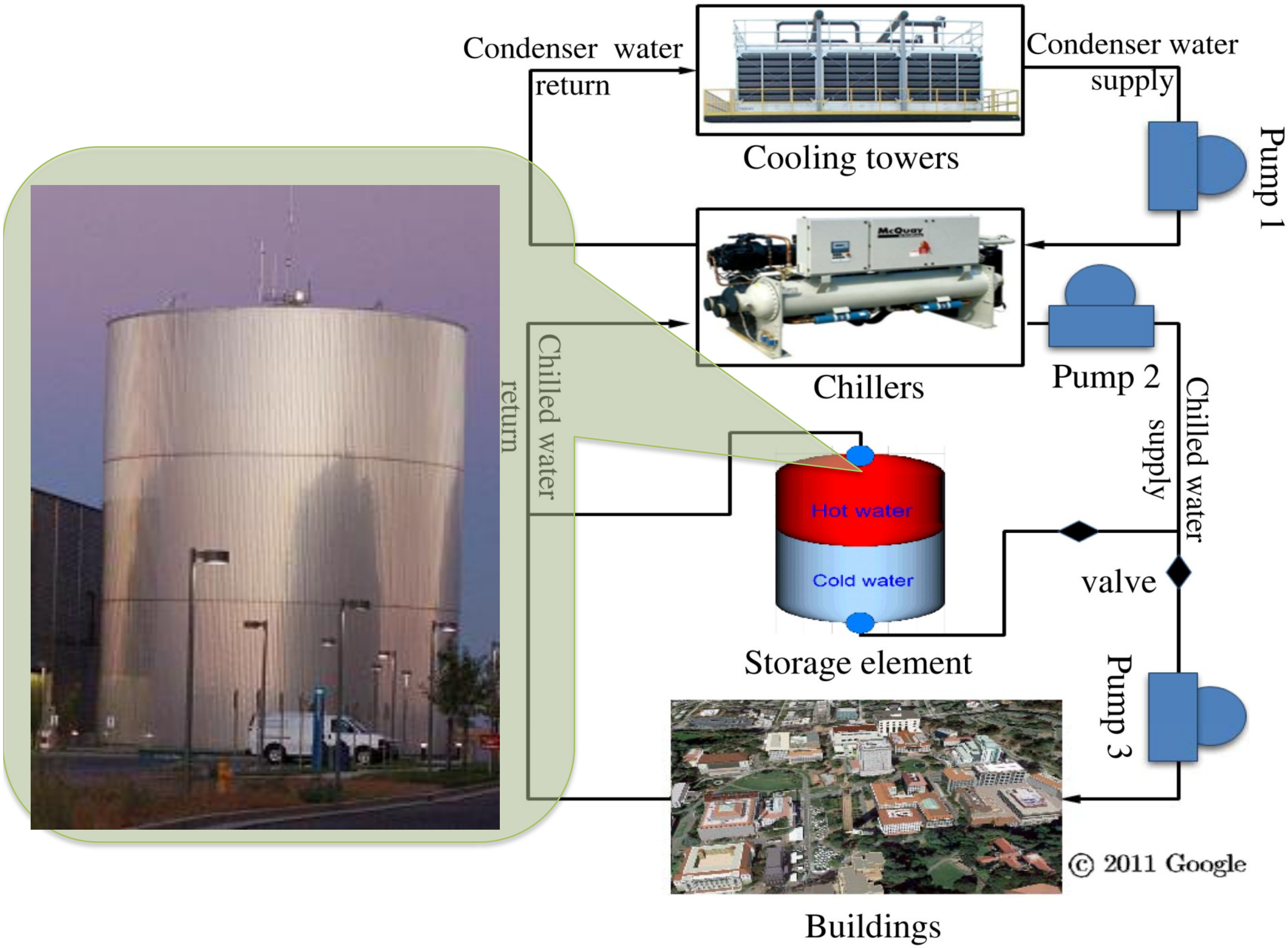
- Proposed model abstractions for building HVAC predictive control design
- Developed distributed MPC implementable on distributed low-cost computation units
- Developed stochastic MPC capable of handling bilinear systems subject to non Gaussian distributions
- Implementations.
  - UC Merced water loop system
  - SMPC in MPC lab HVAC system (HVAC - 1 Zone)
  - CITRIS Building (UC Berkeley)   (HVAC -135 Zones)

Future Plan

- Scalable model-based control design for large scale HVAC system
  - Distributed or decentralized MPC design with primal feasibility guarantee, and bounded optimality gap for a network of bilinear systems
- Data-driven stochastic MPC design for HVAC
  - What to learn and how to learn from trend data to improve the design of MPC for buildings
- Sensitivity analysis of stochastic MPC for energy efficient buildings
  - Analysis of the sensitivity from uncertain parameters to total potential energy savings
- Integration of renewal energy using buildings as energy storages and energy absorber
  - Respond to demand response signals by controlling HVAC systems

Thank you

System Description: Water loop



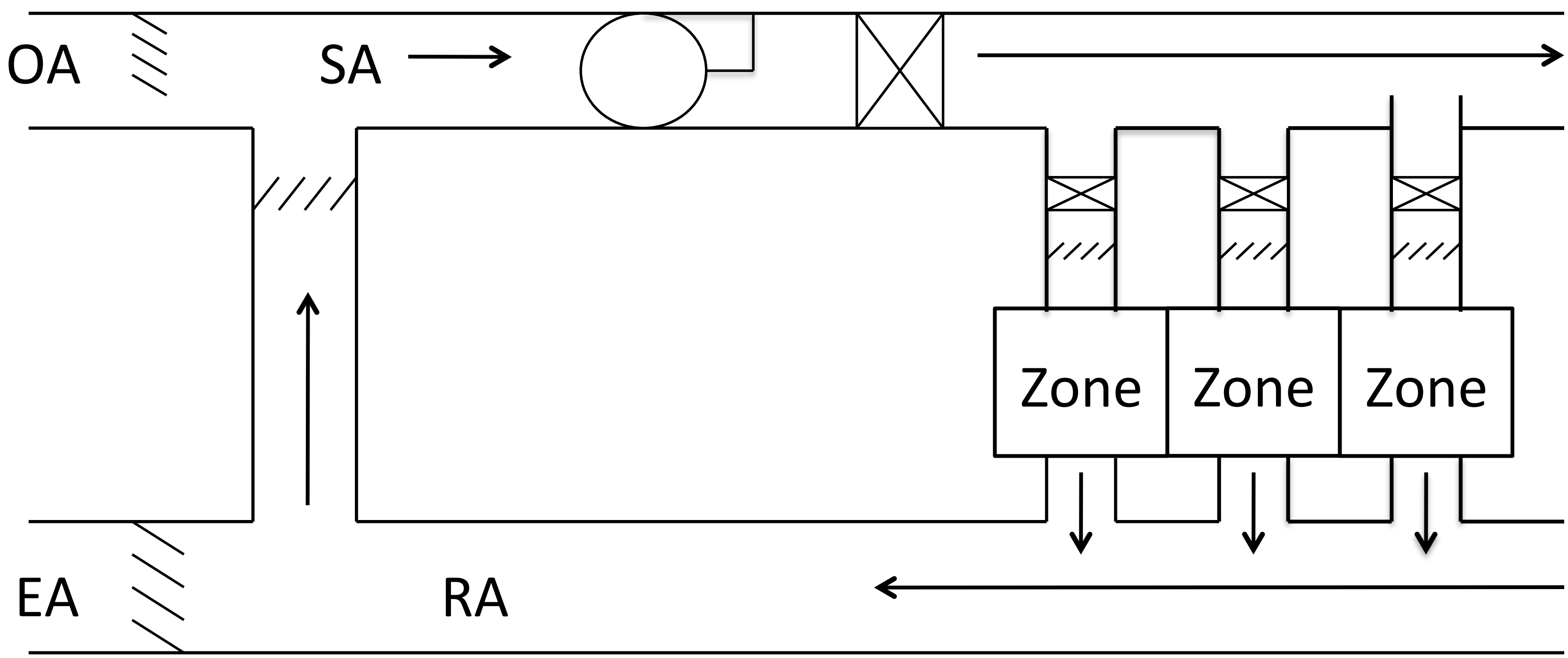
System Description: Air loop

- **Typical in New Constructions**

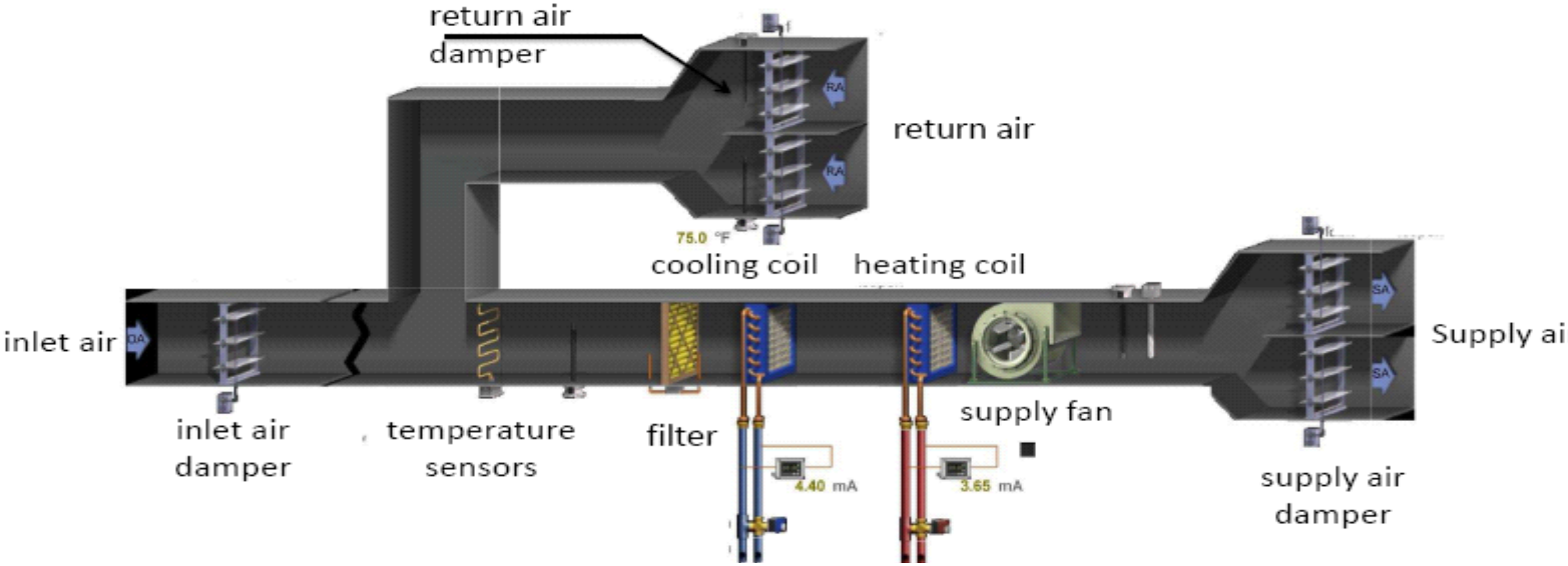
  Variable Air Volume with reheat

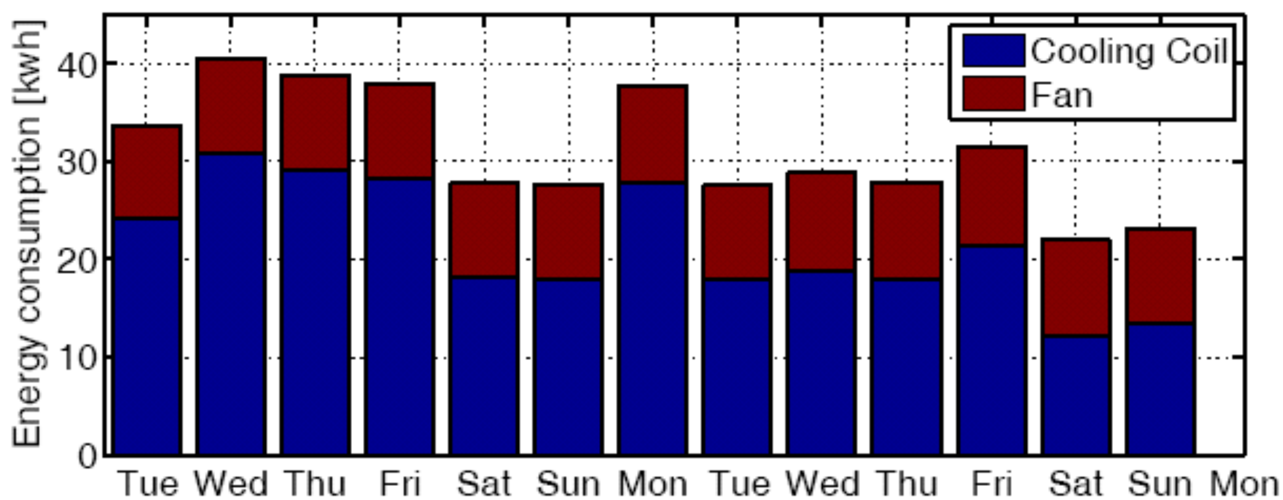
- **Components**

  Supply fan, cooling coil, heating coils, zone dampers



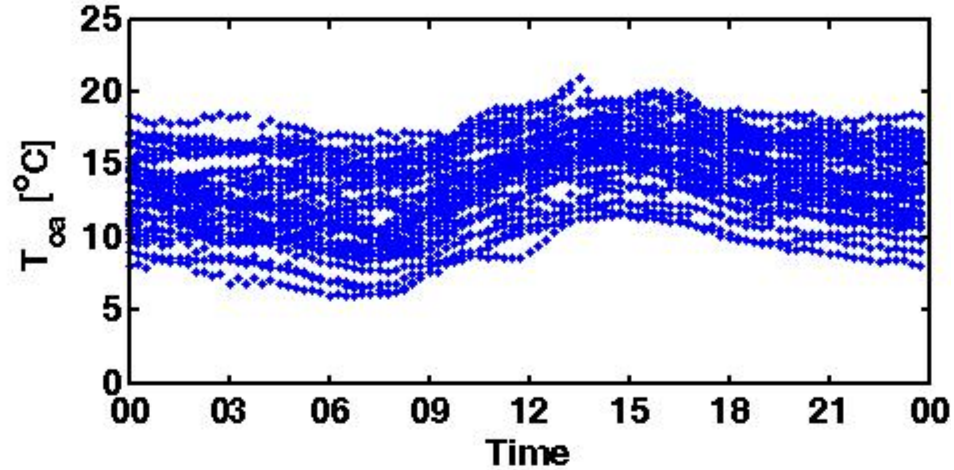
MPC lab implementation



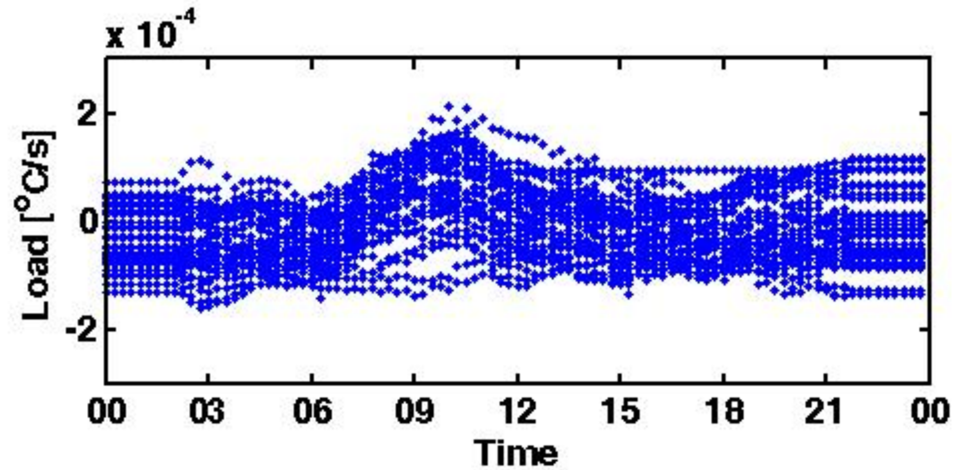


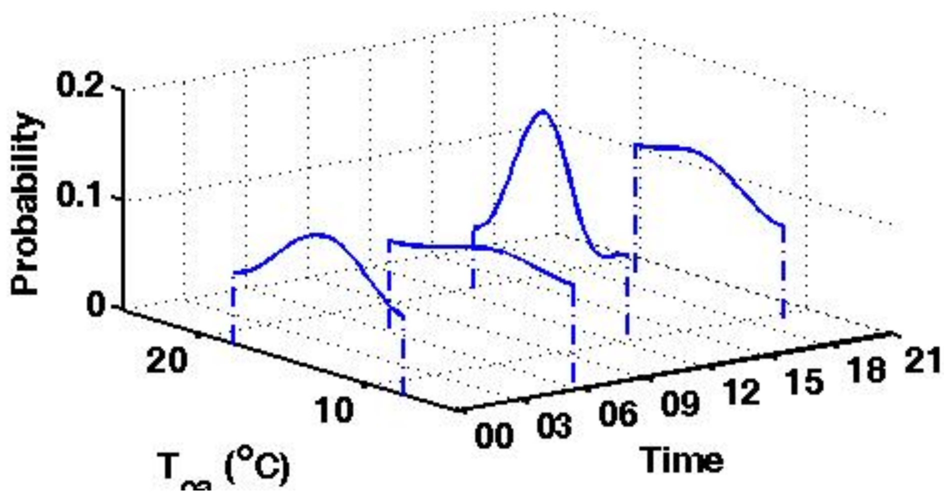
Simulation results

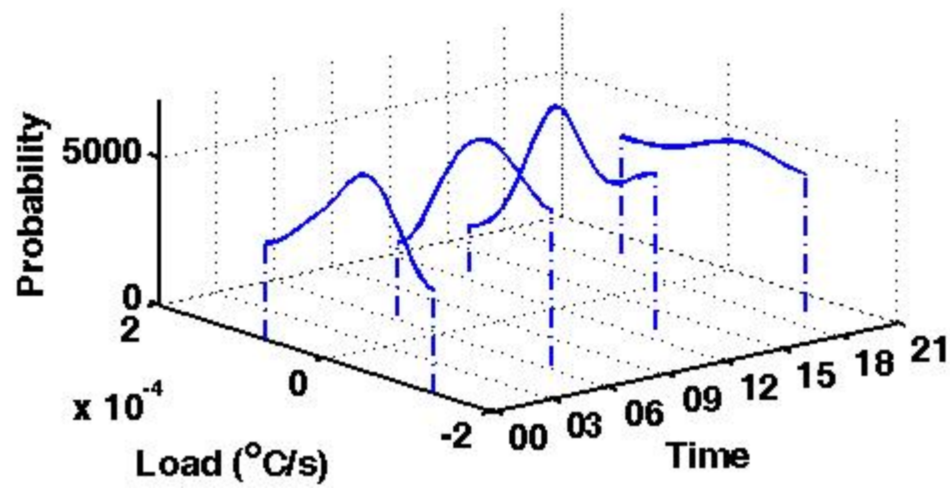
Ambient temperature



Load







Data are collected from DOE library Nov 2011 –Jan 2012