# Statistics in Control

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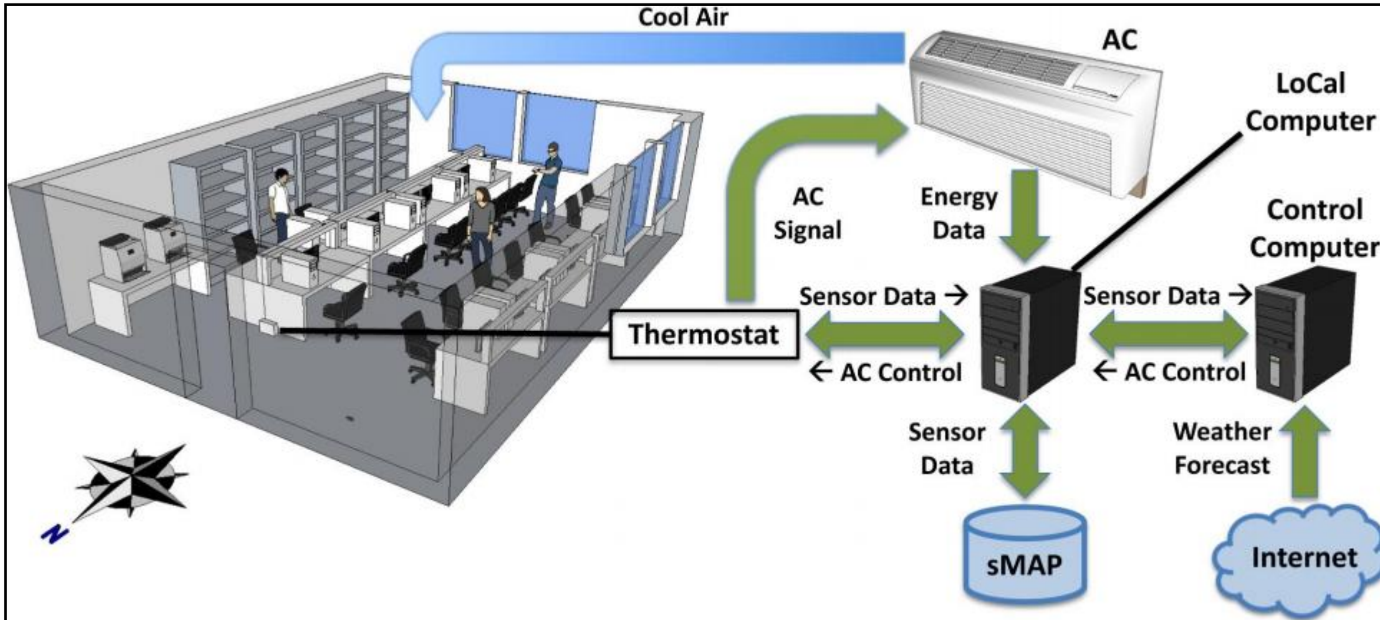
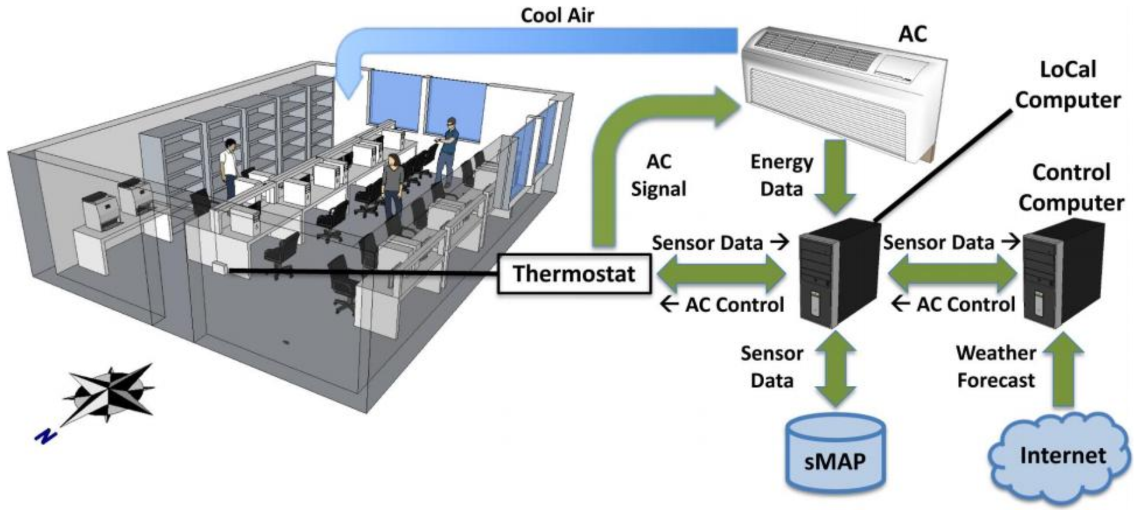
# Berkeley Retrofitted and Inexpensive HVAC Testbed for Energy Efficiency (BRITE)




- Partially engineered living laboratory
  - 640 sq. ft. computer space
  - Networked thermostat
  - Newton's law of cooling with complex heating load from occupant behavior

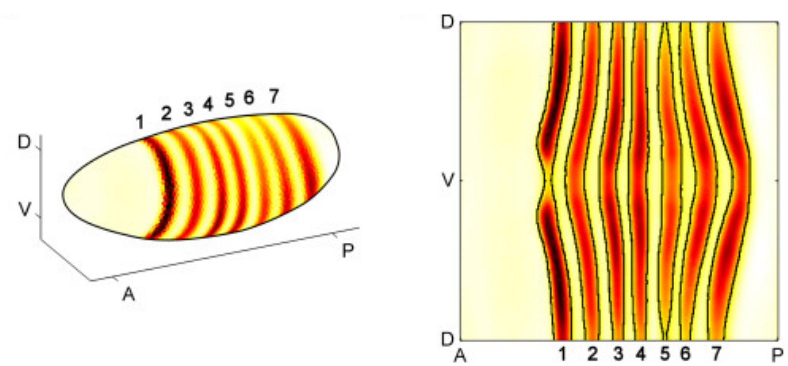
(Aswani, et al., Proc. IEEE, 2011); (Aswani, et al., submitted, 2011)

# Partially Engineered Systems




**Energy-efficient building automation**
(Aswani, et al., Proc. IEEE, 2011)

**Semi-autonomous systems**
(Aswani, et al., submitted, 2011)




**Biology and cancer**
(Aswani, et al., BMC Bioinformatics, 2010)

# Control Paradigms

## Model Based

- Theoretical guarantees
  - Safety and stability
  - Robustness

## Learning Based

- High Performance
  - Adaptation
  - Emergent behavior

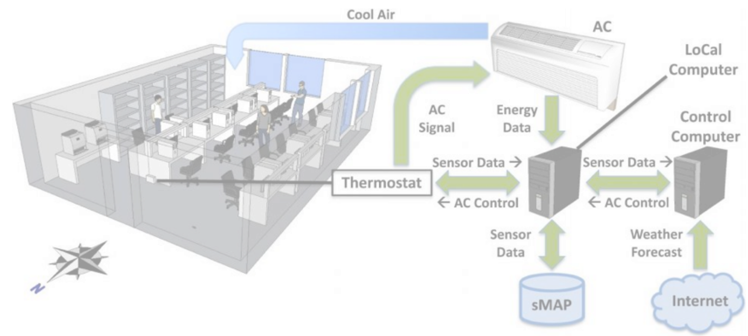
## Learning + Model Based

- Theoretical guarantees from model
- High performance from learning

# Model Predictive Control (MPC)

- Three elements

| Element | Example: BRITE |
|---|---|
| Finite horizon cost | Energy usage and temperature variation |
| Model | Newton's law of cooling |
| Constraints | Room temperature<br>Equipment on-time |



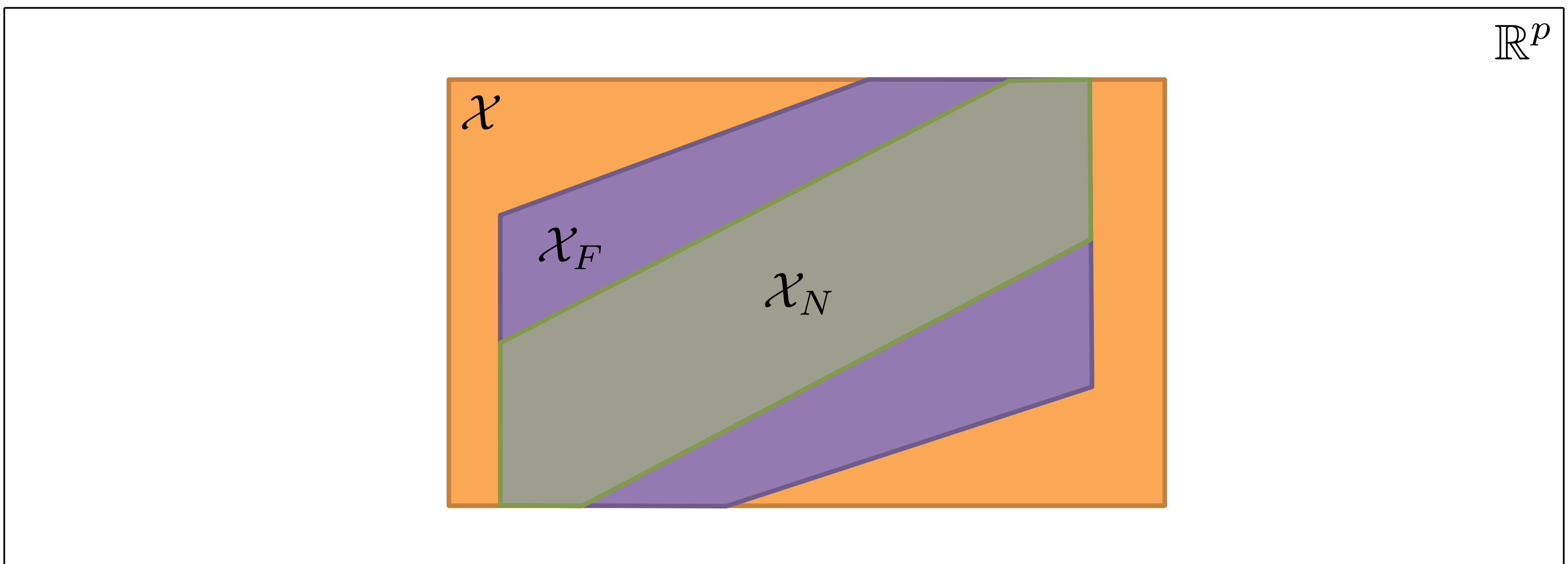

- Optimization solved at each time step

$$u_m^* = \arg\min \; x'_{m+N} P x_{m+N} + \sum_{k=0}^{N-1} \left( x'_{m+k} Q x_{m+k} + u'_{m+k} R u_{m+k} \right)$$
$$\text{s.t. } x_{m+k} \in \mathcal{X};\; x_{m+N} \in \mathcal{X}_N$$
$$u_{m+k} \in \mathcal{U}$$
$$x_{n+1} = A x_n + B u_n$$

(Mayne, et al., 2000); (Borelli, et al., 2009); (Aswani, et al., 2011)

# Solution of MPC

| Element | |
|---|---|
| Non-linear feedback | Minimizer of optimization |
| Value function | Convex for linear problems |
| Enlarged feasible set | $\mathcal{X}_F = \{x : \exists u^*\}$ |




(Mayne, et al., 2000); (Borelli, et al., 2009)

# Modeling for Efficient HVAC


- **Physics given by Newton's law of cooling**
- **Difficult to model heating load**
  - Time-varying nature
  - Lack of direct data

(Aswani, et al., Proc. IEEE, 2011)

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# Identification of System Model

- **Model:** $x_{n+1} = f(x_n, u_n) + x_n$
- **Data:** $\xi_n = x_n + \epsilon; \quad \mathbb{E}(\epsilon) = 0; \mathrm{var}(\epsilon) = \sigma^2$

Input: $$X = \begin{bmatrix} x_0' & u_0' \\ \vdots & \vdots \\ x_N' & u_N' \end{bmatrix}$$

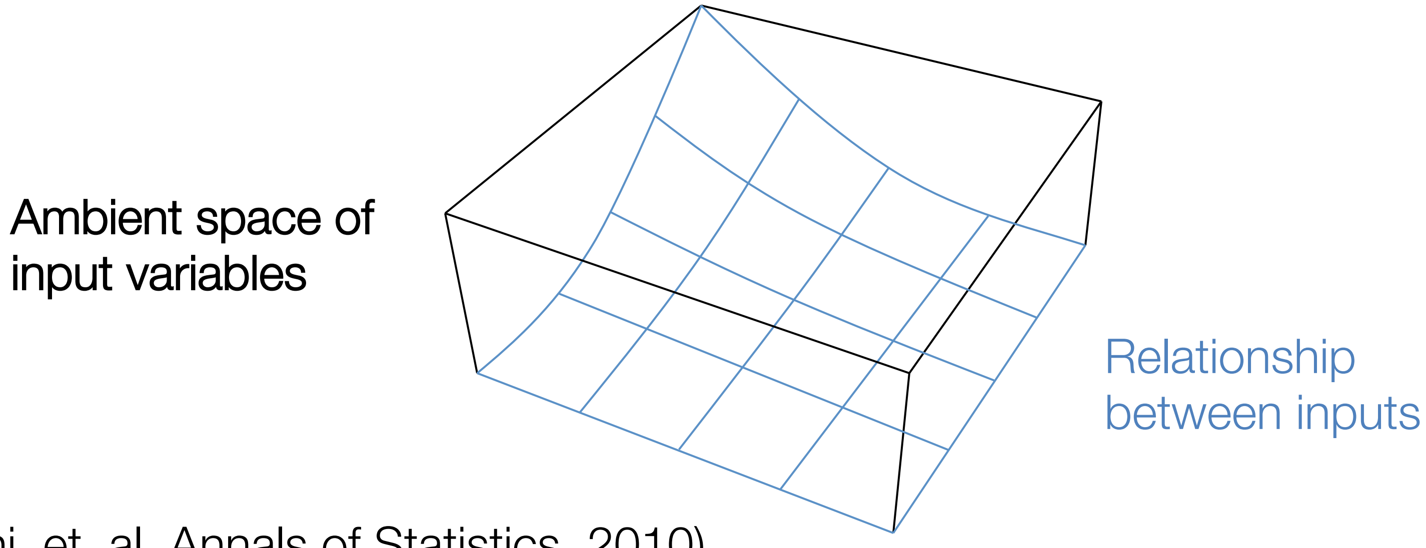
Output: $$Y = \begin{bmatrix} \Delta\xi_0' \\ \vdots \\ \Delta\xi_N' \end{bmatrix}$$

- Regression is ill-posed when
  a) Measured data is collinear
  b) Manifold relationship between input variables
- Can using b) improve identification of ill-posed regression models?

(Aswani, et. al, Annals of Statistics, 2010)

# Piecewise Linear Models

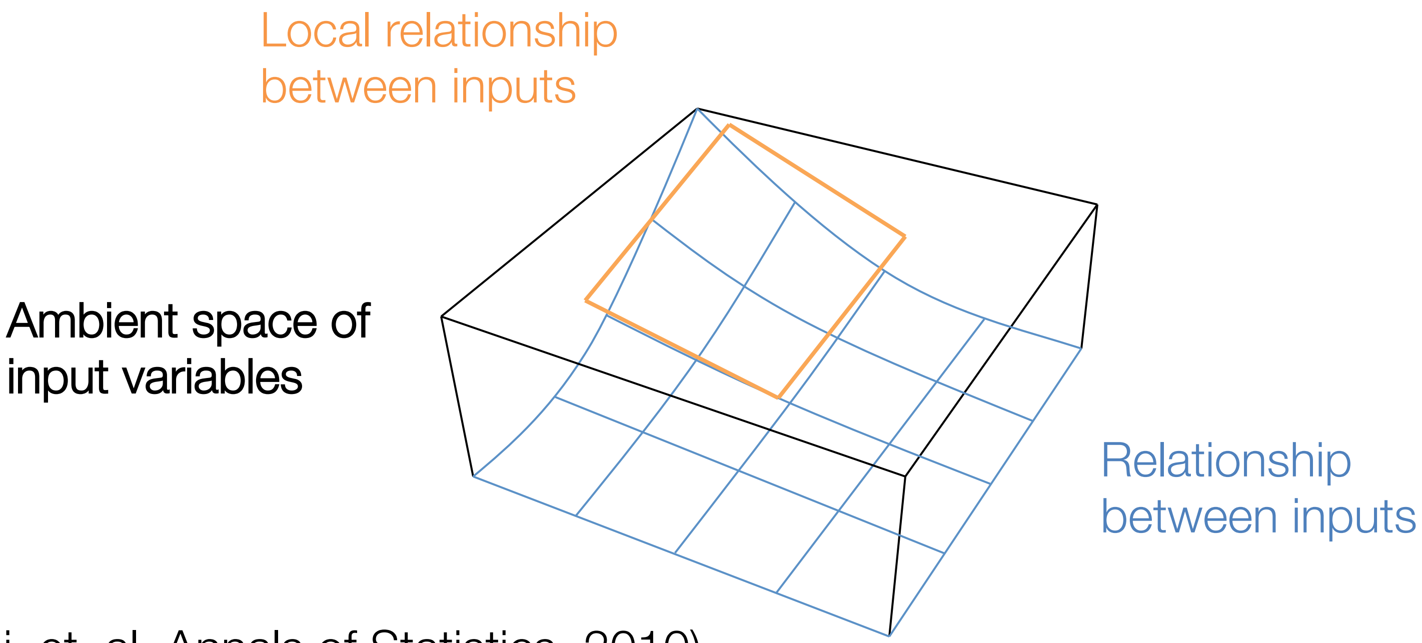
- Exploratory modeling for nonlinear systems




(Aswani, et. al, Annals of Statistics, 2010)

# Piecewise Linear Models

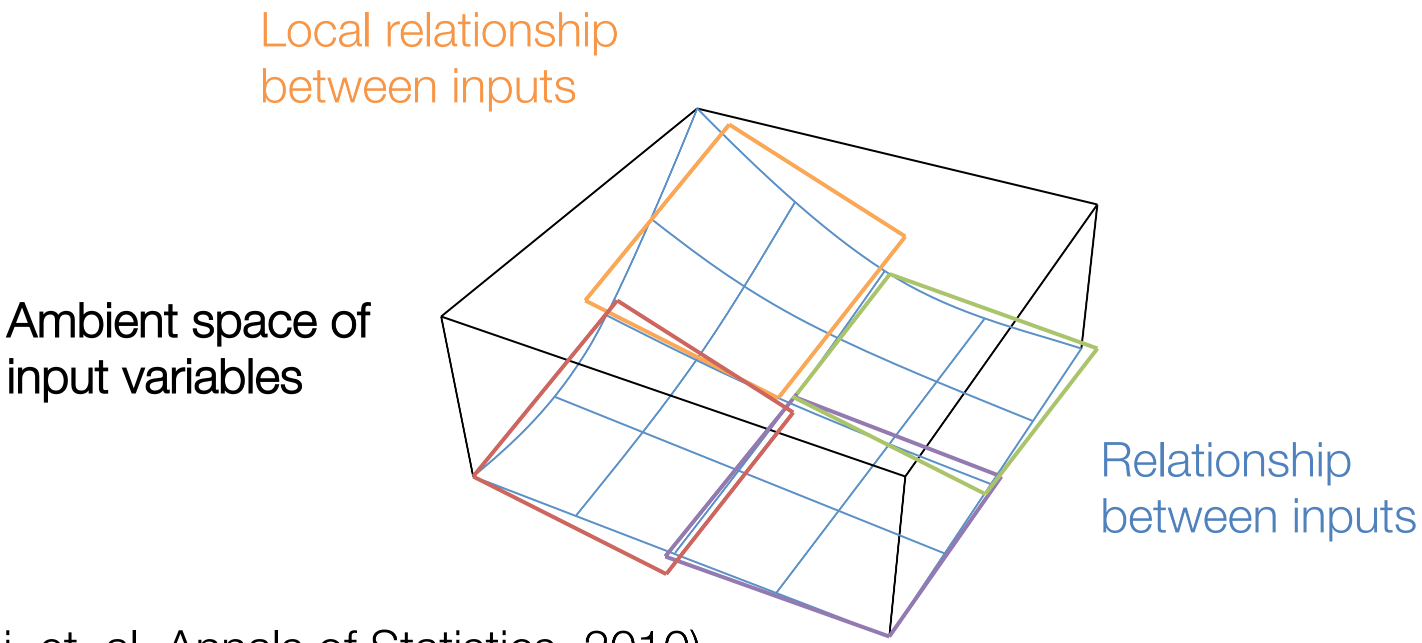
- Exploratory modeling for nonlinear systems
- Indentify local linear models




(Aswani, et. al, Annals of Statistics, 2010)

# Piecewise Linear Models

- Exploratory modeling for nonlinear systems
- Indentify local linear models
- Combine local models to cover space

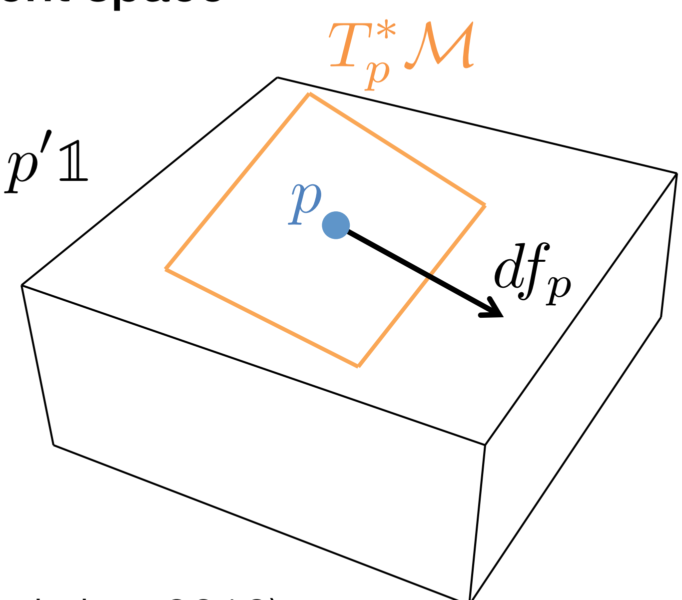



(Aswani, et. al, Annals of Statistics, 2010)

# Manifold Regularization

- For each local model
  - Input variables form plane
  - Outputs linear with respect to inputs
- With differential geometric view
  - Manifold described by cotangent space about a point
  - Exterior derivative
    - Best linear approximation of function
    - Spans cotangent space

$$X_p = X - p'\mathbb{1}$$

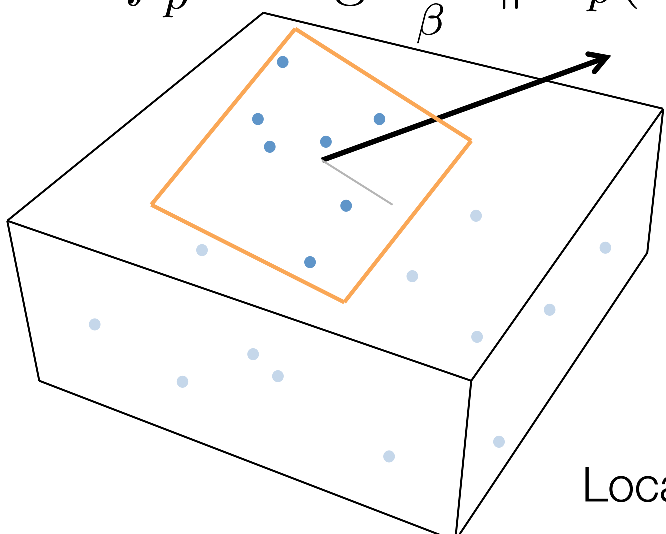



(Aswani, et. al, Annals of Statistics, 2010)

# Manifold Regularization

- Idea: Exploit differential geometric structure
  1) Locally estimate cotangent space
     - Compute local covariance matrix: $\hat{C}_p = X_p' W_p X_p$
     - Take first $d$ principal components
  2) Estimate exterior derivative
     - Penalize deviation of estimate from manifold

$$\hat{df}_p = \arg\min_{\beta} \|W_p(Y - X_p\beta)\|_2{}^2 + \lambda\|\hat{\Pi}\beta\|_2{}^2$$

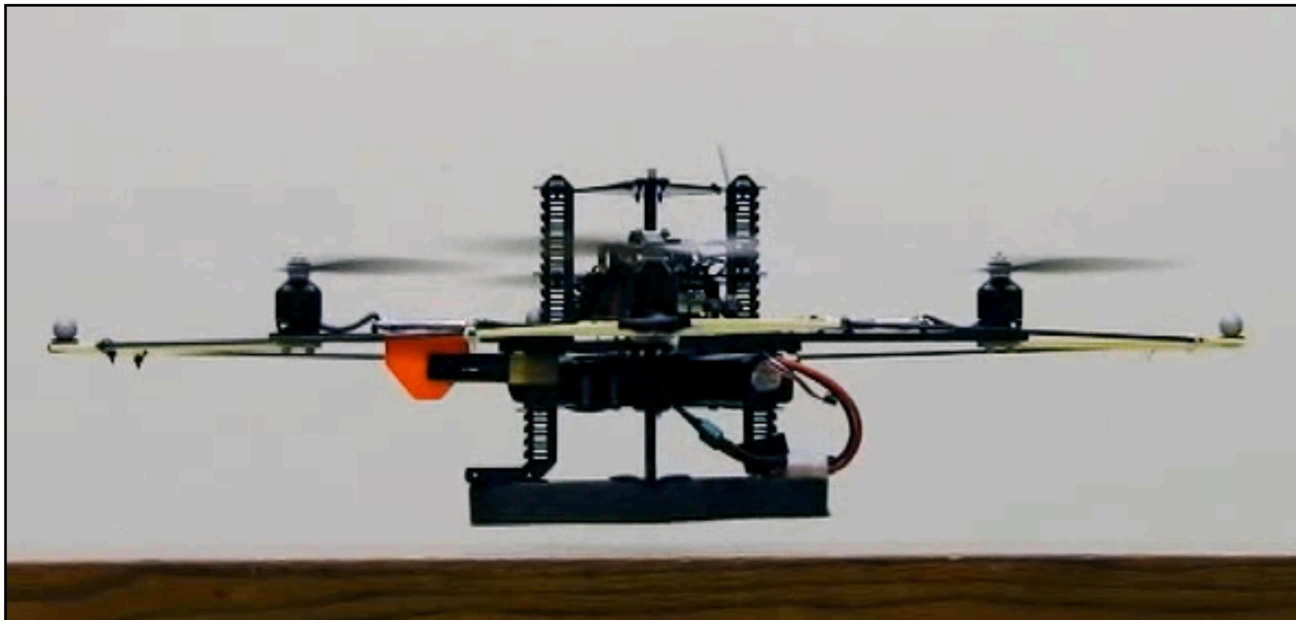


Projection orthogonal to cotangent space

Local linear regression

(Aswani, et. al, Annals of Statistics, 2010)

# Quadrotor Helicopter Testbed



- Partially engineered semi-autonomous system
  - Embedded processor onboard
  - Simple steady-state model
  - Complex physics in dynamic regimes

(Aswani, et al., ICRA, 2009); (Bouffard, et al., submitted, 2011)

# Quadrotor Dataset

- Measurements
  - Position-velocity
  - Angular orientation-velocity
- Online learning simulation
  - Build piecewise linear model with
    $$\{x_i, u_i : 0 \leq i \leq n\}$$
  - Predict position ten steps into future
    $$\{\hat{x}_i^{\text{pos}} : n+1 \leq i \leq n+10\}$$
  - Compare prediction to actual data
    $$\sqrt{1/10 \cdot \sum_{i=n+1}^{n+10} \|\hat{x}_i^{\text{pos}} - x_i^{\text{pos}}\|_2{}^2}$$
- Reduced error with manifold regularization

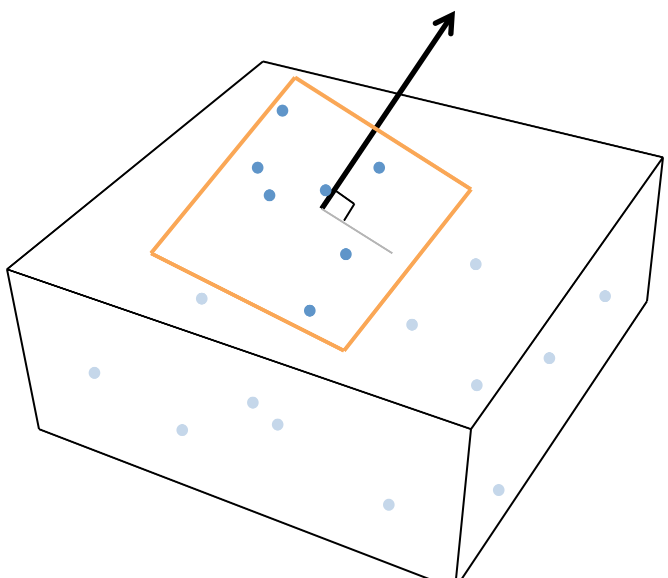
| | Prediction Error | |
|---|---|---|
| Ordinary Least Squares | 0.807 | (3.26) |
| Ridge Regression | 0.165 | (0.07) |
| Elastic Net | 0.166 | (0.08) |
| Partial Least Squares | 0.194 | (0.10) |
| Principal Components Regression | 0.174 | (0.09) |
| Exterior Derivative Estimator | 0.156 | (0.07) |

Averages and standard deviations over 100 steps of online learning

(Aswani, et al., ICRA, 2009)

# Augmentation of Learning

- Consequence: No learning orthogonal to cotangent space of manifold
- Stable control needs more structure
  - Apprenticeship learning uses expert human data
  - Possibility of new technique using physical model



(Abbeel, et al., 2008); (Aswani, et al., Annals of Statistics, 2010)

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# Learning-based MPC (LBMPC)

- Insight: Performance and safety can be decoupled in MPC
- Idea: Maintain two models
  - First updated with learning
  - Second kept fixed
- Learning can be any statistical tool

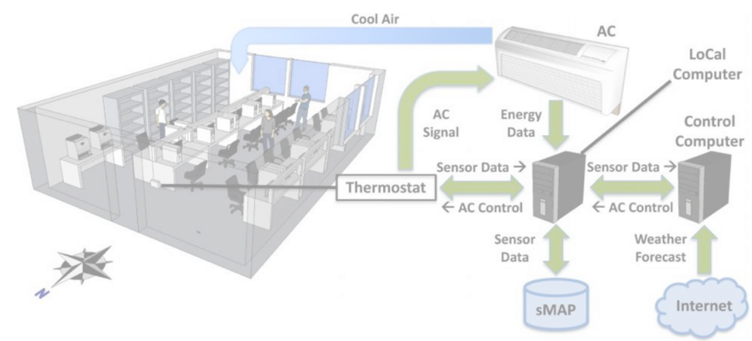
| Performance | Safety |
|---|---|
| • Cost function<br>• Learned model | • Constraints and uncertainty<br>• Original model |

(Aswani, et al., submitted, 2011); (Aswani, et al., Proc. IEEE, 2011)

# Components of LBMPC

- Five elements

| Element | Example: BRITE |
|---|---|
| Finite horizon cost | Energy usage and temperature variation |
| Model | Newton's law of cooling |
| Constraints | Room temperature<br>Equipment on-time |
| Uncertainty | Modeling error<br>Heating load variation |
| Oracle (Learned model) | Learning of heating load |




- Constraints robustified by subtracting out effect of uncertainty

(Aswani, et al., submitted, 2011)

# LBMPC Formulation

- At each time step
  - Optimization solved
  - Oracle updated

$$
\begin{aligned}
u_m^* = \arg\min\ & J(\tilde{x}_{m+1}, \ldots, \tilde{x}_{m+N}, u_m, \ldots, u_{m+N-1}) \\
\text{s.t. } & \tilde{x}_{n+1} = A\tilde{x}_n + Bu_n + \mathcal{O}_m(\tilde{x}_n, u_n) \\
& x_{m+k} \in \mathcal{X} \ominus \mathcal{R}_i; x_{m+N} \in \mathcal{X}_N \ominus \mathcal{R}_N \\
& u_{m+k} = Kx_{m+k} + c_{m+k} \in \mathcal{U} \ominus K\mathcal{R}_i \\
& x_{n+1} = Ax_n + Bu_n
\end{aligned}
$$

(Aswani, et al., submitted, 2011)

# LBMPC Formulation

- At each time step
  - Optimization solved
  - Oracle updated

LBMPC | Performance | Safety

$$u_m^* = \arg\min \; J(\tilde{x}_{m+1}, \ldots, \tilde{x}_{m+N}, u_m, \ldots, u_{m+N-1})$$

$$\text{s.t.} \; \tilde{x}_{n+1} = A\tilde{x}_n + Bu_n + \mathcal{O}_m(\tilde{x}_n, u_n)$$

$$x_{m+k} \in \mathcal{X} \ominus \mathcal{R}_i; x_{m+N} \in \mathcal{X}_N \ominus \mathcal{R}_N$$

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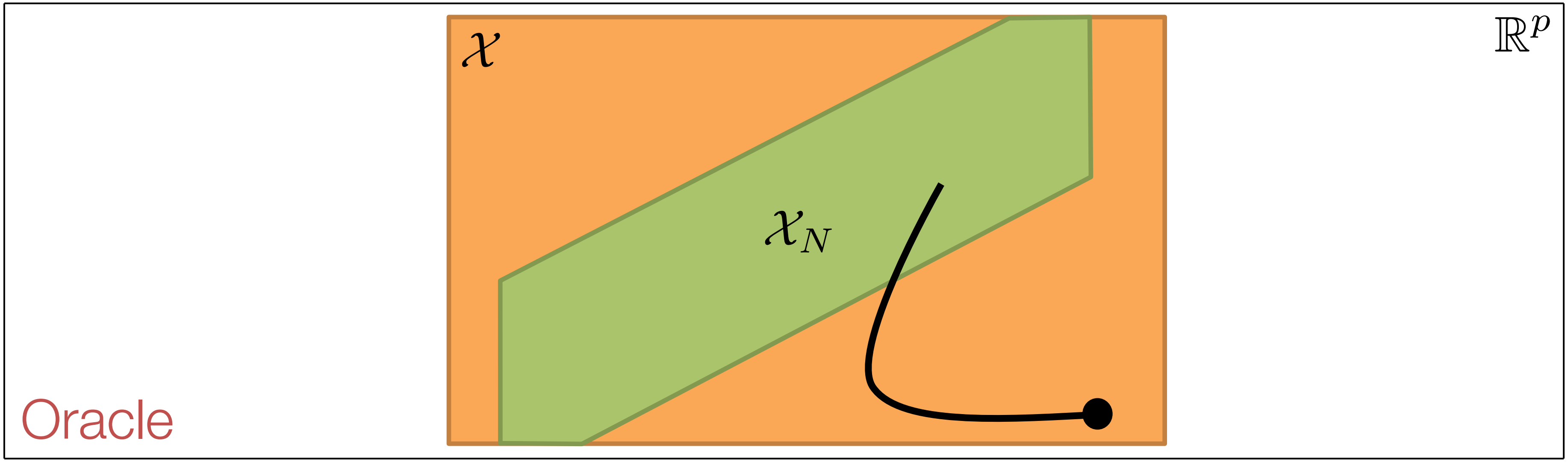
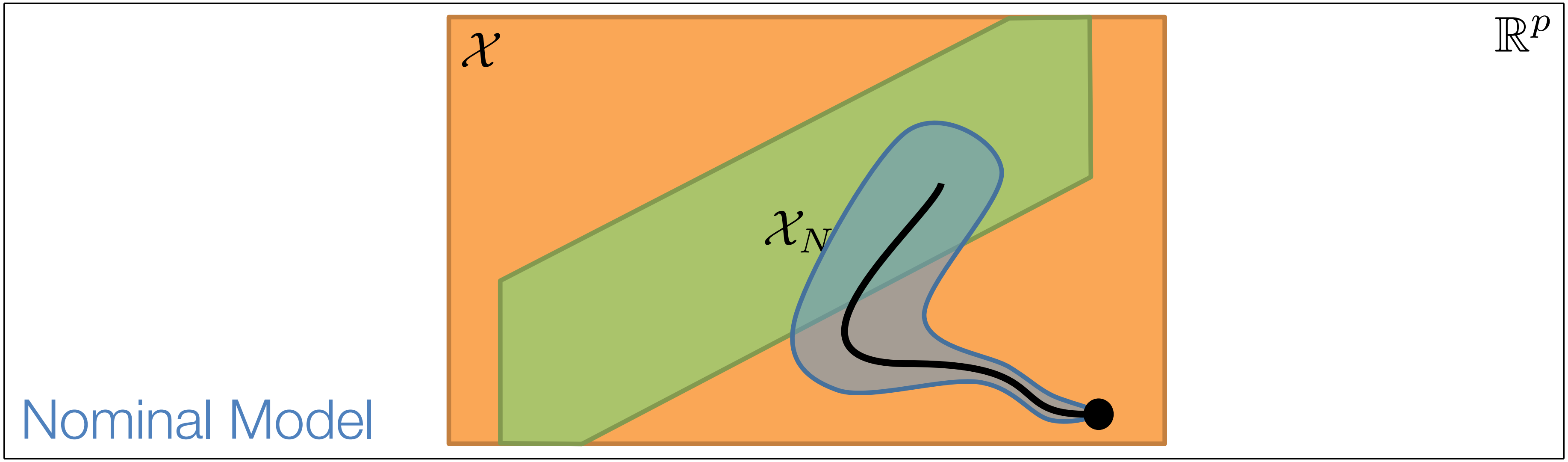
(Aswani, et al., submitted, 2011)

# Solution of LBMPC



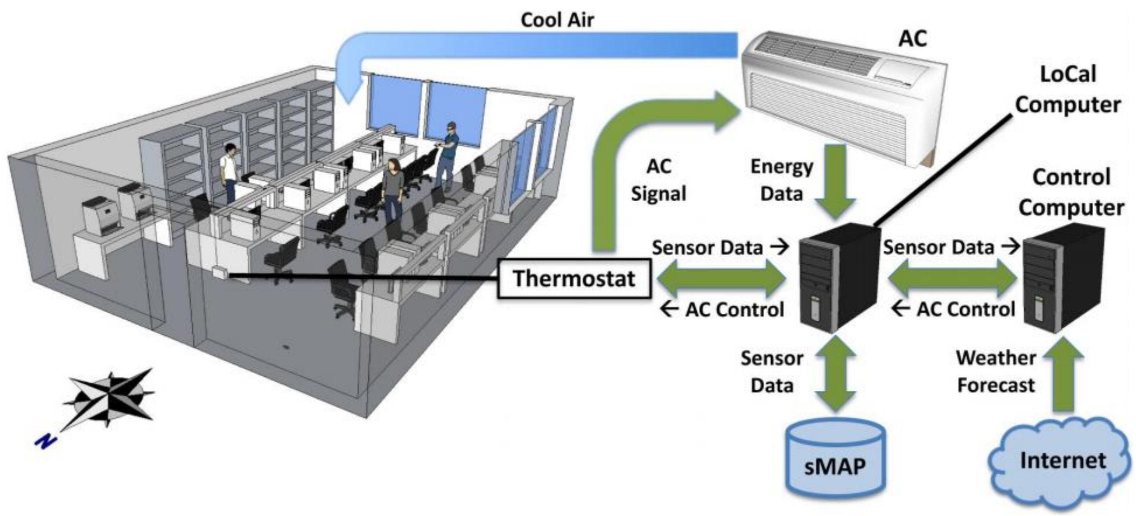




(Aswani, et al., submitted, 2011)

# Theoretical Properties of LBMPC

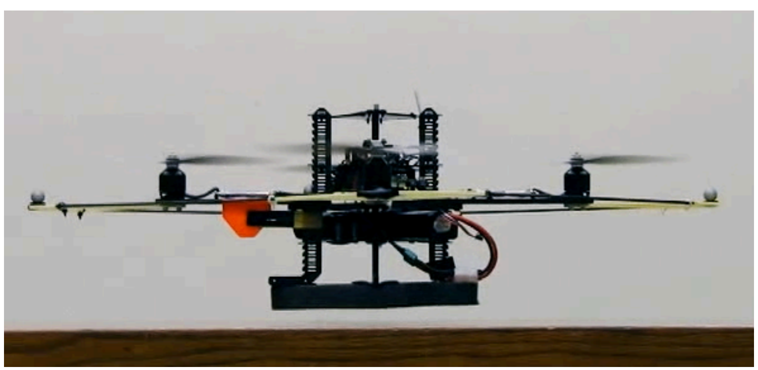
- For bounded modeling error, LBMPC has
  - Deterministic stability
    - Control always computable
    - States remain bounded and in constraints
  - Deterministic robustness
    - Continuous value function
    - Input-to-state stable (ISS) to modeling error
- If system dynamics are sufficiently excited
  - Control law of LBMPC stochastically converges to control law of MPC that knows the true model

# Partially Engineered Systems




**Energy-efficient building automation**

(Aswani, et al., Proc. IEEE, 2011)



**Semi-autonomous systems**

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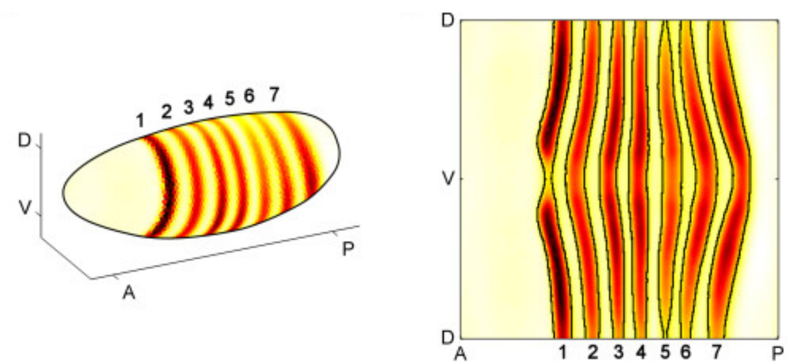



**Biology and cancer**

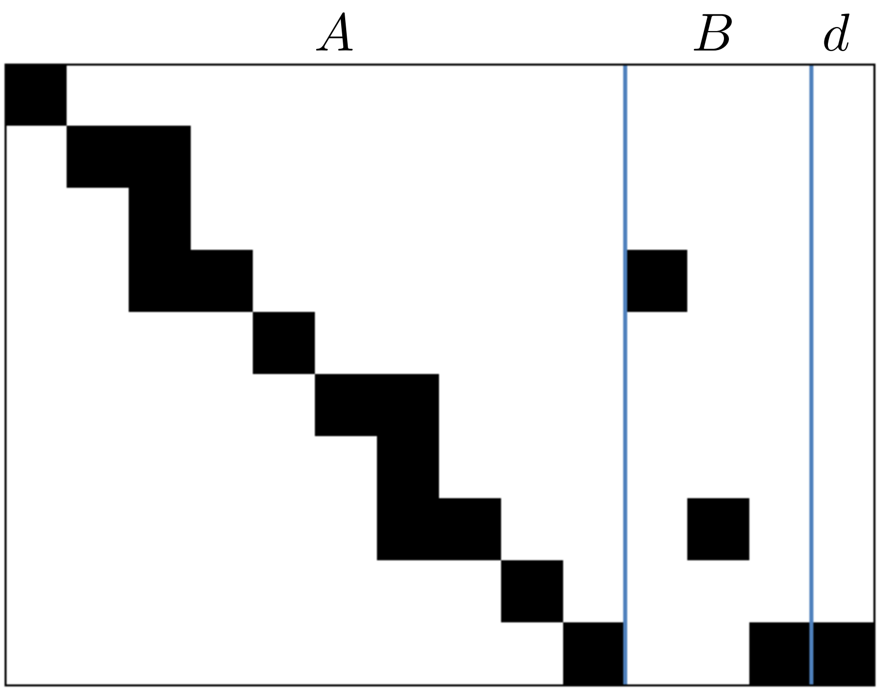
(Aswani, et al., BMC Bioinformatics, 2010)

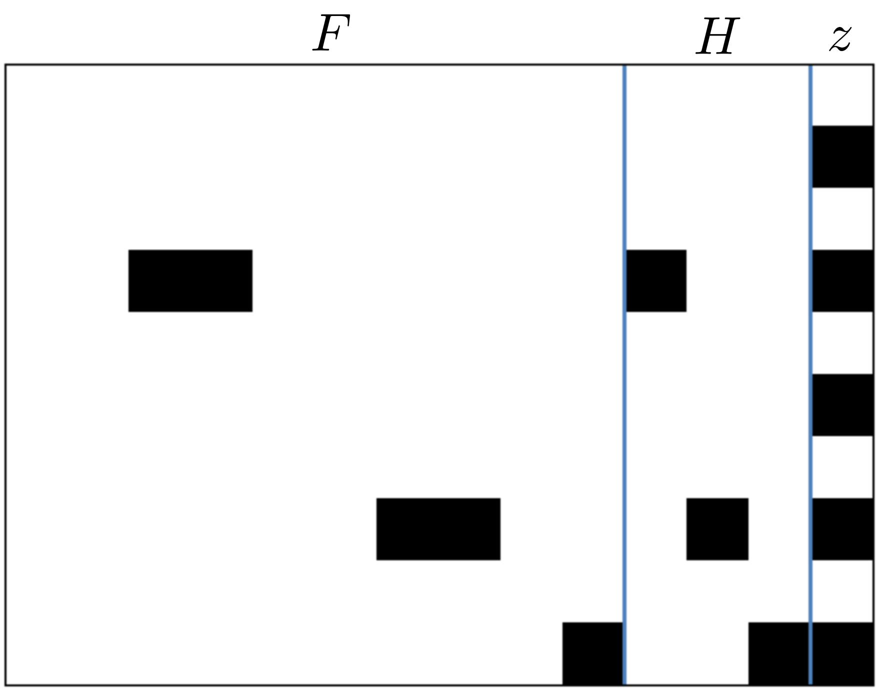
# Quadrotor Helicopter

- Linear model
  - Physics for structure
  - Experimental coefficients
- Physics improve statistics
  - Fewer parameters
  - Less noise

$$x_{n+1} = Ax_n + Bu_n + d$$

$$\mathcal{O}_m = Fx_n + Hu_n + z$$




(Aswani, et al., submitted, 2011); (Bouffard, et al., submitted, 2011)

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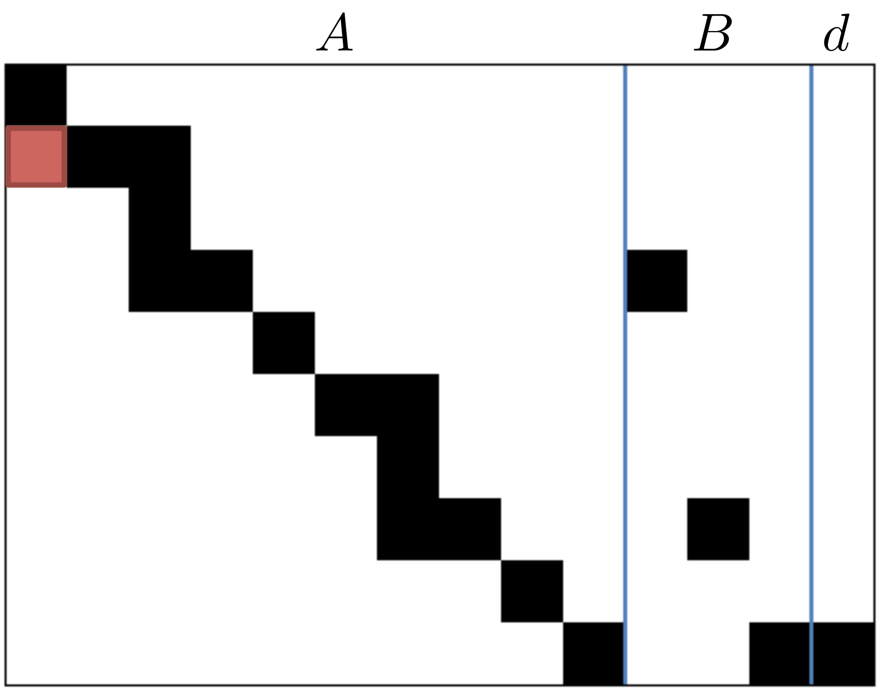


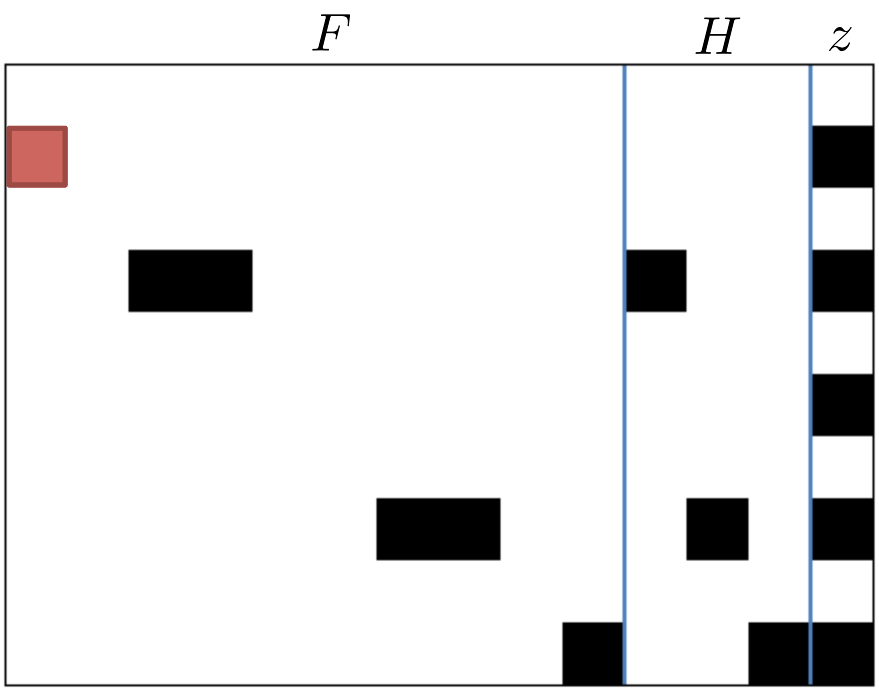


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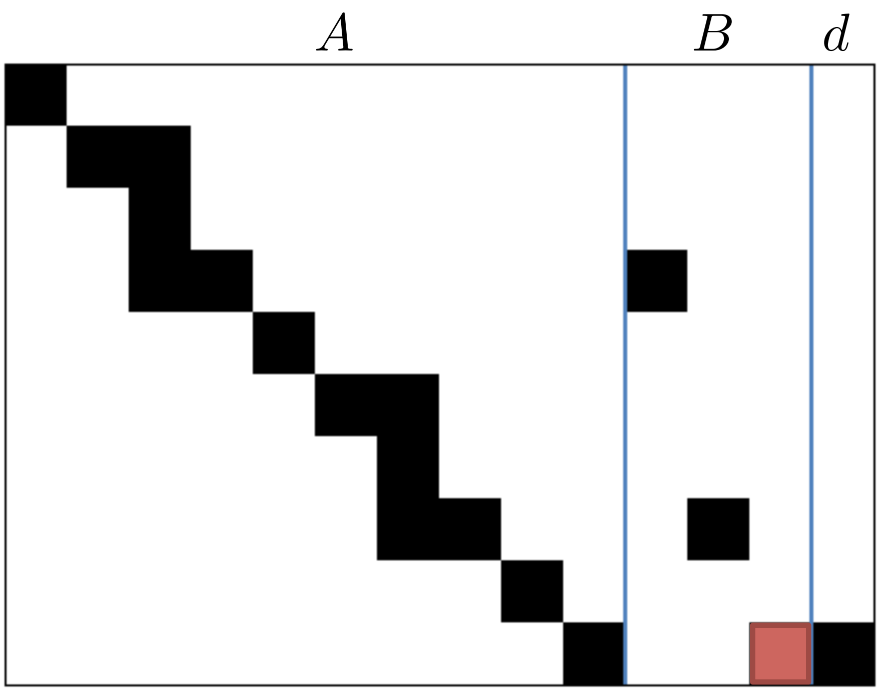
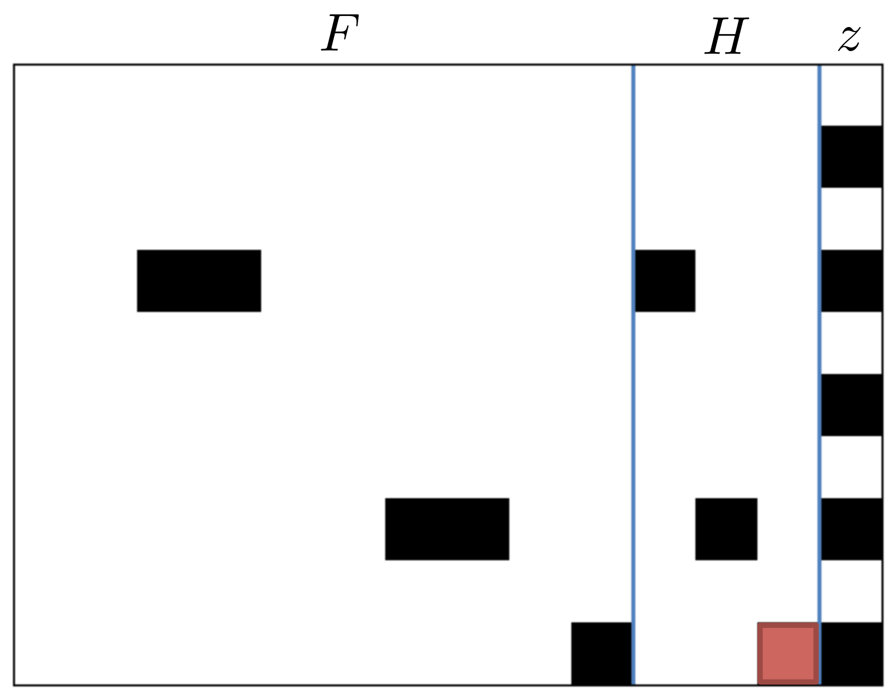



(Aswani, et al., submitted, 2011); (Bouffard, et al., submitted, 2011)

# Quadrotor Experiments

- Implementation with this structure
  - Oracle and state estimation
    - Dual Extended Kalman filter (EKF)
  - LBMPC is quadratic program (QP)
    - Solved using LSSOL solver
- Experiments
  - Learning physical effect
  - Improved performance
  - Robustness under mis-learning
  - High-precision task

Luenberger Observer

$$\hat{x}_{n+1} = (A+F)\hat{x}_n + (B+H)u_n + (k+z) + \hat{K}\zeta_n$$

$$\zeta_n = y_n - C\hat{x}_n$$

$$L_n = P'_{2,n} C' \Xi^{-1}$$

$$P_{2,n+1} = (A+F)P_{2,n} + M_n P_{3,n} - \hat{K}\Xi L'_n$$

$$P_{3,n+1} = P_{3,n} - L_n \Xi L'_n - \delta P_{3,n} P'_{3,n} + \Upsilon$$

$$M_n = \partial(F\hat{x}_n + Hu_n + z)/\partial\beta$$

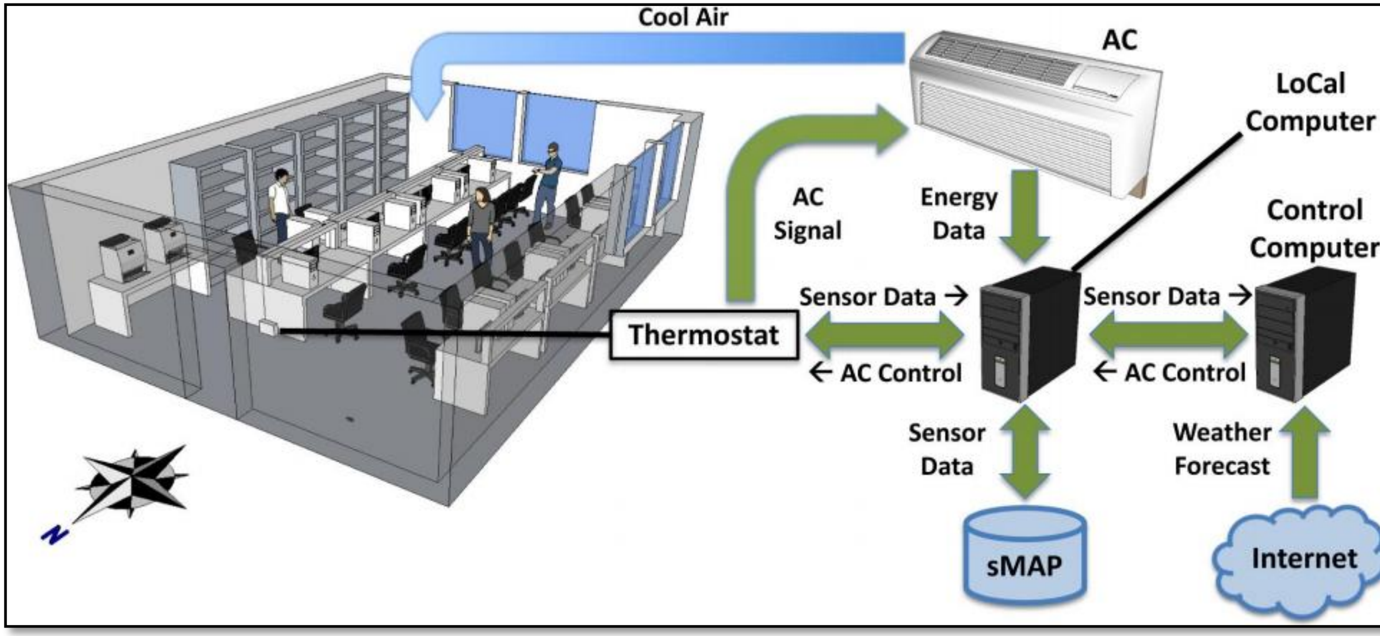
$$\beta_{n+1} = \text{bound}(\beta_n + L_n\zeta_n)$$

Extended Kalman Filter

Dual EKF

(Ljung, 1979); (Aswani, et al., submitted, 2011); (Bouffard, et al., submitted, 2011)

# Berkeley Retrofitted and Inexpensive HVAC Testbed for Energy Efficiency (BRITE)

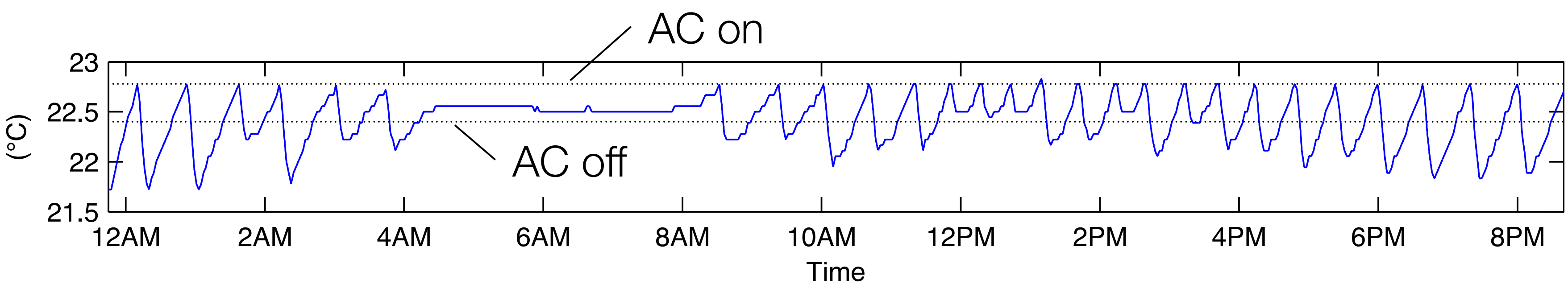



- Partially engineered living laboratory
  - 640 sq. ft. computer space
  - Networked thermostat
  - Newton's law of cooling with complex heating load from occupant behavior

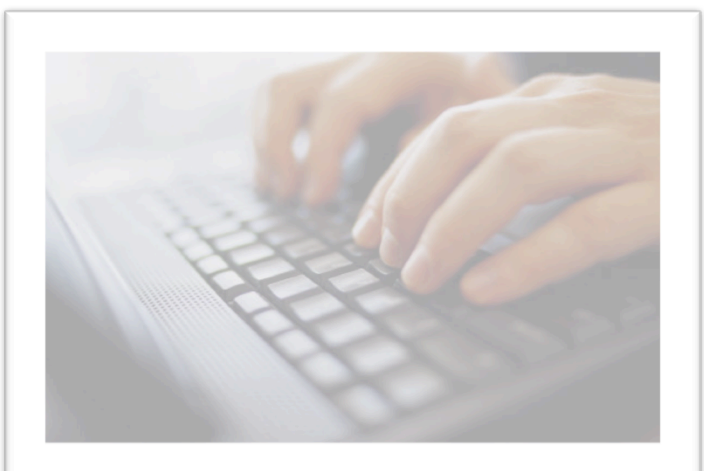
(Aswani, et al., Proc. IEEE, 2011); (Aswani, et al., submitted, 2011)

# Challenges in Efficient HVAC

## Existing control overcools (or overheats)

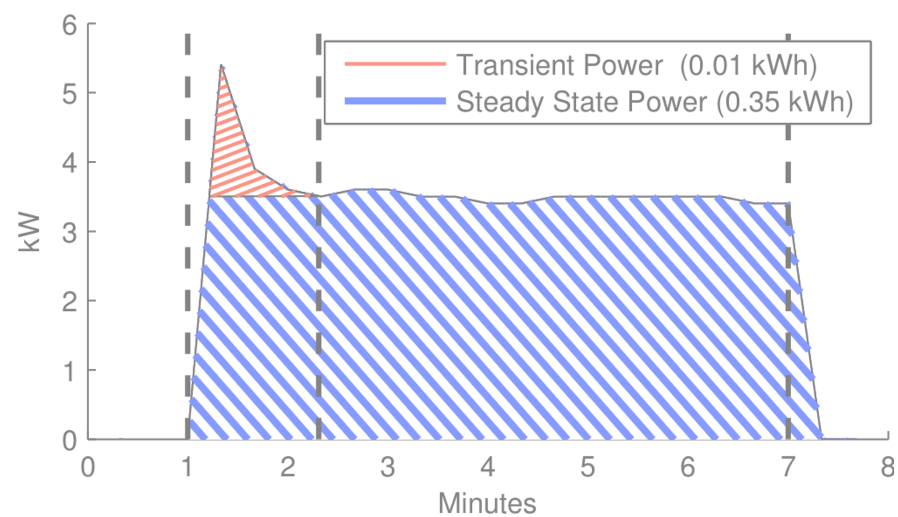



## Time-varying heating load



(Aswani, et al., Proc. IEEE, 2011)

## Complex energy characteristics

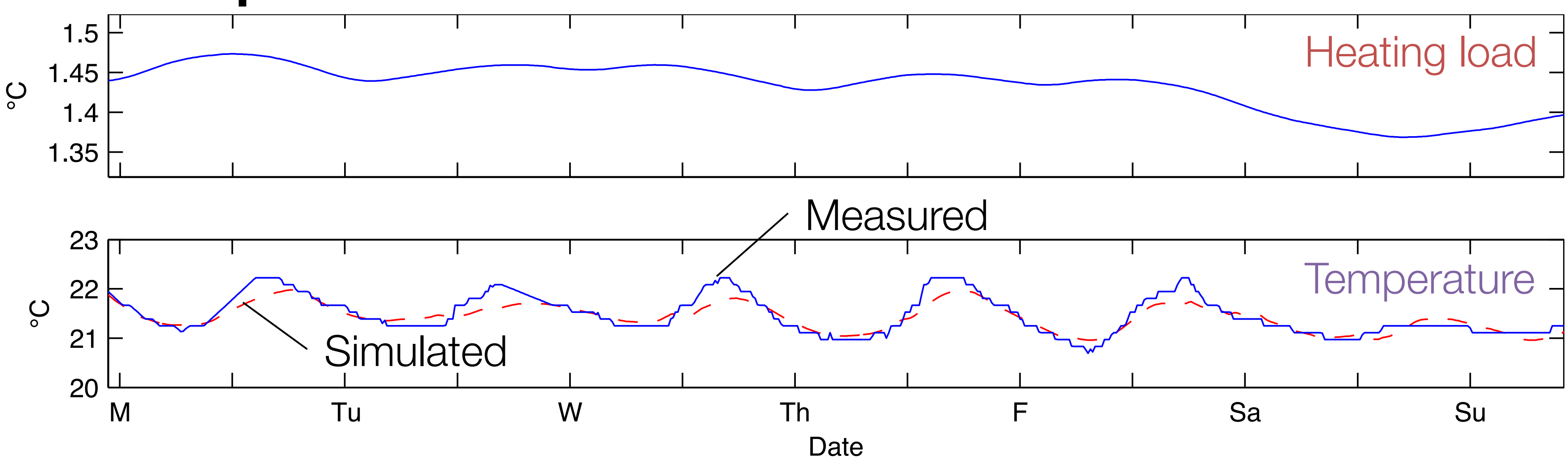


# Temperature Modeling

- Semi-parametric regression modeling
  - Parametric: Newton's law of cooling
  - Nonparametric: Heating load

$$T_{n+1} = AT_n + B_1 u_n + B_2 w_n + q_n$$

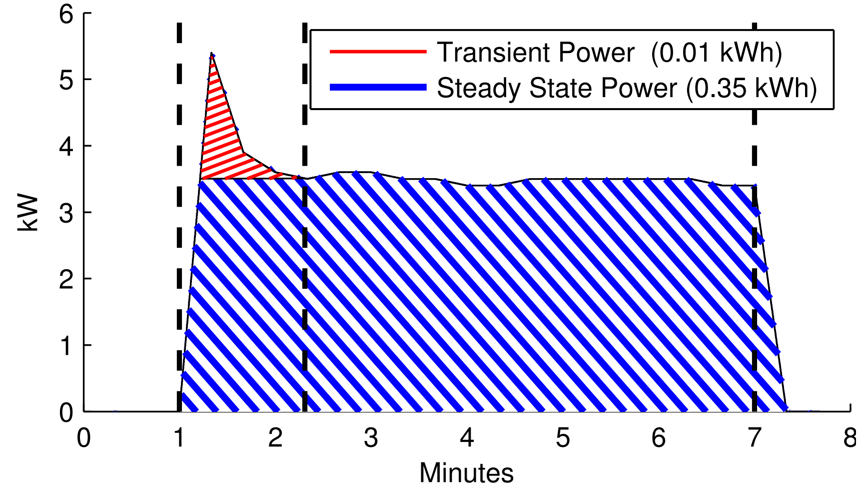
  - Novelty: Estimate heating load using only temperature measurements of thermostat




(Aswani, et al., Proc. IEEE, 2011); (Aswani, et al., submitted, 2011)

# Energy Modeling

- Electrical home AC
  - Transient and steady state power
  - Power independent (on average) of outside




- Energy estimates

$$E_{\text{actual}} = \sum_{k=0}^{N-1} \left[ 0.925 \cdot u_{m+k} + 0.015 \cdot \mathbb{1}(u_{m+k} > 0) \right] \text{ kWh}$$

- Convex relaxation using L1 norm

$$E_{\text{convex}} = \sum_{k=0}^{N-1} (0.925 + 0.015) \cdot u_{m+k} \text{ kWh}$$

(Aswani, et al., Proc. IEEE, 2011)

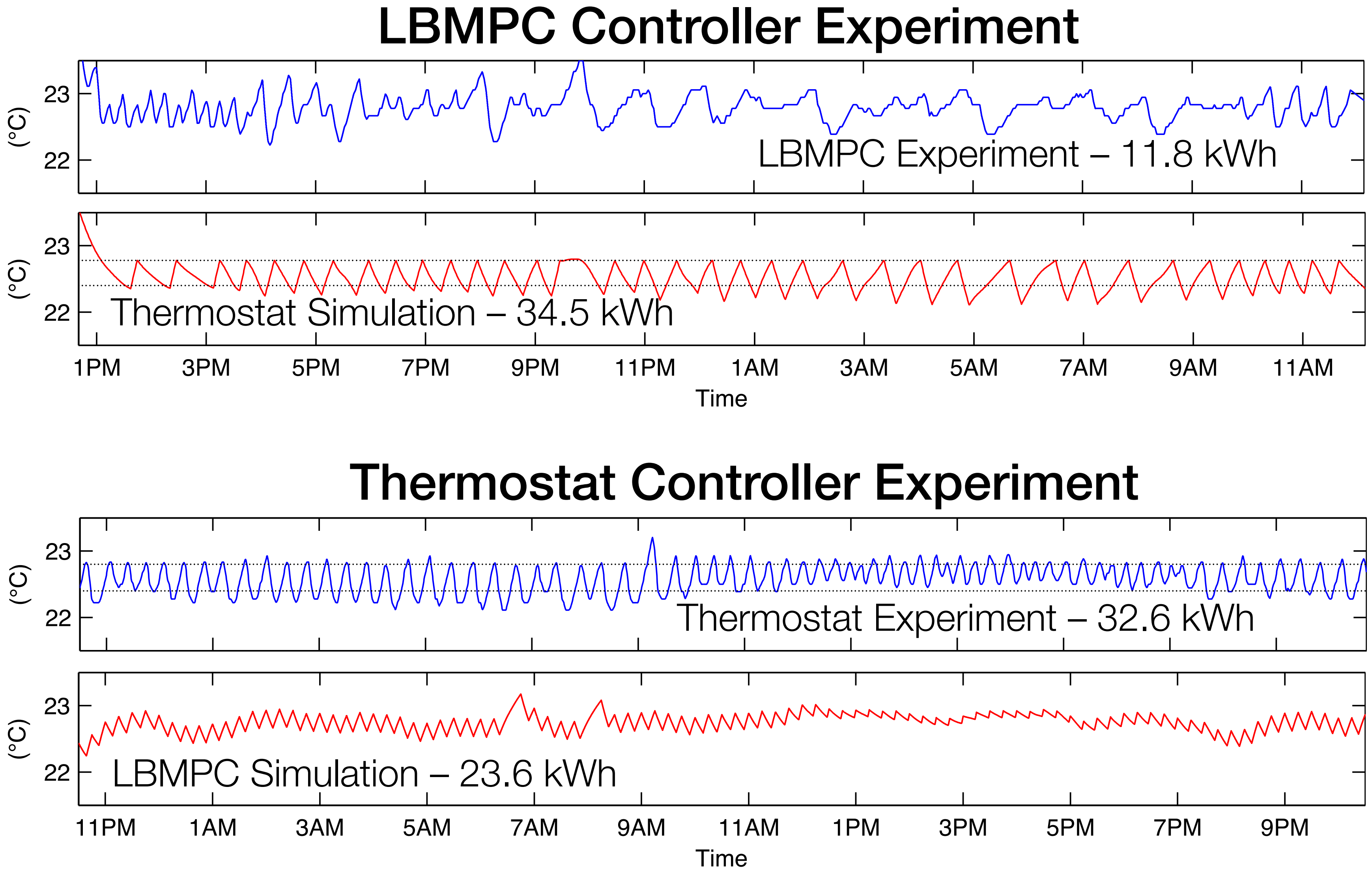
# Experiments on BRITE

- Compare controllers under identical conditions using simulations and experiments
- LBMPC provides 30-70% energy savings

| Experiment | Method | Switches | Energy | Tracking Error | Temperature Variation | Average External Load |
|---|---|---|---|---|---|---|
| Thermostat Controller | LBMPC | 94 | 23.6 kWh | 0.75 °C | 0.13 °C | 11.0 °C |
| | MPC | 96 | 30.5 kWh | 0.62 °C | 0.30 °C | 11.0 °C |
| | Thermostat | 71 | 32.6 kWh | 0.61 °C | 0.20 °C | 11.0 °C |
| LBMPC Controller | LBMPC | 81 | 11.8 kWh | 0.86 °C | 0.17 °C | 8.7 °C |
| | MPC | 70 | 8.6 kWh | 0.93 °C | 0.21 °C | 8.7 °C |
| | Thermostat | 38 | 34.5 kWh | 0.55 °C | 0.19 °C | 8.7 °C |

(Aswani, et al., Proc. IEEE, 2011)

# Experimental Measurements

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# Thank you

Any questions?